# ANALYSIS OF FUSION AND COMBINING FOR WIRELESS SOURCE DETECTION

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## ABSTRACT

In wireless detection scenarios such as sensor networks or cognitive radio, "hidden nodes" can occur due to channel fading, which can be overcome by collaborative sensing. This work considers the performance of collaboration with combining and fusion methods for both static and fading environments, where each node employs energy detection for sensing an unknown source. The case of combined Rayleigh fading and shadowing is also considered, showing that AND fusion can have even lower performance than a single sensor. Our results show that choosing a fusion rule without considering the statistics of the fading environment can lead to very poor performance. It also proposes a new quantized equal gain combining (QEGC) method that strikes a good balance between performance and transmission overhead among the collaborating nodes.

# 1. INTRODUCTION

Source detection is an important component for sensor networks such as surveillance radar, sonar and many civilian applications related to public safety, security, health care, etc. Recently, a new cognitive radio technology has gained significant attention due to its potential to cope with spectrum scarcity [1]. An important component of cognitive radio is primary source detection (sensing) which is aimed at detecting the presence or absence of a primary user in the spectrum of interest. Collaborative sensing has been proposed as a technique to avoid the "hidden node" problem, which arises due to fading [2]. The information from different sensors is combined in order to make a global decision about the presence or absence of the wireless source.

Different combining techniques have been discussed in [3], which include equal gain combining (EGC), selection combining (SC), and switch and stay combining (SSC). Similarly in [4], maximal ratio combining (MRC) has been shown to be the optimal soft combining method whose performance has been compared with equal gain combining. However, for sensing unknown signals, the weighting factor needed for MRC may not be available, due to the lack of channel information. Recent work in [5] indicates that the optimal sensor fusion rule, in the sense of a locally most powerful (LMP) test, is to use EGC for the low SNR regime,

which is a reasonable worst-case assumption when signal and channel characteristics are unknown.

In the presence of small-scale fading, the signal independence required by these soft combining schemes is obtained at sub-wavelength scale separation, allowing easy implementation with nodes having multiple antennas or wired connections. In the case of shadowing, however, fading may be correlated for many wavelengths, requiring nodes to have significant separation for independence, and complicating the transmission of node parameters to the decision center that makes the global decision. In such cases, decision fusion methods can be used which take only the *decisions* of the individual sensors and fuse them to obtain the global decision.

Fusion techniques normally employ a 'k out of n rule' (KNR) where n is the number of sensors. The special cases are OR (k = 1), AND (k = n) or majority gates  $k = \lceil n/2 \rceil$ , where  $\lceil \cdot \rceil$  denotes rounding up to the next integer value. There exist conflicting claims on the superiority of OR fusion and AND fusion [6], [7]. Fefjar in [6] showed the superiority of OR fusion, which was refuted by Stearns in [7], claiming that OR and AND performances intersect, AND being superior for low false alarm probabilities. In [8], the performances of OR and AND fusion are compared, where the decision parameters are assumed to be exponentially distributed with different decay constants for sensors that are not identical, demonstrating that the superiority and intersection all depend upon the global false alarm probabilities as well as the distributions.

As an optimal data fusion technique based on a maximum likelihood ratio test, [9] has proposed a fusion rule in which the decisions 1 or -1 (for existence or absence of the source) are weighted according to sensor reliability, where reliability is a function of the probabilities of false alarm and missed detection. A decision fusion rule with the Neyman-Pearson (N-P) test is described in [10], showing that fusion can provide increased global probability of detection for a fixed or lower global probability of false alarm relative to a single sensor, when more than two sensors are available.

This work compares the performance of different cases of k out of n rules for the decision fusion in unknown source detection employing conventional energy detection at each node. The fusion techniques (hard combining) are also compared with EGC (soft combining), which is the main feasible soft-combining method for an unknown source. Different possible distributions of the energy in individual sensors are considered in static and fading environments. Analytical results are also verified by Monte Carlo simulation for all the cases considered.

The rest of the paper is organized as follows: Section 2 introduces the problem of spectrum sensing, reviews energy detection, and discusses the effect of fading on signal sensing. Section 3 describes different collaborative approaches, such as equal gain combining and fusion methods. Section 4 compares the performance of different combining and fusion techniques in static and fading environments using closed form expressions and Monte Carlo simulations. Conclusion and some future extensions are discussed in Section 5.

