

Improved Autocorrelation-based Sensing Using Correlation Distribution Information

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Abstract—Accurate and efficient spectrum sensing is a critical component of cognitive radio, which is a technology poised to improve dynamic resource management in future wireless networks. Autocorrelation exploitation has been shown to provide improvements in sensing performance over simple methods like energy detection. A new upper bound on the performance of autocorrelation-based detectors based on an NP test under the assumption of correlation distribution information (CDI) is presented, where random parameters of the signal autocorrelation are not known, but their distribution is assumed to be known. Not only does this reveal how close existing ad-hoc autocorrelation-based detectors are to optimal performance, but also it indicates how much performance improvement over energy detection is theoretically possible.

Index Terms—Dynamic radio resource management, cognitive radio, signal detection, Neyman-Pearson criterion, correlation.

I. INTRODUCTION

Dynamic radio resource management is an important part of future wireless communication systems, and cognitive radio is a promising potential solution. Unlicensed cognitive radio users adaptively adjust radio parameters to the network environment, resulting in improved spectral efficiency. Reliable sensing is a major component of cognitive radio, achieved by efficient sensing algorithms combined with collaboration and cooperation among the sensing nodes. Different collaborative diversity schemes like soft combining and hard fusion methods can be used.

Although a common method for unknown primary sensing is energy detection (ED) [1][2], the performance can be improved by exploiting the signal autocorrelation [3][4][5]. In the case of perfect correlation information (PCI), where signal autocorrelation is exactly known, a direct Neyman-Pearson (NP) detector can be applied to achieve optimal performance [3], but for blind sensing, such knowledge is typically not available, and the performance of the NP-PCI detector can be considered a loose upper bound.

Several ad-hoc methods for autocorrelation based sensing that do not require any a priori correlation information have been proposed. For in example, in [5], a decision statistic based on summing the magnitude of non-zero lag autocorrelation values is developed. Also, in [4], we propose a simpler method that jointly exploits both energy and first-lag autocorrelation for improved sensing performance. The major difficulty with analyzing and comparing such methods is that the performance depends critically on the level of autocorrelation present, which can be viewed as a random parameter for blind sensing, hindering a reasonable definition of optimality.

To overcome this difficulty, this work defines a new upper bound on the performance of correlation-based detection (CBD) methods, based on an NP test under the assumption of correlation distribution information (CDI), where random parameters of the signal autocorrelation are not known, but their distribution is assumed to be known. Not only does this reveal how close the ad-hoc autocorrelation-based detectors are to optimal performance, but also it indicates how much performance improvement over ED is theoretically possible, depending on which parameters of the autocorrelation distribution are known (fixed) and which are unknown (random). We propose random correlation models (RCMs) to precisely assess and compare the performance of CBD methods. Further, the fundamental limit of CBD performance is conveniently defined by the performance of an optimal NP detector when the parameters of the RCM (but not the correlation itself) are known a priori. A simple uniform correlation model is considered in this work which is appropriate when correlated signals are present, but the exact level is unknown.

The remainder of the paper is organized as follows: Section II provides background on the system model and energy detection (ED) method, followed by correlation-based ad hoc methods in Section III. Section IV discusses the NP-based methods under the random correlation model (RCM). Section V assesses the performance of existing ad-hoc CBD methods based on the developed RCMs in light of the NP bounds, and concluding remarks are given in Section VI.

II. SIGNAL MODEL AND DETECTION

This section provides the signal model and gives an overview of energy detection (ED). Boldface lowercase and uppercase letters denote vectors and matrices, respectively. Vector transpose and conjugate transpose (Hermitian) are indicated by \mathbf{x}^T and \mathbf{x}^H , respectively. The imaginary unit is indicated by j , and $|\cdot|$ and $\angle\cdot$ take the magnitude and phase of a complex number. The notation $\mathbf{x} \sim \mathcal{Z}$ indicates that the random vector (or variable) \mathbf{x} has the probability distribution \mathcal{Z} . The distribution $\mathcal{CN}(\boldsymbol{\mu}, \mathbf{C})$ is the multivariate complex Gaussian distribution with mean vector $\boldsymbol{\mu}$ and covariance matrix \mathbf{C} . $\mathcal{N}(\boldsymbol{\mu}, \mathbf{C})$ denotes the usual multivariate real Gaussian distribution with mean $\boldsymbol{\mu}$ and covariance \mathbf{C} , and $U(a, b)$ is the uniform distribution on the interval $[a, b]$. Additional notation will be introduced as necessary.

Considering the bandpass noise within a fixed sensing bandwidth to have flat power spectral density (PSD), the noise

is represented as

$$w_{bp}(t) = w_c(t) \cos 2\pi f_c t - w_s(t) \sin 2\pi f_c t, \quad (1)$$

where f_c is the reference frequency, and $w_c(t)$ and $w_s(t)$ are the in-phase and quadrature modulation components, respectively. This bandpass noise is commonly represented in its lowpass equivalent form as

$$w_{bp}(t) = \text{Re} \{ w(t) e^{j2\pi f_c t} \}, \quad (2)$$

where $\text{Re}\{\cdot\}$ takes the real part and $w(t) = w_c(t) + jw_s(t)$ is the complex envelope.

