# UAV Attitude Estimation using Antenna Arrays

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***Abstract*—This paper presents a new algorithm for estimating all three Euler angles that specify the relative attitude between two unmanned aerial vehicles based on multiple-input multiple-output (MIMO) radio transmissions. The algorithm uses direction-of-arrival estimates as well as estimates of the multi-polarized MIMO channel response to construct the coordinate frames describing the UAV attitudes. These coordinate frames then allow specification of the rotations required to align one UAV with the other. Simulations reveal that estimation errors are relatively small even for low signal-to-noise ratio.**

## I. INTRODUCTION

While unmanned aerial vehicles (UAVs) typically estimate their own attitude using on-board sensors, there is growing interest in estimating the relative attitude of a UAV with respect to another node without such sensors. Recent studies on this topic use antenna arrays at the UAV and/or the base station to estimate relative attitude in terms of Euler rotation angles. However, these methods require angle specification in terms of a fixed order of rotation for roll, pitch, and yaw and often assume one of the Euler angles is already known [1], [2]. While the method in [3] estimates all Euler angles, it requires a 10×10 antenna array at the base station, and its estimation accuracy degrades dramatically with increasing range.

This work considers a scenario in which a *tracking* UAV performing a specific task – such as image acquisition – is to be replaced with a *handoff* UAV that must estimate the relative attitude of the tracking UAV in order to assume the task. We assume that both UAVs are capable of performing direction-of-arrival (DOA) estimates. These DOA estimates are combined with an estimate of the multiple-input multiple-output (MIMO) channel response using arrays having polarization diversity to compute the relative orientation of coordinate frame unit vectors that unambiguously specify relative attitude. Once these coordinate frame vectors are known, the Euler rotations (for any rotation order) can be constructed to align the handoff UAV with the tracking UAV.

## II. ATTITUDE ESTIMATION

Fig. 1(a) shows the system model considered in this analysis. A local body coordinate frame defined by the unit-norm vectors $(\hat{x}_\mathrm{i}, \hat{y}_\mathrm{i}, \hat{z}_\mathrm{i})$ is defined for each UAV, where $\mathrm{i} \in \{\mathrm{h}, \mathrm{t}\}$ for the handoff and tracking UAV, respectively. We assume that each UAV is equipped with an array and processing algorithms that can be used to estimate the bearing to the other UAV. An estimation procedure to find $(\hat{x}_\mathrm{t}, \hat{y}_\mathrm{t}, \hat{z}_\mathrm{t})$ with respect to the $(\hat{x}_\mathrm{h}, \hat{y}_\mathrm{h}, \hat{z}_\mathrm{h})$ coordinate frame is outlined below:

- The handoff UAV uses DOA techniques to construct the unit vector $\hat{k}_\mathrm{h}$ pointing to the tracking UAV. The handoff UAV then establishes a new coordinate frame $(\hat{u}_\mathrm{h}, \hat{v}_\mathrm{h}, \hat{k}_\mathrm{h})$, where $\hat{u}_\mathrm{h}$ is the orthogonal projection of $\hat{k}_\mathrm{h}$ onto the local $x$-$y$ plane and $\hat{v}_\mathrm{h}$ is given by the cross product $\hat{v}_\mathrm{h} = \hat{k}_\mathrm{h} \times \hat{u}_\mathrm{h}$.

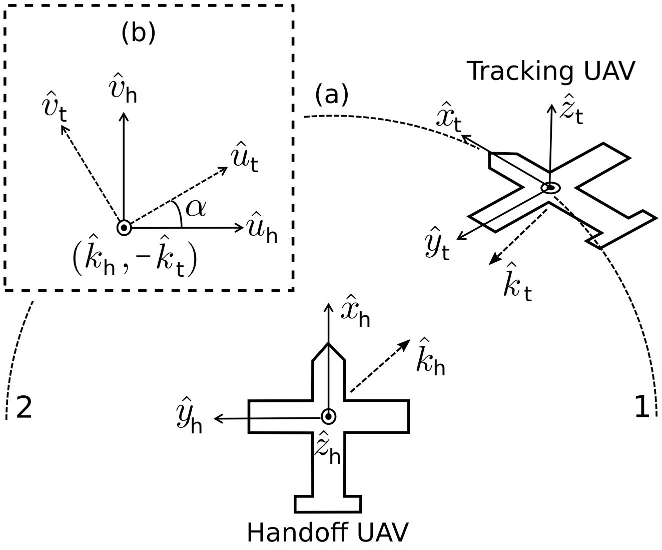

Fig. 1. Scenario for UAV attitude estimation: (a) system model consisting of a tracking and handoff UAV and their corresponding body coordinate frames, (b) relationship between the coordinate frames at the two UAVs based on DOA estimates.