## 2. SINGLE SENSOR DETECTION

## 2.1. Energy Detector-Based Sensing

First we review energy detection, being the most common method of spectrum sensing, due to its low computational complexity and ease of implementation [3], [11], [12].

Considering the bandpass noise within a fixed sensing bandwidth to have flat power spectral density (PSD), the noise is represented as

$$\eta(t) = \eta_c(t)\cos 2\pi f_c t - \eta_s(t)\sin 2\pi f_c t, \qquad (1)$$

where  $f_c$  is the reference frequency, and  $\eta_c(t)$  and  $\eta_s(t)$ are the in-phase and quadrature modulation components, respectively. If the bandpass noise in (1) has bandwidth W,  $\eta_c(t)$  and  $\eta_s(t)$  have bandwidth W/2. The variance of  $\eta_c(t)$ ,  $\eta_s(t)$  and  $\eta(t)$  are all equal to the noise power.

The energy in a continuous sample of finite duration T is often approximated as [11]

$$U = \int_{0}^{T} \eta^{2}(t) \ dt \approx \frac{1}{2W} \sum_{i=1}^{TW} (a_{ci}^{2} + a_{si}^{2}) = N_{0} \sum_{i=1}^{TW} (b_{ci}^{2} + b_{si}^{2}),$$
(2)

where  $a_{ci}$  and  $a_{si}$  are the *i*th discrete samples (at a rate of 1/W) of  $\eta_c(t)$  and  $\eta_s(t)$ , respectively,  $b_{ci} = a_{ci}/\sqrt{\sigma_i^2}$ ,  $b_{si} = a_{si}/\sqrt{\sigma_i^2}$ ,  $\sigma_i^2 = Var(a_{ci}) = Var(b_{si}) = 2N_0W$ , and  $N_0$  is a two-sided PSD. Scaling U in (2) by defining  $U' = U/N_0$ ,

$$U' = \sum_{i=1}^{TW} b_{ci}^2 + \sum_{i=1}^{TW} b_{si}^2, \qquad (3)$$

which follows a central chi-square  $(\chi^2)$  distribution with degree of freedom 2TW.

When the signal is present, U' follows non-central chisquare distribution with 2TW degrees of freedom and a non-centrality parameter  $\gamma'$ , which is the ratio of source signal energy to noise PSD (two-sided) [11]. In terms of SNR  $(\gamma)$ , which is the ratio of source signal to noise power,  $\gamma'$  can be expressed as  $2TW\gamma$ . The decision statistic for this detector can be described compactly as

$$U' \sim \begin{cases} \chi^2_{2TW} & \mathsf{H}_0, \\ \chi^2_{2TW}(\gamma') & \mathsf{H}_1, \end{cases}$$
(4)

where hypotheses  $H_0$  and  $H_1$  refer to the absence or presence of the source, respectively. The probability of false alarm  $P_f$  and the probability of correct detection  $P_d$  for this scheme can be calculated as [3]

$$P_{f} = Pr\left(U' > \lambda | \mathsf{H}_{0}\right) = \frac{\Gamma\left(u, \frac{\lambda}{2}\right)}{\Gamma\left(u\right)}$$
(5)

and

$$P_d = Pr\left(U' > \lambda | \mathsf{H}_1\right) = Q_u\left(\sqrt{\gamma'}, \sqrt{\lambda}\right), \qquad (6)$$

where  $\lambda$  is the decision threshold,  $\Gamma(., .)$  is the incomplete gamma function, u = TW, and  $Q_u(., .)$  is the generalized Marcum Q-function. As expected,  $P_f$  is independent of  $\gamma'$ since under H<sub>0</sub> there is no transmission from the source. For longer sensing duration the distributions in both hypotheses can be approximated as Gaussian. The decision statistic can be described as

$$U' \sim \begin{cases} N\left(\mu_n, \sigma_n^2\right) & \mathsf{H}_0, \\ N\left(\mu_{sn}, \sigma_{sn}^2\right) & \mathsf{H}_1, \end{cases}$$
(7)