If the bandpass noise in (1) has bandwidth W its variance is $\sigma_{wp}^2 = 2WN_{02}$, where N_{02} is a two-sided PSD of the bandpass noise. Now both $w_c(t)$ and $w_s(t)$ will have bandwidth $W/2$. Let $w_c[n]$ and $w_s[n]$ be the n th samples (at an interval of $1/W$) of $w_c(t)$ and $w_s(t)$, respectively. The complex envelope in discrete time notation will be $w(n) = w_c[n] + jw_s[n]$.

Let $\sigma_{w_c}^2$ and $\sigma_{w_s}^2$ are the variance of in-phase and quadrature components respectively. Also, $\sigma_{w_c}^2 = \sigma_{w_s}^2 = \sigma_{wp}^2 = 2WN_{02}$. The variance of the complex envelope $w(t)$ is $\sigma_w^2 = 2\sigma_{w_c}^2$.

For a waveform $x(t)$ over the interval $-\infty < t < \infty$, the autocorrelation is defined as

$$\phi_x(\tau) = \text{E} \{ x(t) x^*(t - \tau) \}, \quad (3)$$

where $*$ denotes the complex conjugate and τ represents the time shift(lag). We can normalize this autocorrelation function as $\rho_x(\tau) = \phi_x(\tau) / \phi_x(0)$ where $\rho_x(\tau)$ can also be termed as an autocorrelation coefficient.

In discrete time the notation $\phi_x[k] \equiv \phi_x(\tau=k/W)$ is adopted, which gives

$$\phi_{x,k} = \text{E} \{ x_i x_{i-k}^* \} \quad (4)$$

where k is the integer shift (lag). Consequently, the normalized autocorrelation (autocorrelation coefficient) can be written in discrete form as $\rho_{x,k} = \phi_{x,k} / \phi_{x,0}$.

Since Nyquist sampling is employed, noise samples are uncorrelated, since the autocorrelation for sample lag k is

$$\phi_{w,k} = \text{E} \{ w_i w_{i-k}^* \} = 4WN_{02} \text{sinc}(k) = \sigma_w^2 \delta[k] \quad (5)$$

where $\text{sinc}(x) = \sin(\pi x) / (\pi x)$ and $\delta[k]$ is the Kronecker delta function, and the last equality comes since k is an integer.

Bandpass signal with carrier frequency offset Δf and phase offset θ (relative to the reference frequency f_c) has an analogous form to (2), or

$$g_{bp}(t) = \text{Re} \{ g(t) e^{j2\pi f_c t} \}, \quad (6)$$

with complex envelope

$$g(t) = g_0(t) e^{j(2\pi\Delta f t + \theta)}, \quad (7)$$

where $g_0(t)$ is the complex envelope corresponding to perfect frequency and phase synchronization. $g(t)$ is also assumed to be zero mean, Gaussian random process with variance σ_g^2 and it can also be represented in discrete time as $g[n]$ by taking the samples at the interval $1/W$ where $\mathbf{g} \sim \mathcal{CN}(\mathbf{0}, \mathbf{C}_g + \sigma_g^2 \mathbf{I})$. The signal autocorrelation is given as $\phi_{g,k} = \text{E} \{ g_i g_{i-k}^* \}$.

In this work, the received waveform is modeled in a simple form as $x(t) = g(t) + w(t)$, which can be written in vector

representation as $\mathbf{x} = \mathbf{g} + \mathbf{w}$. Thus, when only noise is present (referred to as hypothesis H_0) we have $\mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I})$, and the pdf for this case is denoted $p(\mathbf{x}; H_0)$.

When signal is present (hypothesis H_1), the receiver measures signal plus noise and the pdf of \mathbf{x} for this case is denoted $p(\mathbf{x}; H_1)$.

The detection involves the hypotheses

$$\mathbf{x} \sim \begin{cases} \mathcal{CN}(\mathbf{0}, \sigma_w^2 \mathbf{I}) & \text{under } H_0 \\ \mathcal{CN}(\mathbf{0}, \mathbf{C}_g + \sigma_w^2 \mathbf{I}) & \text{under } H_1. \end{cases} \quad (8)$$

The correlation-based detectors studied in this work exploit the fact that signal autocorrelation $\phi_{g,k}$ can differ significantly from $\phi_{w,k}$. Normalized autocorrelation will be denoted $\rho_{r,k} = \phi_{r,k} / \phi_{r,0}$ with $r \in \{x, n, s\}$.

If the covariance of the signal \mathbf{C}_g is completely known then optimal detection is based on NP method. But here we assume that the covariance is not known, but it is a random variable which is a function of random vector of parameters and is completely determined by the (joint) pdf of the random vector.

A Gaussian distributed signal with exponential autocorrelation function is considered in this work. This is a simple but useful model to represent the autocorrelation at an arbitrary lag in terms of the first lag autocorrelation. For the real exponential autocorrelation function, we can write

$$\phi_{g,k} = \sigma_g^2 \exp(-|k|/\zeta) = \sigma_g^2 \rho_{g,1}^{|k|}, \quad (9)$$

where ζ is the decay constant which is high for highly correlated signal and approaches zero for uncorrelated signal.

Assuming that the correlation function is complex having its magnitude exponential, we can write

$$\phi_{g,k} = \sigma_g^2 |\rho_{g,1}|^{|k|} e^{j2\pi\varphi_1 k} = \sigma_g^2 |\rho_{g,1}|^{|k|} e^{j\angle\rho_{g,1} k}, \quad (10)$$

where $|\rho_{g,1}|$ is the random variable representing the magnitude of the correlation coefficient for 1-lag, φ_1 is a random variable between 0 and 1 so that the phase $\angle\rho_{g,1}$ lies between 0 and 2π .

