Optimization of Parasitic Reconfigurable Aperture Antennas with a Hybrid Direct-Newton Approach

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***Abstract*—Reconfigurable aperture (RECAP) antennas consist of a regular array of reconfigurable elements whose state can be changed dynamically, supporting beamforming, null-steering, adaptive matching, frequency agility, etc. Finding the required state of the reconfigurable loads for a given application is difficult, due to the non-convex and non-linear nature of the optimization problem, and generic unstructured global optimization methods such as genetic algorithms and particle swarm optimization have been proposed. Although appropriate for off-line optimization, such unstructured methods are computationally intensive and may not be appropriate for dynamic in-situ optimization, indicating the need for methods that exploit the problem structure directly. To this end, a novel method for optimization of parasitic RECAPs for beamforming and null-steering is presented, which combines direct optimization of a reduced-order reflection model of the RECAP with efficient Newton root optimization. Initial simulations presented herein suggest that the developed method may be orders of magnitude more efficient for beamforming and null-steering than unstructured optimization methods.**

I. Introduction

Reconfigurable aperture (RECAP) antennas can be viewed as a generalization of reconfigurable antennas or parasitic antenna arrays, and consist of a single feed element surrounded by other elements loaded with programmable reactances. RECAPs and parasitic arrays have received significant attention [1]-[3] and have many useful functions including beamforming, null-steering, interference suppression and adaptive matching. Higher spectral efficiency provided by these RECAP structures is also of high interest considering the limited availability and expensive nature of radio frequency (RF) spectrum. In contrast to smart antenna architectures, which require multiple RF chains and expensive DSP capability, RECAPs need only a single RF and DSP channel and at the same time provide similar functionality as the smart antenna concept.

One important challenge of RECAP structures is the problem of optimizing the reconfigurable loads, which suffers from local minima and maxima and often requires a global search of the domain space to find the best solution. Significant research has been performed in using intelligent techniques and derivative-free methods such as genetic algorithms (GAs), particle swarm optimization (PSO), ant colony optimization, and various other techniques to find an optimal solution for the parasitic loads [4],[5]. Although these techniques have been successfully implemented, they typically require extensive time to find an acceptable solution. The derivative-free methods do not directly exploit the structure of the problem and follow an off-line optimization procedure which leads to greater time complexity than what might be necessary. Furthermore there is no way to ascertain whether the solution found by these algorithms is indeed optimal or not.

Reconfigurable antennas are especially beneficial if the array elements can be optimized dynamically [6], and in such a scenario RECAPs can adapt in-situ to new applications and environmental conditions that are unforeseeable in the design stage. Due to their computational complexity, techniques such as the GA and PSO can only be expected to have limited value for dynamic optimization in a real system. The development of novel methods for RECAP optimization that are significantly more efficient than unstructured global optimization methods is therefore an important research topic.

In this paper, a novel method for direct optimization of RECAPs is presented which directly exploits the structure of the EM radiation problem. By combining direct solution of a first-order approximation of RECAP radiation with efficient local optimization, orders of magnitude reduction in computational complexity is achieved compared to existing unstructured optimization methods.

II. Direct RECAP Optimization

This section describes the model of the RECAP antenna and develops an efficient optimization procedure for the beamforming and null-steering applications.

A. Parasitic Antenna Array

Figure 1 shows an N-port antenna array where the feed element is represented by Port 1 while Port 2 to N represent the parasitic loads. The scalars a_{F}, b_{F}, $e_{\mathrm{F}}(\hat{s})$, represent the input wave, output wave, and the single polarization embedded radiation pattern of the feed element in direction $\hat{s}$, respectively. The vectors $\mathbf{a}_{\mathrm{R}}$, $\mathbf{b}_{\mathrm{R}}$, $\mathbf{e}_{\mathrm{R}}(\hat{s})$, represent the input wave, output wave and the embedded radiation pattern of the parasitic ports in direction $\hat{s}$, respectively. The diagonal input reflection matrix $\mathbf{\Gamma}$ is associated with the reactive reconfigurable elements (REs) with which the parasitic ports are terminated. The input-output relationship of the RECAP is given by

$$\begin{bmatrix} b_{\mathrm{F}} \\ \mathbf{b}_{\mathrm{R}} \end{bmatrix} = \begin{bmatrix} s_{\mathrm{FF}} & \mathbf{s}_{\mathrm{FR}} \\ \mathbf{s}_{\mathrm{RF}} & \mathbf{S}_{\mathrm{RR}} \end{bmatrix} \begin{bmatrix} a_{\mathrm{F}} \\ \mathbf{a}_{\mathrm{R}} \end{bmatrix}. \quad (1)$$

