Efficient Newton-Based Pattern Synthesis of Reconfigurable Parasitic Arrays

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***Abstract*—Dense reconfigurable parasitic arrays have many applications including beamforming, null-steering, adaptive matching and frequency agility. Due to the non-convex nature of the objective function, global search methods, such as genetic algorithms and particle swarm optimization have traditionally been employed. However, the computational cost of such algorithms may be prohibitive for dynamic in-situ optimization. It was previously demonstrated that a very efficient method, consisting of a geometric solution followed by Newton-based optimization, produced moderate (suboptimal) main-beam gain and specified pattern nulls. Herein a nearly optimal, yet efficient Newton-based method is developed for dense parasitic arrays based on subspace separation of beam and null optimization. The suitability of the optimization algorithm is demonstrated via numerical examples.**

I. Introduction

Reconfigurable parasitic antenna arrays [1–3] are versatile structures, consisting of a single feed element in close proximity with other parasitic elements loaded with programmable reactances. These reconfigurable elements (REs) are varied to modify the antenna radiation characteristics. Dense parasitic arrays can be seen as a generalization of the reconfigurable antenna concept and provide similar functionality as smart antennas while only requiring one radio frequency (RF) and digital signal processing (DSP) channel, leading to reduced cost and power consumption.

A potential difficulty with parasitic arrays is that the objective function is non-convex, and global search methods have typically been used to optimize these structures [4, 5]. Due to their computational complexity, unstructured global search methods can only be expected to have limited value for dynamic optimization in a real system. In previous work, we demonstrated that a pattern consisting of a single direction of higher gain (main-beam) can be formed using a simple direct solution based on a geometric interpretation of RE-scattered signals. Additionally, multiple deep nulls can be formed by a very efficient Newton-based search [6, 7]. However, a problem with this approach is that the null optimization can significantly degrade main-beam gain when too many nulls are required.

In this work we propose an alternative method that jointly optimizes the main-beam gain and null depth using a modified Newton-based approach. The key to this method is to decouple the two optimization problems by operating on two different subspaces. The method is appropriate for dense parasitic arrays that have *excess* reconfigurability, i.e., where the number of reconfigurable elements is larger than that needed to generate the desired nulls. In numerical examples, we show that this method can achieve main-beam gain that is near the theoretical optimum while maintaining deep nulls in prescribed directions.

II. Parasitic Antenna Array

A network model of an N-port parasitic antenna array is shown in Figure 1(a). Port 1 represents the feed port while Port 2 to N represent ports that are terminated with parasitic elements. The parasitic array is analyzed with the S-parameter method, where the input and output waves at the feed are represented by $a_{\rm F}$ and $b_{\rm F}$ while the input and output waves at the parasitic loads are denoted by the vectors $\mathbf{a}_{\rm R}$ and $\mathbf{b}_{\rm R}$. The matched circuit (Z_0 terminated) embedded radiation pattern at the feed and the parasitic ports are given by $e_{\rm F}(\hat{s})$ and $\mathbf{e}_{\rm R}(\hat{s})$, respectively. The diagonal input reflection matrix $\mathbf{\Gamma}$ represents the reactive reconfigurable elements (REs) that terminate the parasitic ports.

The input-output relationship of the RECAP can be defined in terms of the S-parameter matrix, expressed as

$$\mathbf{S} = \begin{bmatrix} S_{\rm FF} & \mathbf{s}_{\rm FR} \\ \mathbf{s}_{\rm RF} & \mathbf{S}_{\rm RR} \end{bmatrix}. \tag{1}$$

The radiated far-fields for a single polarization of the parasitic array into direction $\hat{s}$ is [8]