Since $|\rho_{g,1}|$ and φ_1 are both random variables, $\rho_{g,k}$ in exponential distribution depends upon these two variables. Different assumptions for the distribution of $|\rho_{g,1}|$ and φ_1 are theoretically possible, but here we assume a simple uniform distribution for both of them i.e., $|\rho_{g,1}| \sim U(0,1)$ and $\varphi_1 \sim U(0, 2\pi)$, where $0 \leq z \leq 1$. The reasonable assumption of the value of z depends upon the sensing scenario. Here we consider two scenarios: fully blind (random) sensing and known bands (guided) sensing. When the sensing is fully blind with no knowledge on the parameter (bandwidth, center frequency) of the signal present on that band, $z = 1$ is the reasonable assumption. However when center frequency of the sensed signal is fully known and the PSD of complex envelope is symmetric about the y-axis, $\rho_{g,k}$ becomes real and $z = 0$ is to be chosen so that $\varphi_1 = 0$. When the actual center frequency of sensed signal is different from f_c (but not fully blind sensing), the offset causes $\rho_{g,k}$ to be complex with small imaginary part and φ_1 no more remains zero. In such case the suitable value of z depends upon the offset. Although we mainly assume $z = 1$ (blind sensing) and $z = 0$ (guided sensing) under the uniform RCM, the effect of frequency offset in real correlation-based sensing will also be discussed.

A. Energy Detection

Although Energy detection (ED) is not correlation based detection it is included here for performance comparison as it is the most common method of spectrum sensing, due to its low computational complexity and ease of implementation [1][2][6].

The test statistic \mathcal{T} for the ED detection when a waveform $x_{bp}(t)$ is received is the normalized energy defined as [1]

$$\mathcal{T} = \frac{1}{N_{02}} \int_0^T x_{bp}^2(t) dt, \quad (11)$$

where T is the sensing duration.

Let us consider the case when there is not signal present (hypothesis H_0). The test statistic can be approximated as

$$\mathcal{T} = \int_0^T \frac{w_{bp}^2(t)}{N_{02}} dt \approx \sum_{i=1}^N \frac{(w_c^2[i] + w_s^2[i])}{2WN_{02}} = \sum_{i=1}^{TW} (b_c^2[i] + b_s^2[i]), \quad (12)$$

where the samples are normalized by defining $b_c[i] = w_c[i]/\sqrt{\sigma_{w_c}^2}$, $b_s[i] = w_s[i]/\sqrt{\sigma_{w_s}^2}$. The test statistic \mathcal{T} in this hypothesis follows a central chi-square distribution with degree of freedom $2N$ where $N = TW$ is the total number of samples collected during the sensing time T . When primary signal $g_{bp}(t)$ is present (hypothesis H_1), \mathcal{T} follows a non-central chi-square distribution with $2N$ degrees of freedom and a non-centrality parameter $\gamma' = 2N\gamma$, where γ is the signal-to-noise ratio (SNR) [1]. The decision statistic will be

$$\mathcal{T} \sim \begin{cases} \chi_{2N}^2 & H_0, \\ \chi_{2N}^2(\gamma') & H_1. \end{cases} \quad (13)$$

This is, however, not exactly true for the uncorrelated Gaussian signal where $\mathcal{T}/(1+\gamma)$ under H_1 should follow again a central chi-square distribution with degree of freedom $2N$ [3].

The probability of false alarm P_{fa} and the probability of correct detection P_d for some threshold λ can be calculated as [2]

$$P_{fa} = \Pr(\mathcal{T} > \lambda | H_0) = \Gamma\left(N, \frac{\lambda}{2}\right) / \Gamma(N), \quad (14)$$

and

$$P_d = \Pr(\mathcal{T} > \lambda | H_1) = Q_N\left(\sqrt{\gamma'}, \sqrt{\lambda}\right), \quad (15)$$

$\Gamma(\cdot)$ is the gamma function, $\Gamma(\cdot, \cdot)$ is the upper incomplete gamma function, and $Q(\cdot, \cdot)$ is the generalized Marcum Q-function. For zero mean Gaussian signal, P_d can be expressed in similar form as P_{fa} and is given as

$$P_d = \Gamma\left(N, \frac{\lambda}{2(1+\gamma)}\right) / \Gamma(N). \quad (16)$$

The result obtained from both (15) and (16) are very close to each other for low SNR conditions. with sufficiently large value of N . It is also to be noted that P_d obtained from (16) is for the uncorrelated Gaussian signal. The signal model we are considering in this work include correlated Gaussian samples causing the increase in the variance of the decision statistic. Thus the actual P_d in the presence of correlated signal is somewhat less than the one obtained from (16) although the effect is noticeable only in very high correlation.

III. CORRELATION-BASED AD HOC METHODS

A. Covariance-Matrix Based Detection (CAV Method)

The spectrum sensing method in [5] exploits the signal correlation in the form of sample covariance matrix. The covariance absolute value (CAV) method forms a test statistic based on the absolute value of the receive covariance matrix \mathbf{C}_x according to

$$\mathcal{T}(\mathbf{x}) = \frac{1}{\text{Tr}\{\hat{\mathbf{C}}_x\}} \sum_{l=1}^L \sum_{m=1}^L |\hat{C}_{x,lm}|, \quad (17)$$

where $\text{Tr}\{\cdot\}$ is trace, and $\hat{\mathbf{C}}_x$ is the $L \times L$ sample covariance matrix $\hat{C}_{x,lm} = \hat{\phi}_{l-m}$ and $\hat{\phi}_k = (1/N) \sum_{i=1}^N x_i x_{i-k}^*$.