The network analysis is similar to [7] and is treated in detail in section III. The pattern of the radiated far-field for a single polarization in direction $\hat{s}$ is given by

$$e(\hat{s}) = \left[e_{\mathrm{F}}(\hat{s}) + \mathbf{e}_{\mathrm{R}}^{T}(\hat{s})\mathbf{\Gamma}(\mathbf{I} - \mathbf{S}_{\mathrm{RR}}\mathbf{\Gamma})^{-1}\mathbf{s}_{\mathrm{RF}}\right] a_{\mathrm{F}}. \quad (2)$$

B. Beamforming and Null-steering

In this paper we restrict our attention to beamforming and null-steering, which are two important applications of reconfigurable structures. In order to create a main beam or a null in an arbitrary direction $\hat{s}$ we would like to maximize or minimize $|e(\hat{s})|^2$ in (2) with respect to the diagonal reflection matrix $\mathbf{\Gamma}$ of the reconfigurable loads. In this work it is assumed that the reconfigurable loads are lossless reactances, meaning that the diagonal terms have unit magnitude and are given by $e^{j\theta}$, so that only the phases of the load reflections can be varied.

Due to the presence of the inverse term in (2), direct optimization of $\mathbf{\Gamma}$ is difficult. In order to simplify the problem we consider the Neumann series, or

$$(\mathbf{I} - \mathbf{S}_{\mathrm{RR}}\mathbf{\Gamma})^{-1} = \sum_{k=0}^{\infty}(\mathbf{S}_{\mathrm{RR}}\mathbf{\Gamma})^{k}. \quad (3)$$

The first order solutions to this problem would therefore be given by replacing $(\mathbf{I} - \mathbf{S}_{\mathrm{RR}}\mathbf{\Gamma})^{-1}$ in (2) by $(\mathbf{I})$.

Considering an array with $a_{\mathrm{F}} = 1$ the first order expression of (2) can be written as

$$e(\hat{s}) = \underbrace{e_{\mathrm{F}}(\hat{s})}_{\alpha} + \sum_{m=1}^{N-1}\underbrace{e_{\mathrm{R},m}(\hat{s})s_{\mathrm{RF},m}}_{\beta_m}\gamma_m, \quad (4)$$

where $\gamma_m = \Gamma_{mm}$. It can be noted from (4) that when α and $\beta_m\gamma_m$ add in-phase a beam would be expected in that direction since $|e(\hat{s})|$ would be maximized. Similarly, if α and $\beta_m\gamma_m$ add out of phase such that $-\alpha = \sum_{m=1}^{N-1}\beta_m\gamma_m$ a null would be expected in that direction since $|e(\hat{s})|$ would be minimized.

As will be demonstrated, beamforming based on the first-order expression in (4) also produces useful beamforming solutions for the exact expression (2), which might be expected since in-phase addition is somewhat insensitive to small phase perturbations. In the case of null-steering, finding the minimum gain for the first order solution is a useful starting point, but obtaining optimal $\mathbf{\Gamma}$ for the exact expression in (2) requires additional effort. Although an exact cancellation of terms in (4) represents approximate cancellation in (2), the effect of the error from the higher order terms cannot be neglected since the goal is to obtain zero radiated field in direction $\hat{s}$. Nulls are therefore very sensitive to small changes in the value of $\mathbf{\Gamma}$ and an optimal set of $\mathbf{\Gamma}$ values for (4) generally does not result in a null for the exact expression in (2). Below we show that near exact nulls can be found very efficiently by applying Newton's method to the result obtained with the first-order solution.