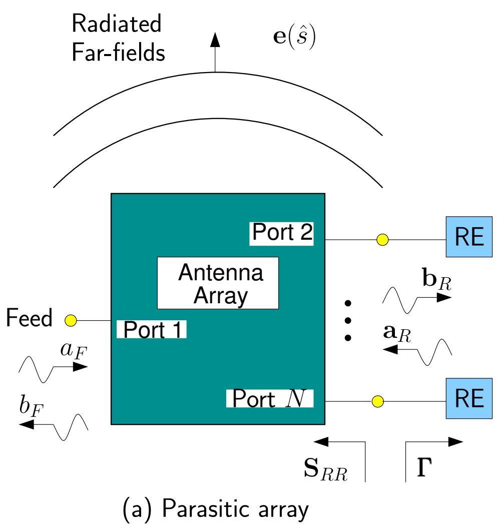
$$e(\hat{s}) = \left[e_{\rm F}(\hat{s}) + \mathbf{e}_{\rm R}^T(\hat{s})\mathbf{\Gamma}(\mathbf{I} - \mathbf{S}_{\rm RR}\mathbf{\Gamma})^{-1}\mathbf{s}_{\rm RF}\right] a_{\rm F}. \tag{2}$$

The dense parasitic array in this study consists of a 5×5 square array of half-wave dipoles with $\lambda/4$ inter-element spacing. The center element serves as the active feeding port while the surrounding elements are terminated with reactive loads. Network parameters and radiation patterns of the structure are found using full-wave analysis with the Numerical Electromagnetics Code (NEC).

III. Newton-Based Optimization

Unlike our previous work where we found nulls in the radiated fields directly using a Newton approach [6, 7], the method proposed in this paper needs to find nulls while constraining the gain in the main-beam direction. For this purpose, the following derivations are required:

1) **Computation of optimal gain:** The theoretically optimal gain in the main-beam direction must be identified, which allows a meaningful objective for main-beam gain to be specified in the subsequent joint optimization.
2) **Newton-based beam optimization:** In our previous work on null optimization, a first-order Taylor series of $e(\hat{s})$



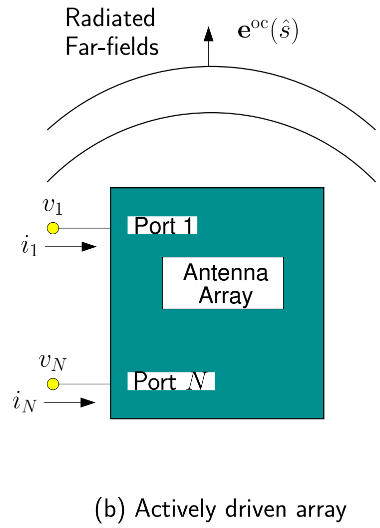


Fig. 1. Network models used for analysis of the dense parasitic array: (a) S-parameter model used for Newton-based optimization of the reconfigurable elements, (b) simplified Z-parameter model used to find the optimal gain of the dense array when driven actively at all ports.

was used to iteratively find nulls. In order to optimize main-beam gain, we must operate on $|e(\hat{s})|^2$, requiring the Taylor series of $|e(\hat{s})|^2$ to be derived.

3) **Subspace decoupling of beam-null optimization:** Joint optimization of the main beam and nulls is faciliated by performing a subspace decomposition of the problem, allowing each optimization problem to be solved in a separate subspace.

A. *Theoretically optimal main beam*

In this section, solutions for the maximum gain in direction $\hat{s}_b$ for an arbitrary array are developed, both with and without pattern nulls. We point out that although the developed method for null-constrained gain maximization is well understood in the array systhesis community, it is included here in the framework of our circuit-based antenna model for completeness. The expressions developed in this section allow a realistic objective to placed on the main-beam gain in subsequent optimization of the parasitic array.

1) Optimization without nulls: We start by studying the case of the theoretically optimal gain in the main-beam direction when no nulls are required. For this purpose the actively driven array in Figure 1(b) is considered, where all ports of the array (feed and RE ports) are driven with active sources. Since no RE terminations need to be considered, a simpler Z-parameter analysis is convenient.

The voltage-current relationship for Figure 1(b) is simply given by

$$\mathbf{v} = \mathbf{Z}\mathbf{i}. \tag{3}$$

The far-field radiation pattern $\mathbf{e}(\hat{s})$ of the array is given by

$$\mathbf{e}(\hat{s}) = \mathbf{e}^{\mathrm{oc},T}(\hat{s})\mathbf{i}, \tag{4}$$

where $\mathbf{e}^{\mathrm{oc}}(\hat{s})$ is the vector of open-circuit field patterns of the antenna elements (for a single polarization) in direction $\hat{s}$. Note that expressions for converting between Z-parameter and S-parameter quantities may be found in [8].