A feature of the CAV method is that normalization by the trace in (17) yields a test statistic under H_0 that is independent of the noise power, placing it in the class of constant false-alarm rate (CFAR) estimators. In the case of noise uncertainty, where accurate knowledge of σ_n^2 is not available, CFAR estimators can exhibit improved performance compared to their non-CFAR counterparts. The trace normalization in (17) unfortunately spreads the noise is known, but one should keep in mind that relative performance can change dramatically when noise uncertainty is an issue. We will consider the performance with trace normalization as well as without normalization.

For a signal with nearly real correlation (symmetric PSD), CorrSum can outperform CAV for two reasons: (i) For limited sample size, both real and imaginary parts of the noise autocorrelation $\hat{\phi}_{n,1}$ are nonzero. Since signal autocorrelation is known to be real, discarding useless information in imaginary part of $\hat{\phi}_{x,1}$ is beneficial. (ii) Under H_0 and limited sample size, the real noise autocorrelation is positive or negative, yet the absolute value in CAV forces this to always be positive, causing more overlap of $p(\mathcal{T}; H_0)$ and $p(\mathcal{T}; H_1)$ and reducing potential performance. However when the correlation is complex and phase is distributed over 0 to 2π , CAV performs better as the absolute value has more information than just real part. CorrSum method is computationally simple as it exploits the energy and the autocorrelation for 1 shift. The detailed comparison is made in section V.

B. Autocorrelation-Based Detection (CorrSum Method)

In [4][7], we have proposed the simple correlation sum (CorrSum) detector that exploits both energy and correlation for improved performance assuming that correlation is real. Here we review the method briefly as it will be also considered here for comparison. Here we also consider the frequency offset scenario, where the correlation tends to be complex.

Using the definition in (3), the autocorrelation of the complex envelope $x(t)$ of the bandpass process $x_{bp}(t)$ is obtained to be

$$\phi_x(\tau) = \phi_{x_c}(\tau) + \phi_{x_s}(\tau) + 2j\phi_{x,sc}(\tau), \quad (18)$$

where $\phi_{x_c}(\tau)$ and $\phi_{x_s}(\tau)$ are the autocorrelation of $x_c(t)$ and $x_s(t)$, respectively, and $\phi_{x,sc}(\tau)$ is the cross-correlation between $x_s(t)$ and $x_c(t)$. Assuming band-pass noise having flat spectrum over bandwidth W , the imaginary part of the

correlation of complex envelope $w(t)$ becomes zero [8]. However, if the integration in (3) is for finite duration, this will only be approximately true. The real correlation of the noise is

$$\phi_w(\tau) = \phi_{wc}(\tau) + \phi_{ws}(\tau), \quad (19)$$

where $\phi_{wc}(\tau)$ and $R_{ws}(\tau)$ are the autocorrelation of $w_c(t)$ and $w_s(t)$, respectively. The autocorrelation can be expressed as

$$\phi_w(\tau) = \phi_w(0) \text{sinc}(W\tau), \quad (20)$$

which can be written in discrete time as $\phi_{w,k} = \phi_{w,0} \text{sinc}[k] = \phi_{w,0} \delta[k]$.

The CorrSum method exploits the fact that the signal autocorrelation $\phi_g(\tau)$ deviates significantly from the sinc shape in (20), depending on the symbol rate, modulation, pulse shape, etc., allowing improved detection performance.

Denoting the autocorrelation of the received signal for sensing duration T as $\hat{\phi}_x(\tau)$ ($x \in \{g, w\}$ for signal present or absent, respectively), this method also exploits $\hat{\phi}_{x,k}|_{k \neq 0}$ in addition to $\hat{\phi}_{x,0}$ as opposed to ED that exploits $\hat{\phi}_{x,0}$ alone. Since the difference of $\hat{\phi}_{g,k}$ and $\hat{\phi}_{w,k}$ is likely to be most pronounced for $k = 1$, but no a priori knowledge on the level of correlation is expected, we investigate the decision statistic

$$\mathcal{T} = \hat{\phi}_{x,0} + \hat{\phi}_{x,1}, \quad (21)$$

that places equal weight on energy and single-shift correlation which was assumed to be real. Note that this summation in discrete time corresponds approximately to integrating $\hat{\phi}_x(\tau)$ for $\tau \in [0, 1/W]$ in continuous time, hence the name *correlation sum* or *CorrSum* for short. Although integration of the envelope of bandpass correlation has been proposed in [4] for implementation, the integration of the baseband correlation gives the same result.