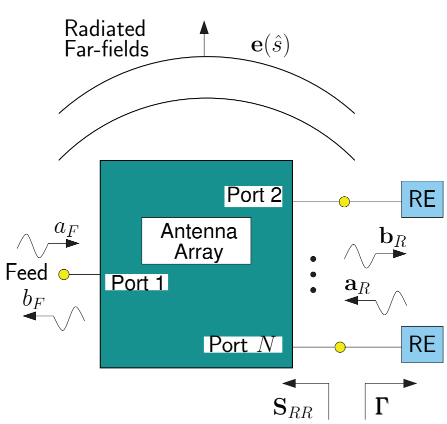

Fig. 1. A general parasitic antenna array

In this iterative approach we first solve for the derivative of (2) with respect to θ_l, where θ_l represents the phase of the lth parasitic element in the array. It can be shown that

$$\frac{\partial e(\hat{s})}{\partial\theta_l} = j\mathbf{e}_{\mathrm{R}}^{T}(\hat{s})(\mathbf{\Gamma}^{-1} - \mathbf{S}_{\mathrm{RR}})^{-1}\mathbf{1}_{ll}(\mathbf{\Gamma}^{-1} - \mathbf{S}_{\mathrm{RR}})^{-1}\mathbf{s}_{\mathrm{RF}}e^{-j\theta_l} \quad (5)$$

where $\mathbf{1}_{ik}$ is an elementary matrix that is all zeros except for a 1 for the ikth element. Given the derivatives with respect to each of the $N-1$ parasitic loads, we form the vector $d_l = \partial e(\hat{s})/\partial\theta_l$. Below we describe two methods for efficiently finding a null using Newton's method.

1) Separate Newton Step: The goal in the optimization is to make both $\mathrm{Re}\{e(\hat{s})\} = 0$ and $\mathrm{Im}\{e(\hat{s})\} = 0$. Newton steps can be applied separately to move in the direction of vanishing $\mathrm{Re}\{e(\hat{s})\}$ and $\mathrm{Im}\{e(\hat{s})\}$ according to

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n - \frac{e_{\mathrm{re}}(\hat{s},\boldsymbol{\theta}_n)\mathbf{d}_{\mathrm{re}}(\boldsymbol{\theta}_n)}{\|\mathbf{d}(\boldsymbol{\theta}_n)\|^2} - \frac{e_{\mathrm{im}}(\hat{s},\boldsymbol{\theta}_n)\mathbf{d}_{\mathrm{im}}(\boldsymbol{\theta}_n)}{\|\mathbf{d}(\boldsymbol{\theta}_n)\|^2} \quad (6)$$

where $e_{\mathrm{re}}(\hat{s}) = \mathrm{Re}\{e(\hat{s})\}$, $e_{\mathrm{im}}(\hat{s}) = \mathrm{Im}\{e(\hat{s})\}$, $\mathbf{d}_{\mathrm{re}} = \mathrm{Re}\{\mathbf{d}\}$, $\mathbf{d}_{\mathrm{im}} = \mathrm{Im}\{\mathbf{d}\}$, and $\boldsymbol{\theta}_n = [\theta_{1,n}\ldots\theta_{N,n}]^T$. Using the first-order solution as a starting point, it is expected that we should be near a null, and only a few Newton steps should be required. Alternatively, we can check whether the first order solution provides a useful starting point by instead seeding the iteration with a random starting point. The iteration in (6) is stopped when the null is "deep enough" for a given application or tolerance.

2) Joint Newton Step: A more rigorous approach is to model $e(\hat{s})$ with the Taylor series

$$e_{\mathrm{re}}(\hat{s},\boldsymbol{\theta}_{n+1}) = e_{\mathrm{re}}(\hat{s},\boldsymbol{\theta}_n) + \mathbf{d}_{\mathrm{re}}^{\mathrm{T}}(\boldsymbol{\theta}_n)(\boldsymbol{\theta}_{n+1} - \boldsymbol{\theta}_n) \quad (7)$$

$$e_{\mathrm{im}}(\hat{s},\boldsymbol{\theta}_{n+1}) = e_{\mathrm{im}}(\hat{s},\boldsymbol{\theta}_n) + \mathbf{d}_{\mathrm{im}}^{\mathrm{T}}(\boldsymbol{\theta}_n)(\boldsymbol{\theta}_{n+1} - \boldsymbol{\theta}_n). \quad (8)$$