Since $|\mathbf{e}(\hat{s}_b)|$ must be maximized with respect to the current $\mathbf{i}$, a realistic constraint is required to avoid a trivial solution (such as infinite input current). We consider the input power constraint on the parasitic array, given by

$$\mathbf{i}^H \underbrace{\mathrm{Re}\left\{\mathbf{Z}\right\}}_{\mathbf{M}} \mathbf{i} = 1, \tag{5}$$

which can be decomposed into

$$\mathbf{i}^H \mathbf{M}^{\frac{1}{2}H} \underbrace{\mathbf{M}^{\frac{1}{2}}\mathbf{i}}_{\mathbf{i}'} = 1. \tag{6}$$

Taking the inverse, the current $\mathbf{i}$ can be written as

$$\mathbf{i} = \mathbf{M}^{-\frac{1}{2}}\mathbf{i}'. \tag{7}$$

Now substituting (7) into (4), total field radiated in the main-beam direction is

$$e(\hat{s}_b) = \underbrace{\mathbf{e}^{\mathrm{oc},T}(\hat{s}_b)\mathbf{M}^{-\frac{1}{2}}}_{\mathbf{e}_b'^{\,T}}\mathbf{i}', \tag{8}$$

where $\mathbf{i}'^H\mathbf{i}' = 1$. The radiated power in direction $\hat{s}_b$ is given by

$$|e(\hat{s}_b)|^2 = e(\hat{s}_b)^* e(\hat{s}_b) = \mathbf{i}'^H \underbrace{\mathbf{e}_b'^{\,*}\mathbf{e}_b'^{\,T}}_{\mathbf{E}_b}\mathbf{i}'. \tag{9}$$

The vector $\mathbf{i}'$ that maximizes (9) for our constraint of $\|\mathbf{i}'\|^2 = 1$ is given by the eigenvector of $\mathbf{E}_b$ corresponding to the single non-zero eigenvalue. The optimal physical current $\mathbf{i}$ can be found by substituting $\mathbf{i}'$ into (7). Finally, the corresponding maximum-gain radiation pattern $\mathbf{e}(\hat{s}_b)$ is obtained by substituting $\mathbf{i}$ into (4).

It should be noted that a danger of analyzing lossless structures with an input power constraint is the possibility of supergain solutions that cannot be practically achieved. To avoid this we include a 1 Ω real part in the diagonal of the impedance matrix $\mathbf{Z}$. This is equivalent to including a series resistance at each of the N ports of the array (reconfigurable elements and feed port).

2) Optimization with nulls: So far it was assumed that the parasitic array was only meant for creating a main-beam. When we search for nulls in addition to finding the main-beam, there is a constraint on the system which needs to be taken into account.

Let the vector $\mathbf{e}_k = \mathbf{e}^{\mathrm{oc}}(\hat{s}_{\mathrm{null},k})$ of size $N\times 1$ represent the open-circuit radiation patterns of the antennas in the direction of the kth desired null ($\hat{s}_{\mathrm{null},k}$), and let K denote the total number of null directions. For the kth null direction, we desire

$$\mathbf{i}^H \mathbf{e}_k^* \mathbf{e}_k^T \mathbf{i} = 0 \tag{10}$$

to achieve a perfect null, which is equivalent to

$$\mathbf{i}'^H \mathbf{e}_k'^* \mathbf{e}_k'^T \mathbf{i} = 0, \tag{11}$$

where $\mathbf{e}_k' = M^{-\frac{1}{2}T}\mathbf{e}_k$. Since the power (11) is non-negative for all k, we can also constrain the sum of (11) over k to be zero, or

$$\sum_{k=1}^{K} \mathbf{i}'^H \underbrace{\mathbf{e}_k'^* \mathbf{e}_k'^T}_{\mathbf{D}_k} \mathbf{i}' = 0, \tag{12}$$

which can be written as

$$\mathbf{i}'^H \underbrace{(\sum_{k=1}^{K} \mathbf{D}_k)}_{\mathbf{D}} \mathbf{i}' = 0. \tag{13}$$