For real $\phi_x(\tau)$, the detection performance for CorrSum method is discussed in [4] and only the closed-form results for P_{fa} and P_d are included here. The false alarm probability is given as

$$P_{fa} = \Pr(\mathcal{T} > \lambda | H_0) = \frac{1}{2} \text{erfc} \left[\frac{(\lambda - \mu_0)}{\sigma_0 \sqrt{2}} \right], \quad (22)$$

where $\mu_0 = 2N$, $\sigma_0^2 = 4N + 2(N-1)$. The detection probability when signal is present is given as

$$P_d = \Pr(\mathcal{T} > \lambda | H_1) = \frac{1}{2} \text{erfc} \left[\frac{(\lambda - \mu_1)}{\sigma_1 \sqrt{2}} \right], \quad (23)$$

where [4]

$$\mu_1 = 2N(1 + \gamma) + 2(N-1)\rho_{g,1}\gamma, \quad (24)$$

$$\begin{aligned} \sigma_1^2 &= (2N-2) [1 + 2\gamma + (\rho_{g,1}^2 + 1)\gamma^2] \\ &+ 4N(1 + 2\gamma) + 8(2N-3)\rho_{g,1}\gamma^2 + 8(2N-3)\rho_{g,1}\gamma \\ &+ 4 \sum_{k=1}^{N-2} \left((N-1-k) \text{Cov}_k + (4N-4k-6)\rho_{g,k}\rho_{g,k+1}\gamma^2 \right) \end{aligned} \quad (25)$$

and

$$\text{Cov}_k = \begin{cases} (\rho_{g,2} + \rho_{g,1}^2)\gamma^2 + \rho_{g,2}\gamma, & k = 1, \\ (\rho_{g,k-1}\rho_{g,k+1} + \rho_{g,k}^2)\gamma^2, & \text{otherwise.} \end{cases} \quad (26)$$

The performance of the CorrSum method is compared with ED using analytical expressions and Monte Carlo simulations for exponential correlation with $\rho_{g,1} = 0.5$ and $2N = 320$, and the result is shown in Fig. 1 (a) in the form of receiver operating characteristics (ROC) curve. For useful levels of P_{fa} , the analytical model faithfully approximates the simulation results. However, for very high correlated signals the simulation results indicate somewhat lower detection than the analytical model. This is because the analytical model used here does not consider the effect due to the correlation of the Gaussian signal. Since this work considers different correlation levels of the signal, the simulation result for ED corresponding to the signal under consideration is taken to be the true detection probability for that scenario. Small degradation in the detection performance of ED is, therefore, expected for the signal involving high correlation.

CorrSum method performs well if carrier frequency and phase of the sensed signal are approximately known such that the correlation of $g(t)$ has negligible imaginary part. In practice, there may be significant frequency mismatch between f_c and the signal carrier frequency such that autocorrelation of $g(t)$ has significant imaginary part as opposed to the assumption in [4]. In such scenario this method can exploit the real part of the complex correlation instead of the real correlation itself. For this, the sensed signal (bandpass) should first be shifted by f_c to baseband and real part of its autocorrelation is to be extracted. The real part obtained in this case will be less than that in the real correlation and the performance degrades. Here we discuss briefly the effect of carrier offset. Using the definition in (3), the signal complex envelope in (7) has the autocorrelation given as

$$\begin{aligned} \phi_g(\tau) &= \phi_{g0}(\tau) e^{j2\pi\Delta f\tau} \\ &= \phi_{g0}[k](\nu_k + j\beta_k), \end{aligned} \quad (27)$$

where $\phi_{g0}(\tau)$ is the autocorrelation of $g_0(t)$, $\nu_k = \cos 2\pi\epsilon k$, $\beta_k = \sin 2\pi\epsilon k$, and $\epsilon = \Delta f/W$.

The autocorrelation of the complex envelope $g(t)$ is now complex, with significant information transferred from real part to the imaginary part due to phase rotation. Since the method only exploits real part the expressions in (23)-(26) can be used by replacing $\rho_{g,k}$ by $\nu_k\rho_{g,k}$. Note from (27) that although frequency offset affects the performance of the CorrSum method, phase offset does not. Results for different carrier offsets ϵ as shown in Fig. 1 (b) indicate only modest performance degradation when the error in carrier frequency estimation is 10% or less, which may be practical for realistic systems when partial primary information is known. Unless the sensing is fully blind, the offset can be kept well below 10% and exploitation of real part collects almost all the correlation present in the signal. It suggests that CorrSum method is the appropriate choice for sensing signal with small frequency offset (almost real autocorrelation).

IV. NP DETECTION OF CORRELATED SIGNAL

A. Case I: Perfect Correlation Information (PCI)

When the correlation of the signal under sensing is perfectly known to the sensing node, the optimum detector exploits the

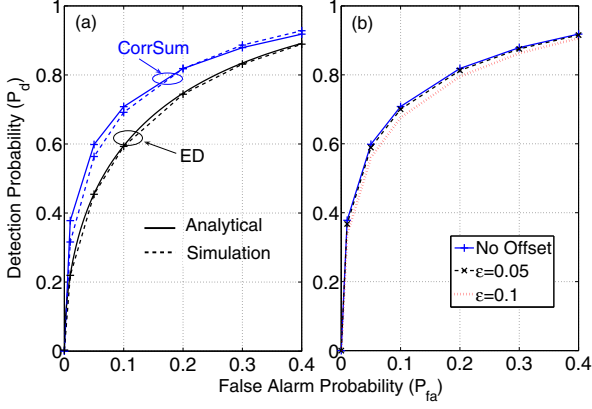


Fig. 1. (a) Performance comparison of CorrSum method with ED with analytical model and Monte Carlo simulation; (b) Effect of carrier frequency offset for CorrSum method

correlation information to maximize the detection probability. We consider here such detector and is referred to as the NP-PCI detector. This assumes an adaptive sensing algorithm that somehow always knows \mathbf{C}_g and σ_w^2 .

