Efficient Optimization Method for a Reconfigurable OTA Chamber

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***Abstract*—A reconfigurable over-the-air chamber (ROTAC) allows the synthesis of arbitrary propagation environments inside a reverberation chamber for wireless device testing by controlling the reflection of waves along the chamber walls with reconfigurable elements. This work proposes an efficient two-step algorithm for realizing desired channels by first optimizing a linearized ROTAC model and then using gradient ascent optimization to refine the reconfigurable reflector states. Representative results demonstrate the effectiveness of the proposed method for power angular spectrum synthesis.**

I. Introduction

Over-the-air (OTA) testing of wireless devices has become increasing popular, with costly but realistic multi-probe anechoic and lower-cost but less flexible reverberation chamber testing methods being most widely used. We recently proposed a reconfigurable OTA chamber (ROTAC), which is a reverberation chamber whose walls are lined with antennas [1]. Using a few of the antennas to excite the chamber and terminating the remainder with reconfigurable elements (REs) that control the chamber wall reflections, the ROTAC is able to emulate a wide range of propagation environments at a reduced cost. However, the optimization of RE states (reflection magnitude and phase) poses a non-convex and nonlinear problem, and prior work has mainly relied on a random search [1]. In this work, we use convex optimization to determine suboptimal RE states based on a linear approximation of the ROTAC model and then refine these solutions using a gradient ascent (GA) algorithm based on the nonlinear ROTAC model. Results show that this algorithm is able to realize a wide range of power angular spectrum (PAS) profiles observed by the device.

II. Analysis

Figure 1(a) shows the cubical ROTAC model (side length of 12 inches) whose top and four side walls are equipped with a grid of 3×3 monopole antennas, the terminals of which are chamber *ports*. We compute the S-parameter matrix and electric field inside the chamber as presented in [1].

The ROTAC is excited using the $N_{\mathrm{F}} = 4$ feed ports located at the center of each wall, while the remaining $N_{\mathrm{R}} = 41$ RE ports are terminated by a tunable impedance. The electric field at a point $\mathbf{r}$ in the chamber can be expressed as [2]

$$e(\mathbf{r}) = \left[\mathbf{e}_{\mathrm{F}}^{\mathrm{T}}(\mathbf{r}) + \mathbf{e}_{\mathrm{R}}^{\mathrm{T}}(\mathbf{r})\boldsymbol{\Gamma}(\mathbf{I} - \mathbf{S}_{\mathrm{RR}}\boldsymbol{\Gamma})^{-1}\mathbf{S}_{\mathrm{RF}}\right]\mathbf{a}_{\mathrm{F}}, \quad (1)$$

where $\boldsymbol{\Gamma}$ is the $N_{\mathrm{R}} \times N_{\mathrm{R}}$ diagonal matrix representing the reflection coefficients on the RE ports, $\mathbf{S}_{\mathrm{RR}}$ and $\mathbf{S}_{\mathrm{RF}}$ are $N_{\mathrm{R}} \times N_{\mathrm{R}}$ and $N_{\mathrm{R}} \times N_{\mathrm{F}}$ S-parameter matrices for coupling between RE ports and feed to RE ports, respectively, $\mathbf{a}_{\mathrm{F}}$ is the $N_{\mathrm{F}} \times 1$ vector of feed port excitations, $\mathbf{e}_{\mathrm{F}}(\mathbf{r})$ and $\mathbf{e}_{\mathrm{R}}(\mathbf{r})$ are $N_{\mathrm{F}} \times 1$ and $N_{\mathrm{R}} \times 1$ vectors containing the electric field at $\mathbf{r}$ based on a unit excitation of feed and RE ports, respectively, and $\{\cdot\}^{\mathrm{T}}$ is the transpose. The REs have a complex reflection coefficient $\Gamma_{ii} = \gamma_i$, where $|\gamma_i| \leq 1$.

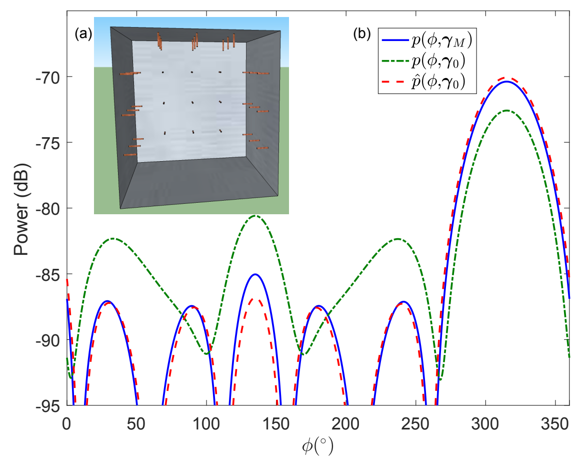

Fig. 1. (a) ROTAC model, where one side panel has been removed to allow visualization of the chamber interior. (b) Power angular spectrum (PAS) for $\phi_{n_0} = 315^\circ$ based on convex optimization using a linear approximation $\hat{p}(\phi, \boldsymbol{\gamma}_0)$, starting point for GA algorithm $p(\phi, \boldsymbol{\gamma}_0)$, and optimized solution $p(\phi, \boldsymbol{\gamma}_M)$.