Any vector $\mathbf{i}'$ that lies in the null-space of the matrix $\mathbf{D}$ will satisfy this equation. Let $\mathbf{D} = \mathbf{U}_D \Lambda_D \mathbf{U}_D^H$ represent the eigenvalue decomposition of $\mathbf{D}$. We can identify the null-space of $\mathbf{D}$ as the $N\times(N-K)$ matrix $\mathbf{U}_{D0}$ containing the eigenvectors (or columns of $\mathbf{U}_D$) corresponding to the $N-K$ zero eigenvalues of $\mathbf{D}$.

Let us define a new vector $\mathbf{i}''$ that always lies in the null-space of $\mathbf{D}$. An arbitrary vector $\mathbf{i}'$ can be projected onto the null-space of $\mathbf{D}$ using the operation

$$\mathbf{i}'' = \underbrace{\mathbf{U}_{D0}\mathbf{U}_{D0}^H}_{\mathbf{Q}} \mathbf{i}'. \tag{14}$$

Using this form of the array input current, we now maximize $\mathbf{i}'^H \mathbf{Q}^H \mathbf{E}_b \mathbf{Q} \mathbf{i}'$ instead of $\mathbf{i}'^H \mathbf{E}_b \mathbf{i}'$ to obtain the optimal main beam with the perfect nulls in the desired directions. Let $\mathbf{E}_b' = \mathbf{Q}^H \mathbf{E}_b \mathbf{Q}$. Taking the eigenvalue decomposition of $\mathbf{E}_b'$, the eigenvector corresponding to the highest (and only non-zero) eigenvalue gives us the optimal value for $\mathbf{i}'$.

The optimal current value $\mathbf{i}$ can now be found by substituting $\mathbf{i}'$ into (7), and the corresponding maximum gain pattern $\mathbf{e}(\hat{s}_b)$ in the main-beam direction is obtained by substituting $\mathbf{i}$ into (4).

B. Newton-based beam optimization

Several methods exist for optimizing the main beam gain by maximizing

$$|e(\hat{s}_b)|^2 = e^*(\hat{s}_b)e(\hat{s}_b). \tag{15}$$

Having found the theoretically optimal main-beam gain, we develop an efficient Newton-based approach that instead attempts to solve

$$|e(\hat{s}_b)|^2 - |e^{\mathrm{opt}}(\hat{s}_b)|^2 = 0, \tag{16}$$

where $e^{\mathrm{opt}}(\hat{s})$ is the optimal gain pattern found using expressions from Section III-A.

A linear approximation of $|e(\hat{s}_b)|^2$ in the neighborhood of $\mathbf{\Gamma}$ is found by computing its first derivative and forming a first-order Taylor series. Lossless reconfigurable elements with programmable phase are considered, where $\Gamma_{\ell\ell} = \exp(j\theta_\ell)$. The derivative of $|e(\hat{s}_b)|^2$ with respect to the phase of the ℓth load is computed using

$$\frac{\partial |e(\hat{s}_b)|^2}{\partial \theta_\ell} = \frac{\partial e^*(\hat{s}_b)e(\hat{s}_b)}{\partial \theta_\ell}. \tag{17}$$

Applying the product rule on (17),

$$\frac{\partial |e(\hat{s}_b)|^2}{\partial \theta_\ell} = e^*(\hat{s}_b)\frac{\partial e(\hat{s}_b)}{\partial \theta_\ell} + e(\hat{s}_b)\frac{\partial e^*(\hat{s}_b)}{\partial \theta_\ell}, \tag{18}$$

which can be written as

$$\frac{\partial |e(\hat{s}_b)|^2}{\partial \theta_\ell} = 2\mathrm{Re}\left\{ e^*(\hat{s}_b)\frac{\partial e(\hat{s}_b)}{\partial \theta_\ell} \right\}. \tag{19}$$