In [3] the generalized NP detection of the random signals with arbitrary covariance matrices is discussed which is also considered for our case. For the detection hypotheses in (8) with known covariance matrix \mathbf{C}_g , the detection problem is to distinguish between the hypotheses H_0 (noise-only) and H_1 (signal+noise). The likelihood ratio is

$$\mathcal{L}(\mathbf{x}) = \frac{p(\mathbf{x}; H_1)}{p(\mathbf{x}; H_0)}, \quad (28)$$

where

$$p(\mathbf{x}; H_1) = \frac{1}{\pi^N \det(\mathbf{C}_g + \sigma_w^2 \mathbf{I})} \exp[-\mathbf{x}^H (\mathbf{C}_g + \sigma_w^2 \mathbf{I})^{-1} \mathbf{x}] \quad (29)$$

and

$$p(\mathbf{x}; H_0) = \frac{1}{\pi^N \sigma_w^{2N}} \exp\left(-\frac{1}{\sigma_w^2} \mathbf{x}^H \mathbf{x}\right). \quad (30)$$

The NP detector decides H_1 if

$$\mathcal{L}(\mathbf{x}) > \lambda, \quad (31)$$

where λ is the threshold. Taking logarithms and retaining only the data-dependent terms and simplification as in [3] gives the decision statistic

$$\mathcal{T}(\mathbf{x}) = \mathbf{x}^T \hat{\mathbf{g}} = \sum_{n=0}^{N-1} x[n] \hat{g}[n], \quad (32)$$

where $\hat{\mathbf{g}} = \mathbf{C}_g (\mathbf{C}_g + \sigma_w^2 \mathbf{I})^{-1} \mathbf{x}$. Thus the NP detector correlates the received data with an *estimate* of the signal, i.e., $\hat{g}[n]$. It is therefore termed an *estimator-correlator*. Evaluating P_{fa} and P_d require either Monte-Carlo simulation or numerical evaluation of Fourier-type integrals [3].

It is to be noted that the performance of NP-PCI is a loose upper bound on the performance of correlation-based detectors, where this optimal detector adapts to the immediate correlation state (assumed to be known). This is useful to consider, since it indicates how much performance is lost by only having distribution (as opposed to instantaneous correlation) information.

B. Case II: Correlation Distribution Information (CDI)

Our goal is to derive the performance of a detector that has information on the distribution of the autocorrelation, but not the autocorrelation itself, providing a tighter bound on blind autocorrelation based detectors. The signal covariance matrix \mathbf{C}_g can be normalized as

$$\mathbf{C}_g = \sigma_g^2 \mathbf{C}'_g = \sigma_w^2 \gamma \mathbf{C}'_g. \quad (33)$$

Although in some cases γ may be estimated, for completely blind sensing this may be unrealistic, and degradations due to incorrect SNR estimates must also be considered. To model the random distribution of autocorrelation, \mathbf{C}'_g is obtained from the deterministic function $\mathbf{C}'_g(\Omega)$, where Ω is a vector of random parameters. Many forms of $\mathbf{C}'_g(\Omega)$ could be considered, and in this work we assume an exponentially correlated signal, such that autocorrelation at all lags can be computed from the first lag autocorrelation providing a simple two-parameter $\Omega = [|\rho_{g,1}| \ \angle \rho_{g,1}]$.

For the known distribution of the correlation (CDI), the covariance matrix is to be expressed as a function of Ω and is denoted as $\mathbf{C}_g(\Omega)$. The likelihood ratio will be different from the known correlation (PCI) case since $p(\mathbf{x}; H_1)$ is to be modified as

$$\begin{aligned} p(\mathbf{x}; H_1) &= \int_{\Omega} p[\mathbf{x}; H_1 | \mathbf{C}_g(\Omega)] f(\Omega) d\Omega \\ &= \int_{\Omega} \frac{1}{\pi^N \det[\mathbf{C}_g(\Omega) + \sigma_w^2 \mathbf{I}]} \times \\ &\quad \exp\left[-\mathbf{x}^H (\mathbf{C}_g(\Omega) + \sigma_w^2 \mathbf{I})^{-1} \mathbf{x}\right] f(\Omega) d\Omega, \end{aligned} \quad (34)$$

and $p(\mathbf{x}; H_0)$ remains the same as given by (30).

The covariance $\mathbf{C}_g(\Omega)$ can be expressed in terms of SNR as

$$\mathbf{C}_g(\Omega) = \sigma_g^2 \mathbf{C}'_g(\Omega) = \gamma \sigma_w^2 \mathbf{C}'_g(\Omega), \quad (35)$$

where $\mathbf{C}'_g(\Omega)$ is the normalized covariance. From (35), assuming the SNR is correctly estimated, (34) can be written as

$$\begin{aligned} p(\mathbf{x}; H_1) &= \int_{\Omega} \frac{1}{\pi^N \det[\sigma_w^2 \gamma \mathbf{C}'_g(\Omega) + \sigma_w^2 \mathbf{I}]} \times \\ &\quad \exp\left[-\mathbf{x}^H (\sigma_w^2 \gamma \mathbf{C}'_g(\Omega) + \sigma_w^2 \mathbf{I})^{-1} \mathbf{x}\right] f(\Omega) d\Omega. \end{aligned} \quad (36)$$

Although $p(\mathbf{x}; H_1)$ may not be evaluated in closed form, it can be computed numerically. After $\mathcal{L}(\mathbf{x})$ is computed substituting (36) and (30) in (28), the detection and false alarm probabilities for this statistic are determined as in the PCI case. However, in this case the distribution of $\mathcal{L}(\mathbf{x})$ as obtained from Monte Carlo simulation is used to compute P_{fa} and P_d .