where

$$\mu_n = 2TW,$$
  

$$\sigma_n^2 = 4TW,$$
  

$$\mu_{sn} = 2TW (1 + \gamma),$$

and

$$\sigma_{sn}^2 = 4TW(1+2\gamma),\tag{8}$$

respectively. For this Gaussian-distributed case,  $P_f$  and  $P_d$  can be calculated as

$$P_f = Pr\left(U' > \lambda | \mathsf{H}_0\right) = \frac{1}{2} \operatorname{erfc}\left[\frac{(\lambda - \mu_n)}{\sigma_n \sqrt{2}}\right]$$
(9)

and

$$P_d = Pr\left(U' > \lambda | \mathsf{H}_1\right) = \frac{1}{2} \operatorname{erfc}\left[\frac{(\lambda - \mu_{sn})}{\sigma_{sn}\sqrt{2}}\right], \quad (10)$$

where  $\lambda$  is the decision threshold and  $\operatorname{erfc}(\cdot)$  is the complimentary error function.

#### 2.2. Sensing in a Fading Channel

In practice, the sensing problem involves both small-scale (multipath) and large-scale (shadow) fading. These two effects are commonly treated as independent processes that combine to produce the overall fading effect [13], and this effect usually degrades the performance of spectrum sensing methods. The hidden node problem in cognitive radio, for example, poses a challenge to spectrum sensing, which happens when the primary source is not detected by a cognitive radio receiver due to fading, but the transmit power from this cognitive radio interferes with a primary receiver.

The distribution of the envelope of received signal in small-scale fading is commonly taken to be Rayleigh distributed, making the received power distribution exponential [14]. The Rayleigh probability density function (pdf) is

$$f_R(r) = \frac{r}{\sigma_r^2} e^{-\frac{r^2}{2\sigma_r^2}}, \quad r \ge 0,$$
 (11)

where  $2\sigma_r^2$  is the average power and r is the faded envelope of the signal.

The slow varying local mean power due to shadowing is commonly assumed to follow the lognormal distribution

$$f_{P_o}(p_o) = \frac{1}{\sqrt{2\pi\sigma p_o}} e^{-\frac{(\ln p_o - \mu)^2}{2\sigma^2}}, \quad p_o \ge 0, \quad (12)$$

where  $\mu$  and  $\sigma^2$  are the mean and variance of the logarithm of local mean power. If the power is expressed in dB, the distribution becomes Gaussian, with the mean and standard deviation reflecting the average power and shadowing variation, both expressed in dB.

The  $P_f$  and  $P_d$  we discussed in 2.1 do not consider the fading effects. If the channel gain is varying due to shadowing or multipath, (6) gives the probability of detection conditioned on the instantaneous SNR  $\gamma'$ . The average probability of detection is obtained by averaging (6) over the fading statistics [2], which is given as

$$P_{d} = \int_{0}^{\infty} Q_{u} \left( \sqrt{\gamma'}, \sqrt{\lambda} \right) f(\gamma') \, d\gamma', \qquad (13)$$

where  $f(\gamma')$  is the pdf of  $\gamma'$  under fading. For a Rayleigh fading channel  $f(\gamma')$  is given as

$$f(\gamma') = \frac{1}{\overline{\gamma'}} e^{-\frac{\gamma'}{\overline{\gamma'}}},$$
(14)

where  $\overline{\gamma'}$  is the average value of  $\gamma'$ . Letting  $x = \sqrt{\gamma'}$ , we obtain

$$P_d = \frac{2}{\overline{\gamma'}} \int_0^\infty Q_u \left( x, \sqrt{\lambda} \right) x \, e^{-\frac{x^2}{\overline{\gamma'}}} \, dx. \tag{15}$$

This can be derived in closed form, using (12) from [15], as

$$P_{d} = e^{-\frac{\lambda}{2}} \sum_{m=0}^{u-2} \frac{1}{m!} \left(\frac{\lambda}{2}\right)^{m} + \left(\frac{2+\overline{\gamma'}}{\overline{\gamma'}}\right)^{u-1} \left[e^{-\frac{\lambda}{2+\gamma'}} - e^{-\frac{\lambda}{2}} \sum_{m=0}^{u-2} \frac{1}{m!} \left(\frac{\lambda\overline{\gamma'}}{2\left(2+\overline{\gamma'}\right)}\right)^{m}\right].$$
(16)

A similar result is also obtained in [2] and [3]. However, note that due to our definition of SNR (a two-sided PSD is used), the expression is slightly modified. Also note that [3] appears to have a missing exponent, which is corrected in [2] and in (16). The analytical expression for  $P_d$  in the combined fading environment, consisting of both Rayleigh fading and lognormal shadowing, is not yet available in the literature, and simulation studies on the effect of this combined fading is an important contribution of this work, providing additional insight on the sensing problem for realistic environments.