It was shown in [7] that the derivative $d_\ell = \partial e(\hat{s})/\partial\theta_\ell$ is given by

$$\begin{aligned}\frac{\partial e(\hat{s})}{\partial \theta_\ell} = & j\mathbf{e}_{\mathrm{R}}^T(\hat{s})(\mathbf{\Gamma}^{-1} - \mathbf{S}_{\mathrm{RR}})^{-1}\mathbf{1}_{\ell\ell} \\ & \times(\mathbf{\Gamma}^{-1} - \mathbf{S}_{\mathrm{RR}})^{-1}\mathbf{s}_{\mathrm{RF}}e^{-j\theta_\ell},\end{aligned} \tag{20}$$

where $\mathbf{1}_{\ell\ell}$ is an elementary matrix which is all zeroes except a one in the $\ell\ell$th element.

Substituting (20) into (19), we obtain

$$\begin{aligned}g_\ell = \frac{\partial |e(\hat{s}_b)|^2}{\partial \theta_\ell} = & 2\mathrm{Re}\{je^*(\hat{s}_b)\mathbf{e}_{\mathrm{R}}^T(\hat{s}_b)(\mathbf{\Gamma}^{-1} - \mathbf{S}_{\mathrm{RR}})^{-1}\mathbf{1}_{\ell\ell} \\ & \times (\mathbf{\Gamma}^{-1} - \mathbf{S}_{\mathrm{RR}})^{-1}\mathbf{s}_{\mathrm{RF}}e^{-j\theta_\ell}\}.\end{aligned} \tag{21}$$

Having found the gradient vector $\mathbf{g}$, the first-order multi-dimensional Taylor series for $|e(\hat{s}_b)|^2$ can now be written as

$$|e(\hat{s}_b, \boldsymbol{\theta}_{n+1})|^2 = |e(\hat{s}_b, \boldsymbol{\theta}_n)|^2 + \mathbf{g}^{\mathrm{T}}(\boldsymbol{\theta}_n)(\boldsymbol{\theta}_{n+1} - \boldsymbol{\theta}_n), \tag{22}$$

where $\boldsymbol{\theta}_n$ is the vector of parasitic load phases at the nth step of the algorithm. Setting $|e(\hat{s}_b, \boldsymbol{\theta}_{n+1})|^2 = |e^{\mathrm{opt}}(\hat{s}_b)|^2$, (22) can be rewritten as

$$\underbrace{|e^{\mathrm{opt}}(\hat{s}_b, \boldsymbol{\theta}_{n+1})|^2 - |e(\hat{s}_b, \boldsymbol{\theta}_n)|^2}_{h} = \mathbf{g}^{\mathrm{T}}(\boldsymbol{\theta}_n)(\boldsymbol{\theta}_{n+1} - \boldsymbol{\theta}_n). \tag{23}$$

The optimal update to the estimate of $\boldsymbol{\theta}$ at iteration n can be found by inverting the relationship to obtain

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n + \delta[\mathbf{g}^{\mathrm{T}}(\boldsymbol{\theta}_n)]^+ h = \boldsymbol{\theta}_n + \delta\frac{\mathbf{g}^*(\boldsymbol{\theta}_n)}{\|\mathbf{g}(\boldsymbol{\theta}_n)\|^2}h, \tag{24}$$

where $(.)^+$ is the pseudo-inverse, and the step size δ is introduced to improve stability of the optimization algorithm.

C. Subspace decoupling of beam-null optimization

In [6, 7] it was shown how Newton-based optimization can be used to find near exact nulls, but at the expense of reduced main-beam gain. Here we propose a method that optimizes nulls without significantly degrading main-beam performance.