- The tracking UAV similarly estimates the unit vector $\hat{k}_\mathrm{t}$ pointing to the handoff UAV and transmits $\hat{k}_\mathrm{t}$ back to the handoff UAV.
- The handoff UAV computes the coordinate frame $(\hat{u}_\mathrm{t}, \hat{v}_\mathrm{t}, \hat{k}_\mathrm{t})$. As depicted in Fig. 1(b) the vectors $-\hat{k}_\mathrm{t}$ and $\hat{k}_\mathrm{h}$ represent the same direction, but there exists an angular rotation of $\alpha$ between $(\hat{u}_\mathrm{h}, \hat{v}_\mathrm{h})$ and $(\hat{u}_\mathrm{t}, \hat{v}_\mathrm{t})$.
- The MIMO channel is used to estimate $\alpha$ (outlined below), allowing computation of the rotated vectors

$$\hat{u}_\mathrm{t} = \hat{u}_\mathrm{h} \cos\alpha + \hat{v}_\mathrm{h} \sin\alpha, \tag{1}$$

$$\hat{v}_\mathrm{t} = -\hat{u}_\mathrm{h} \sin\alpha + \hat{v}_\mathrm{h} \cos\alpha. \tag{2}$$

- Finally, the estimate of $(\hat{x}_\mathrm{t}, \hat{y}_\mathrm{t}, \hat{z}_\mathrm{t})$ is computed by projecting $(\hat{u}_\mathrm{t}, \hat{v}_\mathrm{t}, \hat{k}_\mathrm{t})$ onto the coordinate frame $(\hat{u}_\mathrm{h}, \hat{v}_\mathrm{h}, \hat{k}_\mathrm{h})$. The handoff UAV can now compute pitch, roll, and yaw angles required to move from its initial orientation $(\hat{x}_\mathrm{h}, \hat{y}_\mathrm{h}, \hat{z}_\mathrm{h})$ to the estimated orientation $(\hat{x}_\mathrm{t}, \hat{y}_\mathrm{t}, \hat{z}_\mathrm{t})$ by using any order of Euler angle rotations.

The unknown parameter $\alpha$ can be extracted from the MIMO channel response between the two UAVs when the arrays

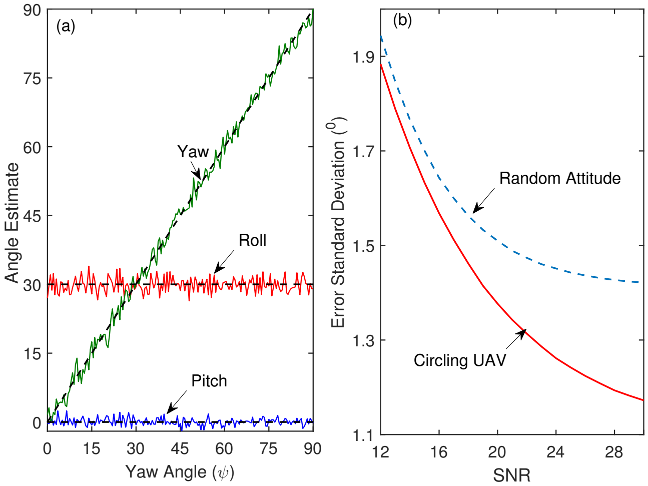

Fig. 2. Performance of the simulated method: (a) estimated pitch, roll, and yaw (solid lines) compared with exact values (dashed lines) for the circling UAV scenario, (b) standard deviation of the estimation errors averaged over pitch, roll, and yaw angles for the circling UAV scenario (solid line) and randomly positioned and oriented UAVs (dashed line).

exhibit diverse polarizations. To this end, we assume each UAV has six monopole antennas pointing outward from the left wing ($\hat{y}_{\mathrm{i}}$), right wing (-$\hat{y}_{\mathrm{i}}$), nose ($\hat{x}_{\mathrm{i}}$), tail (-$\hat{x}_{\mathrm{i}}$), top ($\hat{z}_{\mathrm{i}}$), and bottom (-$\hat{z}_{\mathrm{i}}$). Assuming that the tracking UAV is transmitting, the handoff UAV estimates the $6 \times 6$ channel matrix $\tilde{\mathbf{H}}$. Knowing the radiation patterns of the antennas, the handoff UAV can compute the ideal channel matrix $\tilde{\mathbf{H}}(\tilde{\alpha})$ based on $(\hat{u}_{\mathrm{t}}, \hat{v}_{\mathrm{t}}, \hat{k}_{\mathrm{t}})$ and $(\hat{u}_{\mathrm{h}}, \hat{v}_{\mathrm{h}}, \hat{k}_{\mathrm{h}})$, where $\tilde{\alpha}$ is an assumed rotation angle. The handoff UAV estimates $\alpha$ as the value of $\tilde{\alpha}$ that satisfies