# 3. MULTIPLE SENSOR DETECTION

#### 3.1. Fusion (Hard Combining Methods)

Fusion techniques normally employ k out of n rule (KNR), where n is the number of sensors. In this fusion rule the global decision is 1 if any k or more sensor has output 1, where the decisions 1 or -1 refer to presence or absence of the source. Therefore, KNR fusion is represented mathematically as

$$\mathcal{F}(u_1,\dots,u_n) \sim \begin{cases} 1, & \sum_{i=1}^n u_i \ge 2k-n, \\ -1, & \text{otherwise.} \end{cases}$$
(17)

For OR fusion, at least one output must be 1 to have the global decision 1, so the decision rule is given as

$$\mathcal{F}(u_1, \cdots, u_n) \sim \begin{cases} 1, & \sum_{i=1}^n u_i \ge 2 - n, \\ -1, & \text{otherwise.} \end{cases}$$
(18)

Similarly, in AND fusion the decision is 1 only if all sensors have output 1, or

$$\mathcal{F}(u_1,\dots,u_n) \sim \begin{cases} 1, & \sum_{i=1}^n u_i = n, \\ -1, & \text{otherwise.} \end{cases}$$
(19)

It is obvious that OR and AND cases are the special cases of KNR with k = 1 and k = n, respectively. In the majority gate rule, k is taken to be  $\lceil n/2 \rceil$ .

$$\mathcal{F}(u_1, \cdots, u_n) \sim \begin{cases} 1, & \sum_{i=1}^n u_i \ge 0, \\ -1, & \text{otherwise.} \end{cases}$$
(20)

The decision fusion rules discussed above do not consider the distribution of the decision statistics, the individual  $P_f$ and  $P_d$ , nor the global desired false alarm probability  $P_{f,T}$ when making the global decision. The global receiver operating characteristics (ROC) obtained from theses rules may not be optimal for all values of  $P_{f,T}$ . The optimal data fusion rule given in [9] is

$$\mathcal{F}(u_1, \cdots, u_n) \sim \begin{cases} 1, & a_0 + \sum_{i=1}^n a_i u_i > 0, \\ -1, & \text{otherwise,} \end{cases}$$
(21)

where the optimum weights are given by

$$a_{0} = \log \frac{P_{1}}{P_{0}}$$
(22a)  
$$a_{i} = \begin{cases} \log \frac{P_{d,i}}{P_{f,i}}, & u_{i} = +1, \\ \log \frac{1 - P_{f,i}}{1 - P_{d,i}}, & u_{i} = -1, \end{cases}$$
(22b)

where  $P_0$  and  $P_1$  are the a-priori probabilities of the two hypotheses. If the a priori probabilities are unknown, then the optimal decision scheme should be based on an N-P test which has been discussed in [10], obtaining a similar result as that in (17), except for the  $a_0$  term. This optimal data fusion rule is different from the majority gate rule in the sense that the value of k here changes according to the  $P_f$ and  $P_d$  pairs under consideration. The global false alarm and detection probabilities with KNR are given as

$$P_{f,T} = \sum_{i=k}^{n} {n \choose i} P_{f}^{i} \left(1 - P_{f}\right)^{n-i}, \qquad (23)$$

and

$$P_{d,T} = \sum_{i=k}^{n} {n \choose i} P_d^i (1 - P_d)^{n-i}, \qquad (24)$$

respectively, where  $\binom{n}{i} = \frac{n!}{(n-i)!i!}$  is the number of combinations of the sensor outputs.