The first order multi-dimensional Taylor series expression for nulls was derived in [6] as

$$\underbrace{[e_{\mathrm{re}}(\hat{s}_k,\boldsymbol{\theta}_n)\ e_{\mathrm{im}}(\hat{s}_k,\boldsymbol{\theta}_n)]^T}_{\mathbf{b}}=-\underbrace{[\mathbf{d}_{\mathrm{re}}(\boldsymbol{\theta}_n)\ \mathbf{d}_{\mathrm{im}}(\boldsymbol{\theta}_n)]}_{\mathbf{A}}^T(\boldsymbol{\theta}_{n+1}-\boldsymbol{\theta}_n) \tag{25}$$

which can be solved iteratively using

$$\boldsymbol{\theta}_{n+1}=\boldsymbol{\theta}_n-(\mathbf{A}^T)^{+}\mathbf{b}, \tag{26}$$

where $[\mathbf{d}_{\mathrm{re}}(\boldsymbol{\theta}_n)\ \mathbf{d}_{\mathrm{im}}(\boldsymbol{\theta}_n)]$ represents the real and imaginary parts of $d_\ell=\partial e(\hat{s}_k)/\partial\theta_\ell$ in (20), for the ℓth parasitic load and a null in the $\hat{s}_k$ direction. Multiple nulls can naturally be accommodated, where for each new null direction $\hat{s}_k$, an additional two columns are included in $\mathbf{A}$ along with the corresponding elements in $\mathbf{b}$.

Given enough reconfigurable elements for a specific number of nulls, $\mathbf{A}^T$ will be a fat matrix. For example, in our numerical examples, we search for a maximum of 8 nulls, and since there are $N-1=24$ reconfigurable elements, $\mathbf{A}^T$ has a maximum dimension of 16×24. The fact that $\mathbf{A}^T$ has a null space means that null and beam optimization can be performed in a decoupled way, where null optimization is performed in the range space of $\mathbf{A}^T$ and beam optimization in the null space of $\mathbf{A}^T$. The basic optimization step, then, consists of two phases. First, $\boldsymbol{\theta}$ is projected onto the null space of $\mathbf{A}^T$ and the main-beam optimization step is applied to this projected version, providing an update to $\boldsymbol{\theta}$. Next, Newton-based null optimization is applied to the updated $\boldsymbol{\theta}$ to obtain the final update for the iteration.

Details of this two-step beam-null iteration are now given. If K denotes the total number of null directions, and $M=N-1$ represents the total number of reconfigurable elements, then the fat matrix $\mathbf{A}^T$ will have dimensions $L\times M$, where $L=2K$. We use the singular value decomposition to identify the range and null spaces of $\mathbf{A}^T$, or

$$\mathbf{A}^T=\mathbf{U}\mathbf{S}\mathbf{V}^H=\mathbf{U}\begin{bmatrix}\mathbf{S}_1 & \mathbf{0}\\ \mathbf{0} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{V}_{\mathrm{rng}}^H\\ \mathbf{V}_{\mathrm{null}}^H\end{bmatrix}. \tag{27}$$

Subspace projections for beam optimization onto the null space of $\mathbf{A}^T$ are given by

$$\boldsymbol{\theta}_{\mathrm{null},n}=\mathbf{V}_{\mathrm{null}}^H\ \boldsymbol{\theta}_n, \tag{28}$$

$$\mathbf{g}_{\mathrm{null},n}=\mathbf{V}_{\mathrm{null}}^H\ \mathbf{g}(\boldsymbol{\theta}_n), \tag{29}$$

and the Newton optimization step uses the modified approximation

$$\underbrace{|e_b^{\mathrm{opt}}(\hat{s}_b)|^2-|e_b(\hat{s}_b,\boldsymbol{\theta}_n)|^2}_{h}=\mathbf{g}_{\mathrm{null},n}^T(\boldsymbol{\theta}_{\mathrm{null},n+1/2}-\boldsymbol{\theta}_{\mathrm{null},n}), \tag{30}$$

where the half index $n+1/2$ is used to denote a half step of the iteration, where only the beam optimization has been applied. Equation (30) is solved as

$$\boldsymbol{\theta}_{\mathrm{null},n+1/2}=\boldsymbol{\theta}_{\mathrm{null},n}+\delta(\mathbf{g}_{\mathrm{null},n}^T)^{+}h. \tag{31}$$

TABLE I
COMPARISON OF OPTIMIZATION PERFORMANCE

	No nulls	2 nulls	4 nulls	8 nulls
Optimal beam	14.66 dB	14.36 dB	14.09 dB	13.85 dB
Newton	14.64 dB	14.36 dB	14.07 dB	13.55 dB

