#### Investigation of Complexity-Constrained Peformance of Planar Reconfigurable Aperture Antennas (RECAPs)

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### Introduction

A reconfigurable aperture (RECAP) antenna [1, 2] is a regular array of reconfigurable elements whose state can be changed dynamically. RECAPs have shown the potential to support many antenna operations (beamsteering, interference suppression, frequency agility, matching etc.) in a single aperture, but the level to which increasing complexity helps in achieving peak performance for a particular application has not been studied in detail. This work investigates the beamsteering performance of a planar RECAP structure in terms of number of reconfigurable ports ( $N_{RP}$ ) and number of possible states per port ( $N_{RS}$ ), demonstrating the dependence of RECAP performance on complexity.

# Simulation of the RECAP Structure

The RECAP structure considered in this paper is a 8×8 planar circular patch array confined to an area of  $1\lambda \times 1\lambda$  as shown in Figure 1. Note that unlike standard patch antennas, here there is no ground plane and the elements are small compared to the operating wavelength. The circular patches have a radius of 0.038 $\lambda$  with a transmission line having a length of 0.056 $\lambda$  and width of 0.015 $\lambda$  as an interconnection between them. The differential feed is approximately at the middle of array. Note that each connecting segment shown in the diagram in Figure 1(a) is a reconfigurable port (RP), meaning a site where a reconfigurable element may be present. Open spaces between circles indicate that the elements are always directly connected with a transmission line. Figure 1(b) shows an actual perspective view of one of the RECAP configurations. Reconfigurable elements at each port can assume one of  $N_{RS}$  reconfigurable states (RSs).

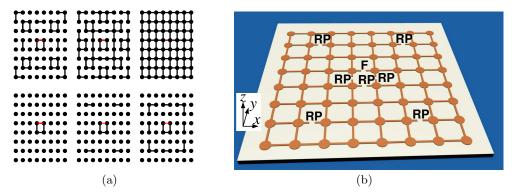


Figure 1: RECAP structure (a) possible layouts for studying limited complexity and (b) a perspective of the actual structure for  $N_{RP} = 8$ , where "F" and "RP" label the feed and reconfigurable ports, respectively

In order to efficiently obtain the input impedance and radiation pattern of the RECAP for many thousands of configurations, a hybrid simulation approach is adopted. First, full-wave simulation is performed using a custom FDTD code, where for each port (location on the structure for the feed or reconfigurable element), an exciting source is placed at that port with all other ports open-circuited. By

computing the induced voltage at all non-excited ports, the Z-parameter matrix of the structure is obtained. Second, efficient network analysis is used to compute the input impedance and synthesized radiation patterns of the array for arbitrary loading as follows. Arranging the impedance matrix such that the feed is Port 1, and Ports 2 through N are RPs, we have  $\mathbf{v} = \mathbf{Z}\mathbf{i}$ , or

$$\begin{bmatrix} v_1 \\ \mathbf{v}_2 \end{bmatrix} = \begin{bmatrix} z_{11} & \mathbf{z}_{12} \\ \mathbf{z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \begin{bmatrix} i_1 \\ \mathbf{i}_2 \end{bmatrix}, \tag{1}$$

where  $v_1$  and  $i_1$  are the scalar voltage and current on the feed,  $\mathbf{v}_2$  and  $\mathbf{i}_2$  are vectors of voltages and currents on the reconfigurable ports, and  $\mathbf{Z}$  has been partitioned appropriately. Terminating port k + 1 with impedance  $z_{L,k}$ ,  $k = 1, \ldots, N - 1$ , we have  $\mathbf{v}_2 = -\mathbf{Z}_L \mathbf{i}_2$ , where  $\mathbf{Z}_L$  is a diagonal matrix with  $Z_{L,kk} = z_{L,k}$ . Combined with (1), we have

$$\mathbf{i}_{2} = -(\mathbf{Z}_{22} + \mathbf{Z}_{L})^{-1} \mathbf{z}_{21} i_{1}, \qquad v_{1} = \underbrace{z_{11} - \mathbf{z}_{12} (\mathbf{Z}_{L} + \mathbf{Z}_{22})^{-1} \mathbf{z}_{21}}_{z_{\text{in}}} i_{1}, \qquad (2)$$

where  $z_{in}$  is the input impedance looking into the feed for the given termination at the RPs. The realized radiation pattern of the array for feed current  $i_1$  is easily found using superposition as

$$\mathbf{E}(\theta,\phi) = \sum_{j=1}^{N} i_j \mathbf{E}_j(\theta,\phi),\tag{3}$$

where currents at the RPs are found with (2). The reflection coefficient looking into the feed will also be considered, given by  $\Gamma = (z_{\rm in} - Z_0)/(z_{\rm in} + Z_0)$ , where  $Z_0$  is a normalizing impedance, which is taken to be the internal impedance of the source driving the antenna.

## Genetic Algorithm

The performance goal considered in this work is to maximize the amount of input power radiated in a sector centered in a specific direction in the elevation (xz) plane, or  $x = \frac{Nz}{Nz}$ 

$$P_{\text{beam}} = \max_{\mathbf{Z}_L} (1 - |\Gamma|^2) \frac{\sum_{n=n_0-N_B/2}^{n_0+N_B/2} |E_{\theta}(\theta_n, \phi = 0^\circ)|^2}{\sum_{n=1}^{N_A} |E_{\theta}(\theta_n, \phi = 0^\circ)|^2},$$
(4)

where elevation is sampled with  $\Delta \theta = 2\pi/N_A$ ,  $\theta_n = n\Delta \theta$ ,  $n_0$  is the desired direction index for the main beam, the main beam occupies indices  $n \in [n_0 - N_B/2, n_0 + N_B/2]$ , and the two-sided beamwidth is  $W = 2N_B\Delta\theta$ . In the simulations that follow, we chose  $N_A = 180$  sample angles, main beams at  $n_0 = 141, 161, 171, 181$ , and beamwidth  $W = 40^\circ$ .