# **3.2.** Decision Parameter Combining (Soft Combining) Methods

Several combining techniques for iid Rayleigh environments are developed in [3], where for the case of equal gain combining, the sum of Rayleigh-distributed random variables is modeled as a Nakagami distribution with suitable parameter values. The important results are also derived here, which is necessary due to our specific definition of SNR. Also, since the threshold after combining should clearly change, we account for this in our derivation.

When equal gain combining is used with n independent sensors, the energy is increased as  $\gamma'_T = \sum_{i=1}^n \gamma'_i$ . Here the concept of equal gain combining is different than that used in multiple antenna diversity schemes. The decision parameter (energy in our case) of the individual nodes is simply added here instead of co-phasing and adding the signals from different antennas in conventional diversity. Diversitylike maximum ratio or equal gain combining is usually not possible for unknown source detection since SNRs are typically very low and there is no cooperation from the source.

Considering energy detection, the total output energy of the combiner again has a non-central chi-square distribution with degree of freedom 2nTW and non-centrality parameter  $\gamma'_T$ . Similarly when no signal is present, the combiner output has a central chi-square distribution with degree of freedom 2nTW. The  $P_{f,T}$  and  $P_{d,T}$  at the equal gain combiner output for the AWGN channel can be evaluated in a manner analogous to (5) and (6) as

$$P_{f,T} = Pr\left(U_T' > \lambda_T | \mathsf{H}_0\right) = \frac{\Gamma\left(nu, \frac{\lambda_T}{2}\right)}{\Gamma\left(nu\right)}$$
(25)

and

$$P_{d,T} = Pr\left(U_T' > \lambda_T | \mathsf{H}_1\right) = Q_{nu}\left(\sqrt{\gamma_T'}, \sqrt{\lambda_T}\right), \quad (26)$$

where  $\lambda_T$  is the decision threshold of the combiner. In the Rayleigh fading channel, the distribution of  $\gamma'_T$  is that of the sum of *n* independent exponentially distributed random variables, each having a mean of  $\overline{\gamma'}$ , which is represented as

$$f(\gamma_T') = \gamma_T'^{n-1} \frac{\frac{-\gamma_T}{\overline{\gamma'}}}{\Gamma(n)\overline{\gamma'}^n},$$
(27)

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where  $\Gamma(n) = (n-1)!$  for integer *n*. The average  $P_{d,T}$  for Rayleigh fading can then be obtained as

$$P_{d,T,Ray} = \int_0^\infty Q_{nu} \left( \sqrt{\gamma_T'}, \sqrt{\lambda_T} \right) f\left(\gamma_T'\right) d\gamma_T'.$$
(28)

Changing the variable  $x = \sqrt{\gamma'_T}$ , we obtain

$$P_{d,T,Ray} = \frac{2}{\Gamma(n)\,\overline{\gamma'}^n} \int_0^\infty Q_{nu}\left(x,\sqrt{\lambda_T}\right) x^{2n-1} e^{-\frac{x^2}{\overline{\gamma'}}} dx$$
$$= \alpha \left[ \Psi_1 + \beta \sum_{i=1}^{nu-1} \frac{\left(\frac{\lambda_T}{2}\right)^i}{2 \times i!} \, _1F_1\left(n;i+1;\frac{\lambda_T}{2}\frac{\overline{\gamma'}}{2+\overline{\gamma'}}\right) \right],$$
(29)

where  $\alpha = 2/[\Gamma(n)\overline{\gamma'}^n]$ ,  $_1F_1$  is the confluent hypergeometric function defined as

$${}_{1}F_{1}(a;b;z) = \sum_{i=1}^{\infty} \frac{(a)_{i} z^{i}}{(b)_{i} i!}$$
(30)

with  $(a)_i$  given as  $(a)_i = a (a + 1) (a + 2) \dots (a + i - 1)$ ,  $\beta = \Gamma(n) \left(\frac{2\overline{\gamma'}}{2 + \overline{\gamma'}}\right)^n e^{-\frac{\lambda_T}{2}}$ , and  $r^2$ 

$$\Psi_1 = \int_0^\infty Q_1\left(x, \sqrt{\lambda_T}\right) x^{2n-1} e^{-\frac{\alpha}{\overline{\gamma'}}} dx, \text{ where } Q_1\left(\cdot, \cdot\right)$$
  