The range-space projected part of $\boldsymbol{\theta}_n$ (used for creating nulls) is not modified in this step, or

$$\boldsymbol{\theta}_{\mathrm{rng},n+1/2}=\boldsymbol{\theta}_{\mathrm{rng},n}=\mathbf{V}_{\mathrm{rng}}^H\boldsymbol{\theta}_n. \tag{32}$$

Transforming back we have

$$\boldsymbol{\theta}_{n+1/2}=\mathbf{V}\begin{bmatrix}\boldsymbol{\theta}_{\mathrm{rng},n+1/2}\\ \boldsymbol{\theta}_{\mathrm{null},n+1/2}\end{bmatrix} \tag{33}$$

$$=\mathbf{V}_{\mathrm{rng}}\mathbf{V}_{\mathrm{rng}}^H\ \boldsymbol{\theta}_n+\mathbf{V}_{\mathrm{null}}\ \boldsymbol{\theta}_{\mathrm{null},n+1/2}. \tag{34}$$

Next, we optimize nulls by applying (25), which in this case becomes

$$\boldsymbol{\theta}_{n+1}=\boldsymbol{\theta}_{n+1/2}-\delta(\mathbf{A}^T)^{+}\mathbf{b}, \tag{35}$$

where it should be noted that the pseudoinverse only operates on the range space of $\mathbf{A}^T$. Iterating on the above procedure we obtain near optimal beam and deep nulls in an efficient manner.

D. Algorithm

The main steps involved in the algorithm are shown in the flowchart in Figure 2. The optimization procedure is initialized by choosing random initial values for $\boldsymbol{\theta}_0$. The total number of iterations N_{I} is specified along with the beam and null directions. Thereafter, we find the derivatives for the beam and null directions. In Step 3, we build $\mathbf{A}^T$ from the derivatives in the null directions. After performing subspace decomposition, the Newton-based beam and null optimization are applied from Step 5 to Step 8. Step 9 is a conditional statement. If the total number of iterations N_{I} is not exhausted, the algorithm returns with the updated $\boldsymbol{\theta}$ to Step 2. Otherwise, the radiation pattern is returned based on the optimized $\boldsymbol{\theta}$ via (2) in Step 10.

IV. RESULTS

Table I shows the optimized gain found in the chosen main-beam direction of $\phi=45°$ using the Newton-based optimization. Also shown is the theoretically optimal beam value considering 1Ω loss at each parasitic antenna in the structure to avoid supergain solutions. For the optimization without nulls, the algorithm is run for 3000 iterations with $\delta=0.001$ in (24). Comparing the values in the table, we note that the Newton-based optimum is quite close to the theoretical optimum.

Figure 3 shows joint beamforming and null-steering for the chosen null angles when searching for 2, 4 and 8 nulls. The Newton-based joint beam and null optimization is run for 1500 iterations with $\delta=0.001$. The result is compared to the reduced order beam optimization with Newton-based null optimization introduced in [7]. It is observed that the proposed approach finds a relatively stronger main-beam performance while keeping the nulls below a threshold value of -20 dB. The result in Table I also shows that the beam optimum found is reasonably close to the reference optimum.