Maximizing (4) with respect to the load states is a non-convex optimization problem, and more investigation is required to find efficient solution methods. Exhaustive searches are not fruitful due to the exponentially increasing search space as complexity increases. Initially, we consider using a genetic algorithm (GA), which appears to find good solutions with modest complexity. The GA operates on a total population size of  $N_P = 500$  individuals, which specify different configurations of the RSs for the RPs. The GA is initialized with  $4N_P$  individuals chosen randomly from the population space, the fitness function is computed for these population vectors, and the best  $N_k = 100$  individuals are retained. This random search process is repeated 10 times, in order to obtain a good starting population for the GA. After this initialization stage, the genetic algorithm proceeds iteratively, where for each iteration the  $N_k$  best individuals of the population are used to generate  $N_P - N_k$ new individuals using crossovers and mutations with different probabilities. After obtaining a population of  $N_P$ , the best  $N_k$  are again extracted and the process repeats. If the GA reaches a point when their is no significant improvement observed in the best individual for 10 iterations, then the best solution is stored, and the GA starts again with a new random population. Figure 2(a) shows the gain pattern when steering the main beam to  $\theta_{n_0} = 161^\circ$  with  $\phi = 0^\circ$  for four different runs of the GA.

## Performance Analysis of RECAP Structure

One important goal of this research is to understand the detailed relationship between RECAP complexity and peak performance, since logically a theoretical bound should exist for a particular application when complexity is limited. We consider a number of different layouts of the RECAPs to study the effect of two kinds of complexity: (1) the number of reconfigurable ports (RPs) where reconfigurable elements may be present, and (2) the number of reconfigurable states (RSs) that each of these elements can assume.

Figure 1(a) depicts six different layouts having 4,8,16,32,64, and 112 reconfigurable ports, respectively, as indicated by the solid connecting lines between circles. Recall that a blank space between circles actually represents a permanent connection between elements. The number of reconfigurable states was varied in powers of 2, having possible values of 2,4,8,16, and 32 states. The impedance of each reconfigurable state is chosen by assuming varactor diodes, whose input reflection coefficient is in  $[-180^{\circ}, 0^{\circ}]$ . Instead of choosing a specific set of quantized capacitance values that the varactors can assume, the set of allowed states is generated randomly once at the beginning of each GA run, avoiding bias due to a poor choice. The GA was performed for each combination of the RP and RS values, where the resulting fitness for each combination was averaged over four different target steering angles  $(\theta_{n_0} = 141^{\circ}, 161^{\circ}, 171^{\circ} \text{ and } 181^{\circ})$  with 15 simulations per angle.

Figure 2(b) shows the average convergence of the GA for increasing  $N_{RS}$  for fixed values of  $N_{RP}$ , as well as the reversed case. Three important regions can be observed in the convergence of the algorithm, each having a characteristic shape. The first part is the initialization phase of GA, where random search is performed. In the second region, a steep rise in fitness function is observed, representing rapid improvement possible with the GA. The last region represents the final convergence of the GA where only modest improvement is obtained. Interestingly, these results suggest that increasing RECAP complexity does not lead to slower convergence of the GA, which might have been expected.

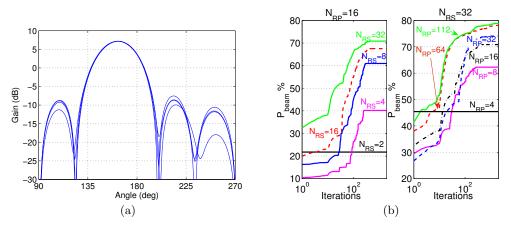


Figure 2: Genetic optimization of the RECAP: (a) Realized gain patterns in xz plane for  $\theta_{n_0} = 161^{\circ}$  with  $N_{RP} = 112, N_{RS} = 32$ , (b) average convergence

Figure 3 shows the average value of percentage power in main beam obtained for each of the various configurations of RPs and RSs. From this figure it is clearly visible that for low complexity ( $N_{RP} \leq 32$ ,  $N_{RS} \leq 16$ ), significant performance improvement is possible by either increasing the number of ports or states. Interestingly, in this same regime, for only 2 states, it appears that increasing to 4 states is better than doubling the number of ports. For a large number of ports  $N_{RP} > 32$  we see that the number of states has very little impact on performance. However, for a large number of states, increasing the number of ports is still beneficial. This suggests that the number of ports has a stronger limiting effect on the performance than the number of reconfigurable states.

# Conclusion

A  $1\lambda \times 1\lambda$  circular planar RECAP structure was analyzed for beamforming performance with respect to constrained complexity, in terms of the number of reconfigurable ports  $N_{RP}$  and allowed reconfigurable states per port  $N_{RS}$ . It was found that diminishing returns of the structure occurred near  $N_{RP} = 64$  reconfigurable ports. Also, increasing the reconfigurable states per port appears to partially compensate for a limited number of ports. Although the structure considered in this work is somewhat idealized, future work will consider more practical RECAP structures that include loss and biasing effects, as well as other applications, such as null-steering and three-dimensional pattern synthesis.

#### References

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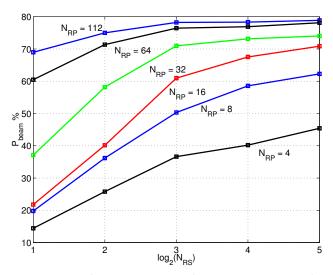


Figure 3: Main beam power with varying  $N_{RP}$  and  $N_{RS}$