is the first order Marcum Q-function.  $\Psi_1$  can be evaluated



Figure 1: Computation of parameters for two-bit energy quantization using the energy distribution of noise

using [15] as

$$\Psi_{1} = \frac{(n-1)!\overline{\gamma'}^{n+1}}{2(2+\overline{\gamma'})} e^{-\frac{\lambda_{T}}{2+\overline{\gamma'}}} \times \left(\sum_{k=0}^{n-2} \left(\frac{2}{2+\overline{\gamma'}}\right)^{k} L_{k} \left(-\frac{\lambda_{T}}{2} \frac{\overline{\gamma'}}{2+\overline{\gamma'}}\right) + \frac{2^{n}}{\overline{\gamma'}} \left(2+\overline{\gamma'}\right)^{2-n} L_{n-1} \left(-\frac{\lambda_{T}}{2} \frac{\overline{\gamma'}}{2+\overline{\gamma'}}\right)\right), \quad (31)$$

where  $L_k(x)$  is known as the Laguerre polynomial of degree k defined as

$$L_k(x) = \frac{e^x}{k!} \frac{d^k}{dx^k} \left(\frac{e^{-x}}{x^k}\right).$$
(32)

The values of  $L_0(x)$  and  $L_1(x)$  are 1 and -x + 1, respectively. Using these values, the value of  $L_k(x)$  can be computed using the recursive formula [16]

$$L_{k+1}(x) = \frac{1}{k+1} \left[ (2k+1-x) L_k(x) - kL_{k-1}(x) \right].$$
(33)

# **3.3.** Decision Parameter Combining with Quantization (QEGC) Method

To improve the performance of fusion techniques without large investment in the cooperating bandwidth, an equal gain combining scheme with quantized decision parameters is proposed. Here the energy of each sensor is quantized using four levels as depicted in Figure 1, where two levels are used above and below the threshold. The identical quantization levels are known by all sensors as they are chosen based only on the noise distribution.

After choosing a suitable maximum energy level  $U'_{\rm max}$ , below which most of the probability falls (for the noise-only case), the midpoints below and above the threshold are computed as  $U'_1 = \lambda/2$  and  $U'_3 = (U'_{\rm max} + \lambda)/2$ . When a node



Figure 2: ROC curves comparing the analytical and simulated results in different scenarios

observes the energy on the interval  $[U'_i, U'_{i+1}]$ , the two-bit binary representation of *i* is transmitted, which is decoded as the quantized level  $\overline{U}'_i = (U'_i + U'_{i+1})/2$  at the combiner. As with EGC, the combiner adds these quantized values from all nodes and compares to a threshold. Due to the combinations of the four levels with different sensors the output of the combiner has a distribution having a larger number of possible outcomes, which can improve performance significantly compared to one bit fusion.

### 4. RESULTS

The analytical performance of energy detection is compared to Monte Carlo simulations for single sensor and 2-sensor parameter combining (EGC) for static and Rayleigh fading channels. The results are shown in Figure 2 in the form ROC curves. It is seen that the analytical model and simulations are in excellent agreement. It is interesting that when  $P_f$  tends to zero, the  $P_d$  for Rayleigh fading is better than in the case without fading. The reason is that for a given SNR, no signal is received above threshold in the static environment, but due to occasional constructive Rayleigh fading, there are still some events where the received signal is above the threshold.

In the following results, the performance of the various sensing methods is obtained both from closed-form expressions (when available) as well as Monte-Carlo simulations. Since the results are virtually identical in all cases, only the analytical results are plotted, except for combined fading, where no analytical expression is available. Since it is speculated that the relative performance of the methods will change as the distribution changes, various distributions of the signal energy are considered, ranging from 20 (low) to 160 (sufficiently high) degrees of freedom, having chisquare and approximately Gaussian distributions, respectively. Note that the degree of freedom depends directly on the sensing time available for a given application.