V. RESULTS

This section presents detection performance results based on an SNR of -9 dB and $N = 160$ sensing samples. Below, some important observations are mentioned.

Comparison of CDI and PCI – The performance of NP-PCI and NP-DCI methods for real and complex correlation is given in Figure 2 and 3, respectively. As expected, there is

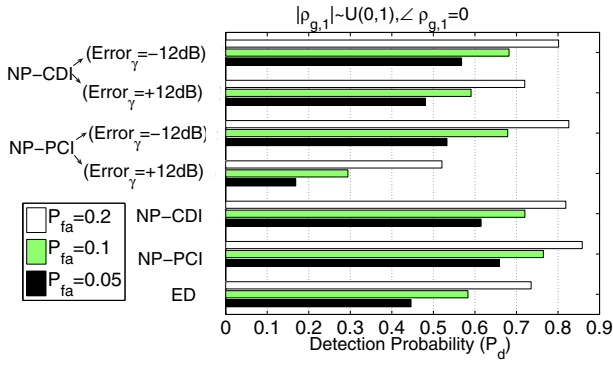


Fig. 2. Detection performance of NP methods for real correlation scenario. The effect of SNR estimation error (Error_γ) is also shown

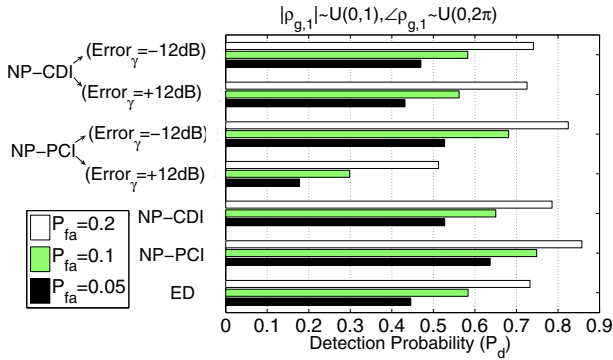


Fig. 3. Detection performance of NP methods in for complex correlation scenario. The effect of SNR estimation error is also shown

a performance degradation in the NP-CDI detector relative to the NP-PCI detector, since less information about the signal distribution is known. However, since the performance of NP-CDI is above both the CorrSum and CAV methods as seen in Figure 4, room for improvement in these two ad-hoc detectors is possible.

NP Detection with SNR Error – When there is error in the estimated SNR, the performance of both NP methods are degraded. In the case of uniformly distributed correlation, negative error (underestimating SNR) has negligible effect, while overestimating SNR can lead to poor performance, especially for NP-PCI. This indicates that a pessimistic estimate of the SNR should be favored. In the case of high correlation, overestimating SNR is favorable for NP-CDI, but at the expense of very poor performance when signals are uncorrelated. Underestimating SNR, both the NP-PCI and NP-CDI detectors have superior performance than ED for low and high correlation as seen in Figure 5.

CAV and CorrSum Methods – As expected, the CAV method in [5] and our CorrSum method are suboptimal compared to NP-CDI. Note that a smoothing factor of $L = 2$ was chosen here, since this was found to give the optimal performance of CAV. The effect of L is studied in detail for CAV (with and without trace normalization) and CorrSum and it was found that for uniform real and complex RCM models as well as fixed correlation of $\rho_{g,1} = 0.5$, or less $L = 2$ is found to be optimal. The effect of L is shown in Figure 6 and 8 for CAV

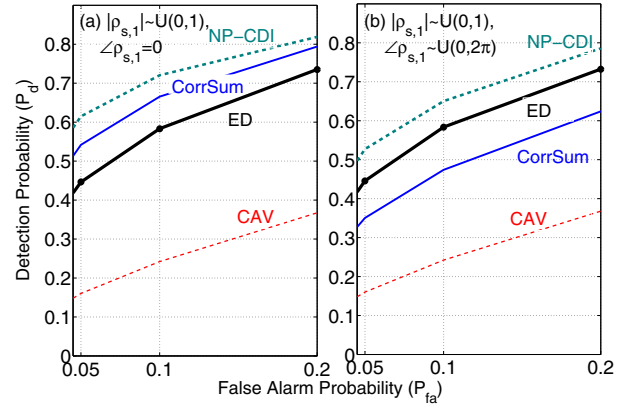


Fig. 4. Comparison of ad-hoc CBD methods for (a) real autocorrelation (guided) and (b) complex autocorrelation (random) sensing

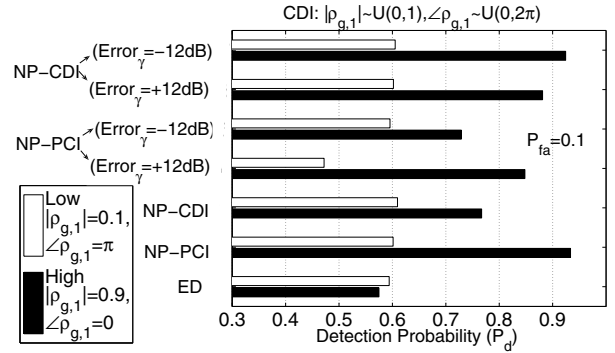


Fig. 5. The Performance of NP-CDI and NP-PCI methods compared with ED for a primary signal having complex correlation with low and high magnitude

without trace normalization and in Figure 7 and 9 for CAV with trace normalization. Larger values of L only help when the correlation of the signal is very high.