$$\alpha = \arg\min_{\tilde{\alpha}} \underbrace{\|\mathbf{H} - \tilde{\mathbf{H}}(\tilde{\alpha})\|_{\mathrm{F}}^2}_{d(\tilde{\alpha})}, \tag{3}$$

where $\tilde{\alpha}$ is swept in the interval $\tilde{\alpha} \in [0, 2\pi)$. Channel estimation errors are included in the simulations according to

$$\mathbf{H} = \mathbf{H}^{'} + \boldsymbol{\eta}, \tag{4}$$

where $\mathbf{H}^{'}$ is the ideal channel, $\boldsymbol{\eta}$ is a noise matrix having zero-mean complex Gaussian random entries, and the error variance is chosen to ensure a specific signal-to-noise ratio (SNR).

We note that in the absence of absolute MIMO phase information, there is an ambiguity of $\pi$ radians in $\alpha$ from (3) irrespective of SNR, which translates into two solutions for $(\hat{x}_{\mathrm{t}}, \hat{y}_{\mathrm{t}}, \hat{z}_{\mathrm{t}})$. One way to remove this uncertainty is to consider the relative movement of two UAVs. We have observed that if the azimuth angle $\phi_{\mathrm{h}}$ defining $\hat{k}_{\mathrm{h}}$ changes by a small angle $\Delta\phi_{\mathrm{h}}$, then the vectors $(\hat{x}_{\mathrm{t}}, \hat{y}_{\mathrm{t}}, \hat{z}_{\mathrm{t}})$ corresponding to the correct $\alpha$ are far less sensitive to changes in $\Delta\phi_{\mathrm{h}}$ than the vectors computed from the incorrect $\alpha$.

## III. RESULTS

In the simulation considered, the tracking UAV moves along a circular path around the handoff UAV from Location 1 to 2, as depicted in Fig. 1(a). To follow this trajectory, the yaw angle $\psi$ changes uniformly from $0°$ to $180°$ with constant roll ($\gamma = 30°$) and pitch ($\beta = 0°$). The DOA estimate at the handoff UAV is given by $(\theta_{\mathrm{h}}, \phi_{\mathrm{h}})$, where azimuth angle $\phi_{\mathrm{h}}$ changes uniformly from $-90°$ to $90°$ with constant elevation ($\theta_{\mathrm{h}} = 90°$). Estimation errors in the DOA are modeled by adding zero-mean Gaussian noise with $1.7°$ standard deviation.

Fig. 2(a) plots estimated pitch, roll, and yaw angles compared to exact values (dashed lines) for 20 dB SNR over a quarter turn of the handoff UAV path, indicating good agreement. The solid curve in Fig. 2(b) plots standard deviation of estimation error (degrees) averaged over the three Euler angles (pitch, roll, and yaw) for increasing SNR. The results show that even for relatively low SNR of the MIMO channel (12 dB), error in the estimated UAV attitude is quite small. We note that mean error in the simulations (not plotted) is very close to zero, suggesting that our estimation method is unbiased. Also, the flattening of the curves with increasing SNR occurs due to the constant DOA estimation error assumed.

Since the circling UAV simulation only accounts for a limited subset of possible Euler angles, another simulation was performed where the Euler angles of the tracking UAV were chosen randomly and uniformly over 5,000 realizations on the intervals yaw $\psi \in [0°, 180°]$, roll $\gamma \in [-45°, 45°]$ and pitch $\beta \in [-45°, 45°]$. Also, the location of the tracking UAV with respect to the handoff UAV was varied randomly and uniformly with $\phi_{\mathrm{h}} \in [0°, 360°]$, and $\theta_{\mathrm{h}} \in [80°, 100°]$. The dashed curve in Fig. 2(b) plots the resulting standard deviation of the error in the estimated attitude for this scenario. The results indicate that estimation is more challenging for the random situation, but that acceptably low estimation error can still be obtained.

## IV. CONCLUSION

This paper presents a novel estimation algorithm that can be used to determine the relative attitude between two communicating UAV nodes, provided that each node is equipped with MIMO antenna arrays with diverse polarization. Simulation results indicate that the proposed method performs well for moderate SNR (channel estimation errors) and in the presence of DOA estimation errors.

## REFERENCES

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