Setting $e_{\mathrm{re}}(\hat{s},\boldsymbol{\theta}_{n+1}) = e_{\mathrm{im}}(\hat{s},\boldsymbol{\theta}_{n+1}) = 0$, we have

$$\underbrace{[e_{\mathrm{re}}(\hat{s},\boldsymbol{\theta}_n)\ e_{\mathrm{im}}(\hat{s},\boldsymbol{\theta}_n)]^T}_{\mathbf{B}} = -\underbrace{[\mathbf{d}_{\mathrm{re}}(\boldsymbol{\theta}_n)\ \mathbf{d}_{\mathrm{im}}(\boldsymbol{\theta}_n)]^T}_{\mathbf{A}}(\boldsymbol{\theta}_{n+1} - \boldsymbol{\theta}_n) \quad (9)$$

which can be solved as

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n - (\mathbf{A}^T)^{+}\mathbf{B}^T, \quad (10)$$

where $(.)^+$ is the pseudo-inverse.

Although somewhat more complex, this method has the advantage of moving in the jointly optimal and minimum norm direction to make both the real and imaginary radiated fields in the specific direction tend to 0.

Having developed two methods for finding a single null, the methods can be naturally extended to find multiple nulls. In

(6) we simply extend the Newton step to be

$$\boldsymbol{\theta}_{n+1} = \boldsymbol{\theta}_n - \sum_k \left[\frac{e_{\rm re}(\hat{s}_{\rm k})\mathbf{d}_{\rm re}(\hat{s}_{\rm k})}{\|\mathbf{d}(\hat{s}_{\rm k})\|^2} + \frac{e_{\rm im}(\hat{s}_{\rm k})\mathbf{d}_{\rm im}(\hat{s}_{\rm k})}{\|\mathbf{d}(\hat{s}_{\rm k})\|^2} \right], \quad (11)$$

where $e(\hat{s})$ and $\mathbf{d}$ are always evaluated at the current step $\boldsymbol{\theta}_n$. Alternatively, in (10) additional columns are added to $\mathbf{A}$ and $\mathbf{B}$ for the additional directions. In this case, it is clear that if too many directions are added, an exact solution to (9) may not be possible, but the method will still find the best step in the least square sense.

It is interesting to also consider how the method can be used to jointly form a main beam and null(s). Experience indicates that a main beam created with the first-order solution is relatively insensitive to small perturbations in $\boldsymbol{\Gamma}$. Therefore a useful approach is to find a main beam using (4), followed by Newton steps to find nulls in the desired directions. Note that the reverse procedure of finding nulls followed by local optimization to find a main beam is very problematic, since nulls are very sensitive to error.

III. Numerical Examples

This section provides some illustrative examples for the developed techniques.

A. Simulation and Network Analysis of RECAPs

Consider a 5×5 square dipole array where the half-wave dipoles are spaced at a distance of $\lambda/4$, similar to that in [7]. We use a single method-of-moments (MOM) simulation for each port to obtain network characteristics and embedded radiation patterns of the antennas. Using the Numerical Electromagnetic Code (NEC) to perform the MOM simulation, we supply unit voltage to the excited antenna with all other ports short circuited and thereby find the admittance matrix $\mathbf{Y}$ and the short circuit radiation patterns $\mathbf{e}^{\rm sc}(\hat{s})$. Matched ($Z_0$-terminated) embedded patterns $\mathbf{e}^{\rm mc}$ are then computed by connecting a source to the kth port and terminating all other ports by the normalizing impedance $Z_0 = 72\Omega$ to obtain

$$\mathbf{e}^{\rm mc}(\hat{s}) = \frac{\mathbf{e}^{\rm sc}(\hat{s})}{\sqrt{Z_0}} \mathbf{Z}(\mathbf{I} - \mathbf{S}). \quad (12)$$

The scattering matrix is given by $\mathbf{S} = (\mathbf{I} + Z_0\mathbf{Y})^{-1}(\mathbf{I} - Z_0\mathbf{Y})$.