To compute the PAS, we place a virtual 8-element uniform circular array (UCA) with an inter-element spacing of 0.39λ, where λ is the free space wavelength, at the chamber center. We apply a Bartlett beamformer to compute the PAS at the N_{A} discrete values $\phi_n = 2\pi n/N_{\mathrm{A}}$ of the azimuth angle. The PAS at ϕ_n can be expressed as

$$p(\phi_n, \boldsymbol{\gamma}) = |\mathbf{a}^{\dagger}(\phi_n)\mathbf{e}(\boldsymbol{\gamma})|^2, \quad (2)$$

where $\mathbf{a}$ is the array steering vector, $\{\cdot\}^{\dagger}$ is a conjugate transpose, and $\mathbf{e}(\boldsymbol{\gamma})$ contains the electric field values on the UCA elements for the realized RE reflection coefficient vector $\boldsymbol{\gamma}$. We want to maximize the PAS in the main beam direction ϕ_{n_0} relative to the side lobes as described by

$$f(\boldsymbol{\gamma}) = \frac{|\mathbf{a}^{\dagger}(\phi_{n_0})\mathbf{e}(\boldsymbol{\gamma})|^2}{\frac{1}{N_{\mathrm{A}} - N_{\mathrm{B}}} \sum_{n} |\mathbf{a}^{\dagger}(\phi_n)\mathbf{e}(\boldsymbol{\gamma})|^2}, \quad (3)$$

where $n \notin [n_0 - N_\mathrm{B}/2, n_0 + N_\mathrm{B}/2]$ and N_B is the number of discrete angles that lie within the main lobe of the PAS. The GA optimization of $f(\boldsymbol{\gamma})$ uses the iterative update

$$\boldsymbol{\gamma}_{m+1} = \boldsymbol{\gamma}_m + \delta \nabla f(\boldsymbol{\gamma}), \tag{4}$$

where m is the iteration index, δ is the step size, and the gradient $\nabla f(\boldsymbol{\gamma})$ is computed using the method in [2].

We use convex optimization [3] on a simplified problem to create a smart initial solution $\boldsymbol{\gamma}_0$ that can be refined by the GA algorithm. We begin by linearly approximating (1) using the first term of the *Neumann* expansion of $(\mathbf{I} - \mathbf{S}_\mathrm{RR}\boldsymbol{\Gamma})^{-1}$, or

$$\hat{e}(\mathbf{r}) = e'_\mathrm{F}(\mathbf{r}) + \mathbf{e}_\mathrm{R}^\mathrm{T}(\mathbf{r})\mathbf{S}'_\mathrm{RF}\boldsymbol{\gamma}, \tag{5}$$

where $e'_\mathrm{F}(\mathbf{r}) = \mathbf{e}_\mathrm{F}^\mathrm{T}(\mathbf{r})\mathbf{a}_\mathrm{F}$ and $\mathbf{S}'_\mathrm{RF}$ is the diagonal matrix with diagonal elements contained in the vector $\mathbf{S}_\mathrm{RF}\mathbf{a}_\mathrm{F}$. The output of the Bartlett beamformer is then

$$\hat{v}(\phi_n) = \mathbf{a}^\dagger(\phi_n)\hat{\mathbf{e}}(\boldsymbol{\gamma}) = \underbrace{\mathbf{a}^\dagger(\phi_n)\mathbf{e}'_\mathrm{F}}_{v_\mathrm{F}(\phi_n)} + \underbrace{\mathbf{a}^\dagger(\phi_n)\mathbf{E}_\mathrm{R}\mathbf{S}'_\mathrm{RF}}_{\mathbf{v}_\mathrm{R}^\mathrm{T}(\phi_n)}\boldsymbol{\gamma}, \tag{6}$$

where $\mathbf{e}'_\mathrm{F}$ and $\mathbf{E}_\mathrm{R}$ are the row-wise stacked versions of $e'_\mathrm{F}(\mathbf{r})$ and $\mathbf{e}_\mathrm{R}^\mathrm{T}(\mathbf{r})$, respectively, for $\mathbf{r}$ at each of the UCA element locations.