In all of the environments, the performance of EGC is found to be the highest. For a 2-sensor system in a static environment, OR and AND intersect each other with OR performing better for high  $P_{f,T}$  values and AND performing better for lower  $P_{f,T}$  (Figure 3). For a higher number of sensors, the performance of OR and AND show the same behavior (Figures 4 and 5), which becomes more pronounced for higher degrees of freedom (Gaussian approximation) as seen in Figure 6. Among the fusion techniques with more than 2 sensors, the performance of majority rule is the best for both the chi-square and Gaussian distributed cases (Figures 4, 5 and 6). For the similar sensors, the optimal (Chair) rule proposed in [9] ensures performance improvement compared to a single sensor giving the best performance in particular value of  $P_{f,T}$ , but it does not give the best performance for all global false alarm probability  $(P_{f,T})$  values.

An interesting outcome of this study is that the detection performances are quite different for Rayleigh fading compared to the static case, which results from the difference in the distributions of the energy. In Rayleigh fading, OR fusion gives the best performance for almost all  $P_{f,T}$  values, whereas AND fusion performs the worst (Figures 7, 8, and 9). Moreover, ROCs of AND and OR never intersect in Rayleigh fading, OR being far better than AND for all  $P_{f,T}$ values. Also, in Rayleigh fading the performance of OR fusion is the best of all fusion rules and this approaches that of EGC when sensors are decreased to two. At the same time the AND fusion loses its performance and does not help at all when sensors are reduced to two.

The poor performance of AND fusion is higher when there is shadowing in addition to Rayleigh fading. This is apparent from Figure 10 where the collaboration with AND fusion degrades the performance rather than improving.

These results suggest that it is inappropriate to choose any fusion method without knowledge of the distribution of the decision parameter (energy in our case) in the collaborating sensors and the desired global false alarm probability  $P_{f,T}$ .

The performance improvement to fusion by implementing EGC with 2-bit quantization is shown in Figures 11 and 12, where  $U'_{\text{max}} = 45$  was chosen such that  $Pr(U' > U'_{\text{max}}) = 0.001$ . This simple choice of the quantization regions gives performance much closer to optimal EGC than the best performing fusion rule. This result is encouraging given the simplicity of the method, and further gains are likely to be possible by optimizing the quantization regions. Note that for three sensors, the same false alarm probability was obtained for more than one threshold, and the threshold giving larger detection probability must be chosen.

# 5. CONCLUSIONS

This work presented different collaboration techniques for signal sensing in fading environments. Comparisons were



Figure 3: Performance of EGC and fusion techniques for low degree of freedom with 2-sensors



Figure 4: Performance of EGC and fusion techniques for low degree of freedom with 4-sensors



Figure 5: Performance of EGC and fusion techniques for higher degree of freedom



Figure 6: Performance of EGC and fusion techniques for sufficiently high degree of freedom (Gaussian distribution for energy)



Figure 7: Performance of EGC and fusion techniques in a Rayleigh fading environment with 4-sensors and low degree of freedom



Figure 8: Performance of EGC and fusion techniques in a Rayleigh fading environment with 2-sensors and higher degree of freedom



Figure 9: Performance of EGC and fusion techniques in Rayleigh fading environment with 8-sensors



Figure 10: Performance of EGC and fusion techniques in a combined fading environment ( $\sigma = 5 \text{ dB}$ ) with 2-sensors



Figure 11: Comparison of the performance of 2-bit quantized EGC with other combining and fusion methods for 2sensor collaboration



Figure 12: Comparison of the performance of 2-bit quantized EGC with other combining and fusion methods for 3sensor collaboration

made of the various techniques for both analytical and simulation results. The results indicated that the superiority of a particular fusion method fully depends upon the distribution of the decision parameter at the sensors. Since this distribution in turn depends on the fading environment, a method that is superior for one environment may actually be inferior for a different environment. It appears that in general, AND fusion does not help in a Rayleigh fading environment. As the number of sensors is reduced to two, AND fusion does not give any performance gain, whereas OR fusion performs the best, approaching equal gain combining. Since OR fusion is simpler than EGC, it should be preferred for Rayleigh fading environments with two sensors. This result is even more pronounced for combined fading with increasing levels of lognormal shadowing. Finally, the fusion performance is significantly improved using a quantized EGC method.

In future work, we plan to investigate extending the fusion methods to find optimal performance with small increase in the data overhead. We also plan to study the performance of the these methods with more realistic pathbased propagation channel models and actual measured channel data.

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