Network analysis using (1) is applied to find the array radiation pattern for arbitrary loads. By observing Figure 1 we see that $\mathbf{a}_{\rm R} = \boldsymbol{\Gamma}\mathbf{b}_{\rm R}$. Using (1) along with this information,

$$\mathbf{a}_{\rm R} = \boldsymbol{\Gamma}(\mathbf{I} - \mathbf{S}_{\rm RR}\boldsymbol{\Gamma})^{-1}\mathbf{s}_{\rm RF}a_{\rm F}. \quad (13)$$

The matched patterns of the ports are partitioned as $\mathbf{e}^{\rm mc}(\hat{s}) = [e_{\rm F}(\hat{s}) \ \ \mathbf{e}_{\rm R}(\hat{s})]^T$, and using superposition the radiation pattern of the complete array is

$$e(\hat{s}) = a_{\rm F}e_{\rm F}(\hat{s}) + \sum_{k=1}^{N-1} a_{\rm R,k}\mathbf{e}_{\rm R,k}(\hat{s}). \quad (14)$$

Substituting (13) into (14) we obtain

$$e(\hat{s}) = [e_{\rm F}(\hat{s}) + \mathbf{e}_{\rm R}^T(\hat{s})\boldsymbol{\Gamma}(\mathbf{I} - \mathbf{S}_{\rm RR}\boldsymbol{\Gamma})^{-1}\mathbf{s}_{\rm RF}]a_F, \quad (15)$$

which is identical to (2).

B. Illustrative Results

Figure 2 shows the application of the joint Newton method seeded with the first-order solution, where the 5×5 square array with inter-element spacing of $\lambda/4$ is considered. The plot shows the antenna gain in dB for a steering angle of $\phi = 120°$. Three iterations were required to achieve a null in direction $\phi = 120°$ with gain lower than the threshold of -25 dB. It should be noted that the optimization time required for the proposed method is much shorter than unstructured methods. Our Matlab implementation requires only 122ms for the three iterations, whereas a GA or PSO would take 10s of seconds or more to find a similar solution.

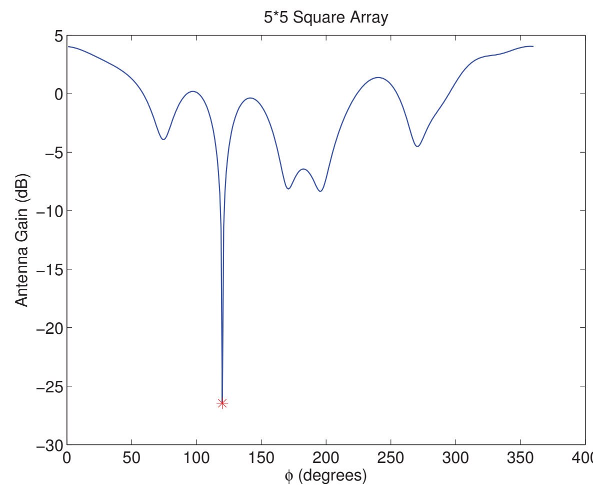

Fig. 2. Application of the joint Newton method seeded with first order solution. A null is created for an angle of $\phi = 120°$.

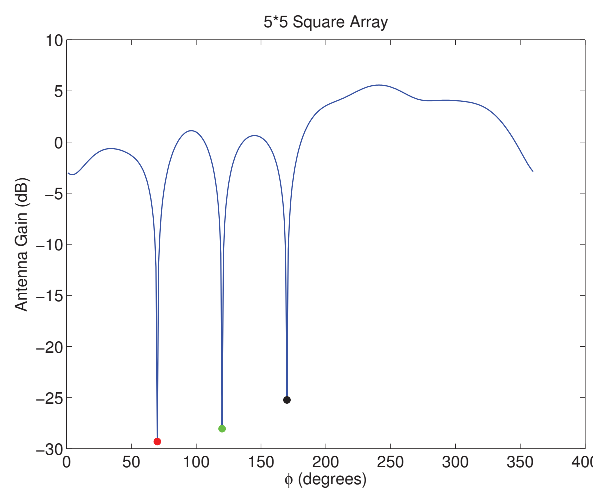

Fig. 3. Joint Newton method is applied with random seeding to create multiple nulls at steering angles $\phi_1 = 70°$, $\phi_2 = 120°$ and $\phi_3 = 170°$.

Using the same array topology, we also consider forming multiple nulls. Although we are developing methods for multiple null formation based on the first-order expression in (4), these are beyond the scope of the present paper. Instead we form multiple nulls with random seeding for steering angles $\phi_1 = 70°$, $\phi_2 = 120°$, $\phi_3 = 170°$ using the joint Newton method as shown in Figure 3. Finding multiple nulls below the threshold of -25 dB takes slightly longer, amounting to a total of 116 iterations taking 8.2 seconds. The increased simulation time and number of iterations result not only from the conflicting constraints, but also due to random seeding which does not fully exploit the problem structure. Note, however, that finding multiple nulls with a GA or PSO using a similar Matlab implementation may require several minutes to find an acceptable solution.