For the case of real correlation (center frequency known and signal spectrum symmetric), CorrSum has superior performance to CAV, which is reasonable since the imaginary part of correlation used by CAV contains useless information. CAV without trace normalization performs better than CorrSum for complex correlation (since the imaginary part of the correlation can be exploited in terms of absolute value) but the performance is only marginally better than ED (not plotted here). The performance of CAV with trace normalization is, however, worse than CorrSum (Figure 4) although it is expected to perform better under noise uncertainty [5].

Blind Detection with Complex Correlation – Since the Although autocorrelation does not help for blind sensing using ad hoc methods (CorrSum and CAV), the performance of the NP-CDI detector is significantly better than ED, indicating that correlation is still valuable for increasing detection performance.

Robustness of Correlation-Based Detection – For high correlation, the NP-based methods perform significantly better than ED, even in the presence of SNR error and mismatch of the assumed and true level of correlation. Interestingly, for low correlation, the performance of the methods is still slightly

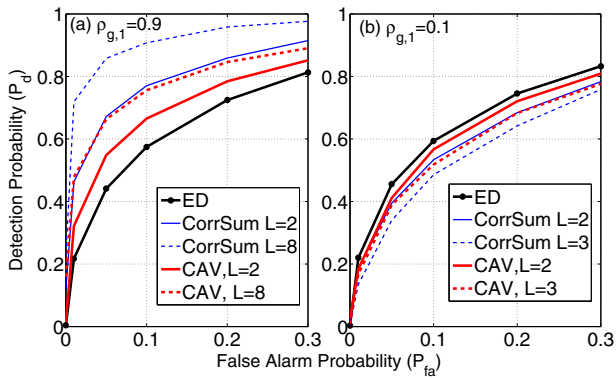


Fig. 6. Performance of ED, CorrSum, and CAV (without trace normalization) for (a) high correlation and (b) low correlation

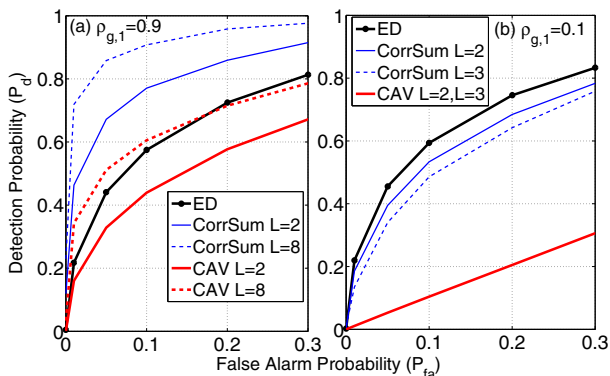


Fig. 7. Performance of ED, CorrSum, and CAV for (a) high correlation and (b) low correlation

higher than ED, with the exception of NP-CDI detection with overestimated SNR.

VI. CONCLUSION AND FUTURE WORK

This work has explored the optimality of correlation-based detectors by developing an NP detector that exploits distribution knowledge of the randomly occurring correlation, as opposed to the correlation itself. This new detector defines an upper bound on the performance of blind correlation detectors, allowing the performance of ad-hoc methods to be classified. Analysis of correlation-based methods indicates that the highest performance improvement is possible when the signal has real correlation. In the case of complex correlation (blind sensing) higher performance is still possible, but only marginal improvements are seen with absolute correlation detection. It was also observed that for unknown SNR, choosing the SNR pessimistically (assuming low SNR) should be preferred for NP-CDI detection. Although the work in this paper assumes a very simple model for the random autocorrelation, in the future we will apply the methods to correlation distributions extracted from spectrum measurements in true urban and sub-urban environments, providing a more realistic view of the potential and limitations of correlation-based detection.

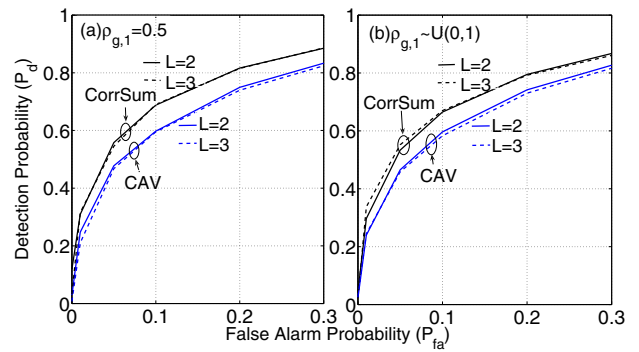


Fig. 8. Comparison of ad-hoc CBD methods for (a) constant correlation and (b) random correlation assuming a simple uniform RCM. CAV without trace normalization is considered

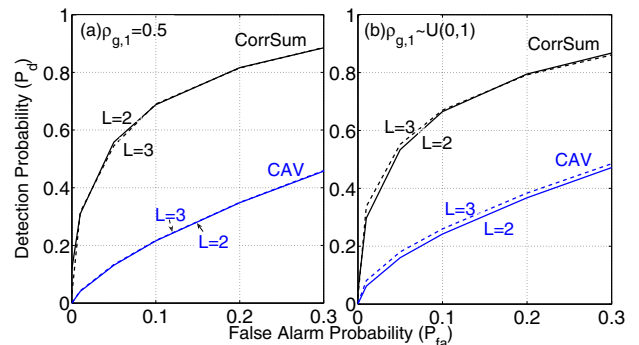


Fig. 9. Comparison of ad-hoc CBD methods for (a) constant correlation and (b) random correlation assuming a simple uniform RCM

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