To make our optimization convex, we must also simplify the fitness function (3). We therefore specify the array response in the main beam direction as $\hat{v}(\phi_{n_0}) = v_0$ and then find the vector $\boldsymbol{\gamma}$ that minimizes the side lobe level (SLL). The vector $\boldsymbol{\gamma}$ that satisfies (6) is given as

$$\boldsymbol{\gamma} = \frac{\mathbf{v}_\mathrm{R}(\phi_{n_0})^*}{\|\mathbf{v}_\mathrm{R}(\phi_{n_0})\|^2}(v_0 - v_\mathrm{F}(\phi_{n_0})) + \mathbf{U}\mathbf{g}, \tag{7}$$

where $\{\cdot\}^*$ is the conjugate, $\mathbf{U}$ is an $N_\mathrm{R} \times (N_\mathrm{R}-1)$ orthogonal complement to $\mathbf{v}_\mathrm{R}(\phi_{n_0})^\dagger$, and $\mathbf{g}$ is the $(N_\mathrm{R}-1)\times 1$ vector to be determined. For all optimizations, we use $v_0 = v_\mathrm{F}(\phi_{n_0})$. The convex optimization problem can now be specified as

$$\begin{aligned} \underset{\mathbf{g}}{\text{minimize}} \quad & \alpha \\ \text{subject to} \quad & \hat{v}(\phi_n)\hat{v}(\phi_n)^* \leq \alpha \\ & \gamma_i \gamma_i^* \leq 1 \quad \forall i \end{aligned}$$

where α is the SLL. Note that the main beam power is fixed for this optimization, whereas it can vary in the GA optimization due to the fitness function (3). Our experience is that the main beam power can change as much as 3-4 dB during the GA optimization.

III. Results

We use CVX, a package for solving convex problems [3], to compute $\boldsymbol{\gamma}_0$ for a specified main beam direction ϕ_{n_0}. We then run the GA algorithm for $M = 250$ iterations to obtain the optimized RE reflection coefficient vector $\boldsymbol{\gamma}_M$. Figure 1(b) plots the approximate PAS $\hat{p}(\phi, \boldsymbol{\gamma}_0) = |\hat{v}(\phi)|^2$ from (6), the actual initial PAS $p(\phi, \boldsymbol{\gamma}_0)$ from (2), and the final optimized PAS $p(\phi, \boldsymbol{\gamma}_M)$. These results demonstrate that $\boldsymbol{\gamma}_0$ is a good solution assuming the linear model, but that the actual achieved PAS with $\boldsymbol{\gamma}_0$ is degraded significantly.

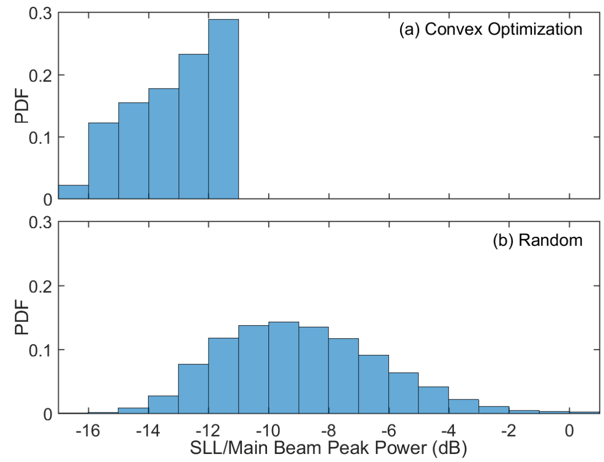

Fig. 2. Probability density function of the SLL relative to the main beam peak power (dB) for $\phi_{n_0} \in [0, \pi/4]$ based on convex optimization and random initialization of gradient ascent algorithm.

However, applying the GA algorithm to refine $\boldsymbol{\gamma}_0$ produces a good final result.

Since the SLL depends generally on the main beam direction ϕ_{n_0}, we sweep ϕ_{n_0} over the interval $[0, 45°]$ (due to the symmetry of the chamber). Figure 2(a) plots the probability density function (pdf) of the SLL normalized by the main beam peak power $p(\phi_{n_0}, \boldsymbol{\gamma})$ when the convex optimization result is used for GA initialization, revealing a worst case relative SLL of -11 dB. Figure 2(b) shows the same pdf when 1000 random initializations $\boldsymbol{\gamma}_0$ are generated for each main beam direction ϕ_{n_0}. These results reveal that although the random initializations produce an average relative SLL of -9 dB, the spread in final achieved SLL is significant, motivating use of the convex optimization to initialize the GA algorithm.

IV. Conclusion

This paper proposes a computationally efficient algorithm for optimization of the RE states to achieve a desired PAS observed by a wireless device in a ROTAC. Results demonstrate that the approach is able to achieve highly directional PAS performance. Future work will focus on the application of this method to the synthesis of arbitrary fading statistics and real-time ROTAC optimization using a hardware prototype.

References

[1] R. Mehmood, J. W. Wallace, and M. A. Jensen, "Reconfigurable OTA chamber for MIMO wireless device testing," in *Proc. 10th European Conference on Antennas and Propagation*, Davos, Switzerland, Apr. 2016, pp. 1–4.

[2] P. Baniya, S. Ur Rehman, and J. W. Wallace, "Efficient optimization of reconfigurable parasitic antenna arrays using geometrical considerations," in *Proc. 5th European Conference on Antennas and Propagation*, Apr. 2011, pp. 270–274.

[3] CVX Research Inc., "CVX: Matlab software for disciplined convex programming, version 2.0.," April 2011.