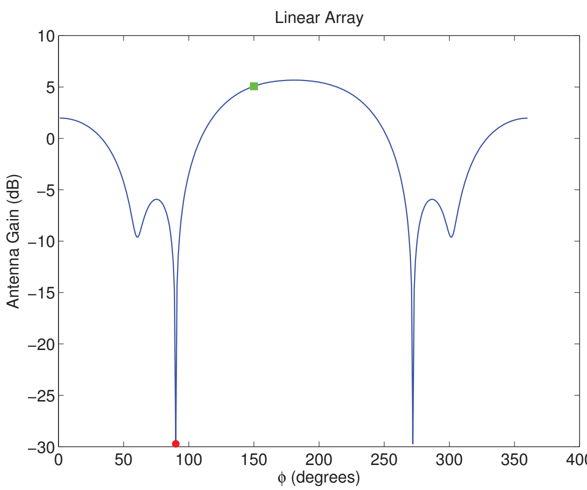

Fig. 4. Joint optimization of a beam and a null using the separate Newton method with the first order solution as the starting point

Joint optimization of beamforming and null-steering is illustrated in Figure 4. In this case the separate Newton method is tested with the first-order solution as the seed. A uniform linear array with 9 elements having inter-element spacing of 0.1λ is used. Steering angles of $\phi_b = 150°$, $\phi_n = 90°$ are chosen for the beam and null respectively. The separate Newton method also provides acceptable performance in finding a null and the $\mathbf{\Gamma}$ value does not change so much during null optimization so as to effect the beam in direction $\phi_b = 150°$.

Figure 5 considers the same uniform linear array and compares the number of iterations needed by the joint and separate Newton methods with random seeds as well as first-order solution as starting points, where the goal is to find a null at a steering angle $\phi = 180°$. As can be seen the combination of the joint Newton method with first-order solution seeding is the fastest to converge with a null at -55 dB after only three iterations. The slowest convergence rate is exhibited by the separate Newton method with random seeding which requires 10 iterations to find a null lower than the threshold of -25 dB. It is also interesting to note that the separate Newton method with the first-order solution as a starting point outperforms the joint Newton method with random seeding, thus illustrating the value of using the first-order approximation over a random starting point.

IV. Conclusion

In this work we have presented a novel method for direct optimization of reconfigurable aperture (RECAP) antennas which

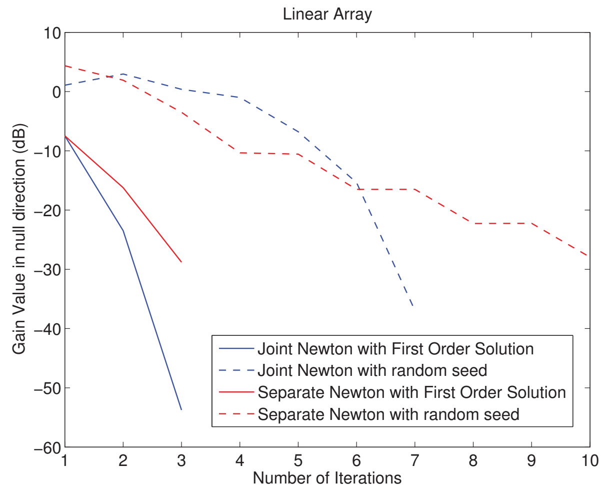

Fig. 5. Gain in null direction $\phi = 180°$ versus number of Newton steps for the joint Newton method and separate Newton method with either first-order solution or random seeding. The search is terminated when the goal of -25 dB is obtained.

directly exploits the structure of the EM radiation problem. The first-order approximation of the RECAP radiation was combined with local optimization based on two variations of Newton's method to achieve efficient optimization of the reconfigurable loads. The optimization procedure was applied on a single null, multiple nulls, and a joint null and main beam problem, illustrating that relatively few iterations are required to find suitable solutions, which may be orders of magnitude more efficient than unstructured global search methods. The separate and joint Newton methods with first-order and random seeding were also compared, indicating that the first-order initial solution is valuable for speeding up null formation. These results indicate that the proposed method should be useful for dynamic in-situ RECAP optimization.

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