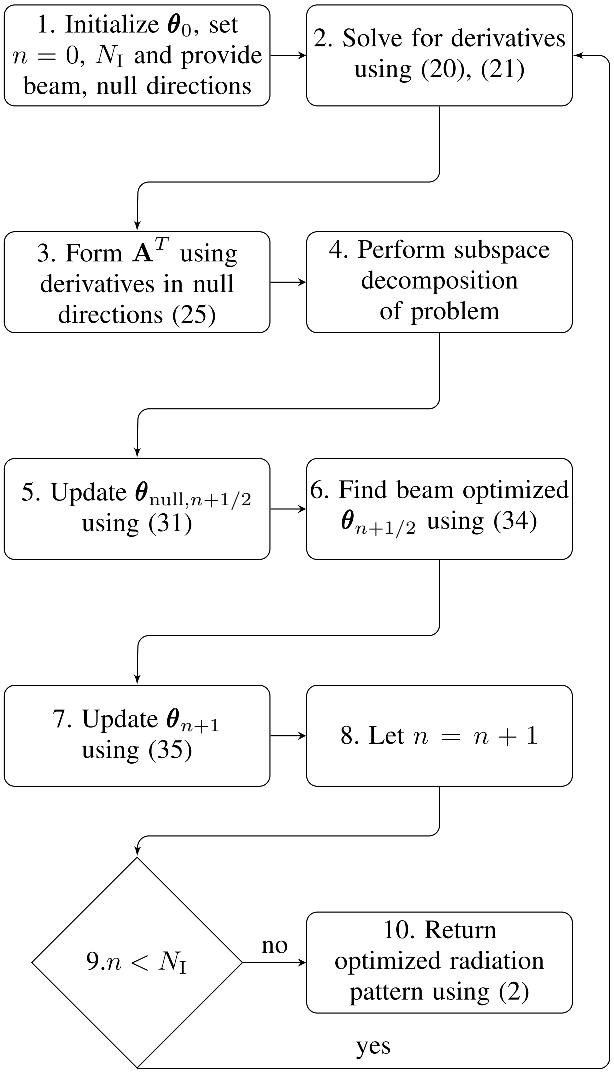


Fig. 2. Flowchart shows the main steps of the efficient Newton-based pattern synthesis algorithm. The algorithm terminates when the threshold for the total iterations N_I is exhausted.

V. Conclusion

An efficient strategy for joint beam and null optimization of dense parasitic arrays has been described. The method combines Newton-based optimization with subspace decomposition of the problem to find near optimal solutions for the main beam while maintaining deep nulls. Due to its efficiency, the approach is likely to be more suitable for in-situ optimization than global search methods.

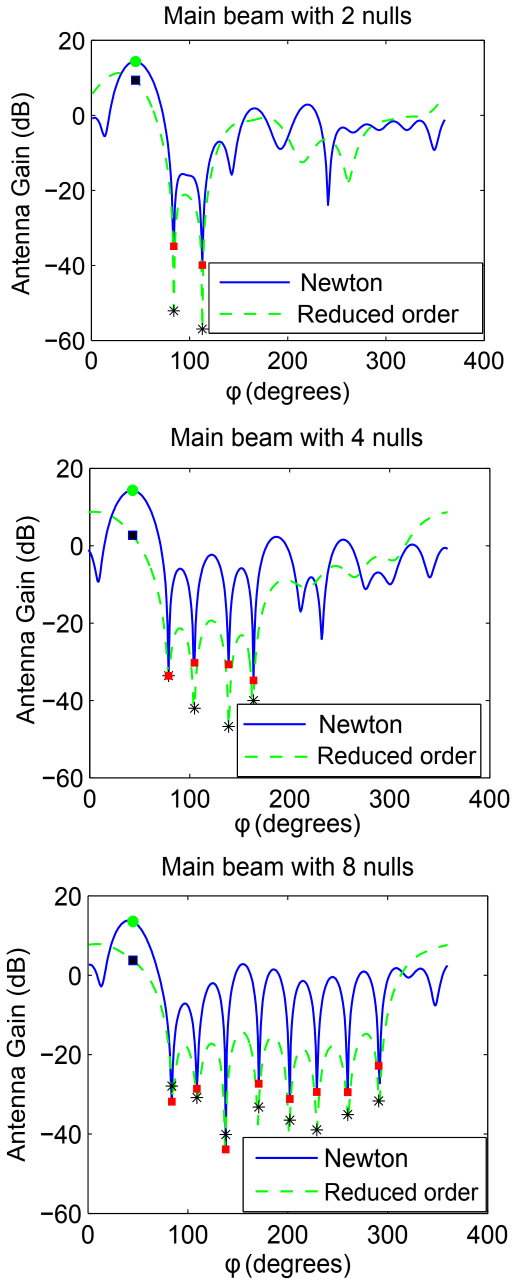


Fig. 3. Joint beam and null formation for 2, 4 and 8 nulls using the reduced order approach [7] and the new Newton method. Prescribed main-beam and null directions are shown by point symbols.

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