Improved Spectrum Sensing by Utilizing Signal Autocorrelation

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***Abstract*—A method that exploits the autocorrelation of transmission signals for primary user detection in cognitive radio is proposed. A closed-form analysis of the method is presented and compared with Monte Carlo simulations. Analytical and simulation results indicate that the method provides significant improvement in the probability of detection for moderate SNR scenarios where fast sensing is required. Simulation results indicate the better performance in fading environment and collaborative sensing as well.**

I. Introduction

Cognitive radio was suggested in [1] to realize a flexible and efficient usage of spectrum. An important component of cognitive radio is spectrum sensing to detect the presence or absence of a primary (licensed) user in the spectrum of interest. Some existing spectrum sensing techniques are matched filtering, waveform-based sensing [2], cyclostationarity-based sensing [3], [4], and energy detection [5]- [8]. Matched filtering and waveform-based sensing can only be used for known signals, which may be unrealistic for cognitive radio. Energy detection is perhaps the most common method for detecting unknown signals. Recently, cyclostationarity-based sensing has gained significant attention due to better immunity to noise uncertainty, especially at very low SNR [4]. Interestingly, at moderate SNR values, energy detection outperforms the cyclostationary method, suggesting that energy detection should be used for fast sensing above -18 dB SNR [4].

In this paper we propose a new autocorrelation-based method (CorrSum method) and compare its performance with energy detection. The benefit of the new method lies in its ability to exploit the difference between the signal spectrum and noise spectrum over the sensing bandwidth, which arises due to higher autocorrelation of the signal. The use of practical modulation schemes and the existence of RF channel guard bands tend to increase signal correlation. Also, certain primary systems are allocated much more bandwidth than the bandwidth they use to transmit symbols (e.g. radar), making the signal spectrum significantly different from flat spectrum of noise. Finally, partially occupied bands (e.g. in multichannel systems) exhibit large deviation in spectrum from that of noise. If the new CorrSum method is employed in such scenarios, significant performance improvement in sensing compared to energy detection can be achieved.

The rest of the paper is organized as follows: Section II introduces the problem of spectrum sensing and reviews energy detection method. Section III describes the proposed autocorrelation-based method (CorrSum method) for spectrum sensing along with closed-form analysis of its performance. Section IV gives some results comparing the CorrSum method with the energy detection method. Conclusion and some further works are discussed in Section V.

II. Spectrum Sensing for Cognitive Radio

A. Energy Detector-Based Sensing

First we review energy detection method since it is the most common method of sensing due to low computational complexity and ease of implementation [5]- [7].

The bandpass noise can be represented as

$$n\left(t\right) = n_c\left(t\right)\cos 2\pi f_c t - n_s\left(t\right)\sin 2\pi f_c t, \tag{1}$$

where f_c is the reference frequency and $n_c\left(t\right)$ and $n_s\left(t\right)$ are the in-phase and quadrature modulation components, respectively. If the bandpass noise in (1) has bandwidth W, $n_c\left(t\right)$ and $n_s\left(t\right)$ have bandwidth $W/2$. The variance of $n_c\left(t\right)$, $n_s\left(t\right)$ and $n\left(t\right)$ are all equal to the noise power.

The energy in a sample of finite duration T is often approximated as [5]

$$U = \int_0^T n^2\left(t\right)\,dt \approx \frac{1}{2W}\sum_{i=1}^{TW}(a_{ci}^2 + a_{si}^2) = N_0\sum_{i=1}^{TW}(b_{ci}^2 + b_{si}^2), \tag{2}$$

where a_{ci} and a_{si} are the ith samples (taken at an interval of $1/W$) of $n_c\left(t\right)$ and $n_s\left(t\right)$, respectively, $b_{ci} = a_{ci}/\sqrt{\sigma_i^2}$, $b_{si} = a_{si}/\sqrt{\sigma_i^2}$, $\sigma_i^2 = Var(a_{ci}) = Var(b_{si}) = 2N_0W$, and N_0 is a two-sided power spectral density (PSD). Scaling U in (2) by defining $U' = U/N_0$, we get

$$U' \approx \sum_{i=1}^{TW} b_{ci}^2 + \sum_{i=1}^{TW} b_{si}^2, \tag{3}$$

which follows a central chi-square $\left(\chi^2\right)$ distribution with degree of freedom $2TW$.

In the case of signal being present, the decision metric U' follows non-central chi-square $\left(\chi^2\right)$ distribution with $2TW$ degrees of freedom and a non-centrality parameter γ', where γ' is the ratio of primary signal energy to noise PSD (two-sided) [5]. Defining SNR (γ) as the ratio of primary signal to noise power, γ' becomes $2TW\gamma$. The decision statistic can be described compactly as

$$U' \sim \begin{cases} \chi^2_{2TW} & \mathsf{H}_0 \\ \chi^2_{2TW}\left(\gamma'\right) & \mathsf{H}_1. \end{cases} \tag{4}$$

The probability of false alarm P_f and the probability of correct detection P_d for this scheme can be calculated as [7]

$$P_f = Pr\left(U' > \lambda_{U'}|\mathsf{H}_0\right) = \frac{\Gamma\left(u, \frac{\lambda_{U'}}{2}\right)}{\Gamma\left(u\right)} \tag{5}$$

and

$$P_d = Pr\left(U' > \lambda_{U'}|\mathsf{H}_1\right) = Q_u\left(\sqrt{\gamma'}, \sqrt{\lambda_{U'}}\right), \tag{6}$$

where $\lambda_{U'}$ is the decision threshold, $\Gamma(. \, , \, .)$ is the incomplete gamma function, $u = TW$, and $Q_u(. \, , \, .)$ is the generalized Marcum Q-function. As expected, P_f is independent of γ' since under H_0 there is no primary signal present.

III. Proposed CorrSum Method

Instead of considering only energy for signal detection, we should exploit any properties which exist in the signal but not in the noise, and one notable property is the autocorrelation. The PSD of the bandpass noise is related to its autocorrelation function as

$$\operatorname{sinc}(W\tau) \quad \cos 2\pi f_c \tau \longleftrightarrow \frac{1}{2W}\left[\operatorname{rect}\left(\frac{f-f_c}{W}\right)+\operatorname{rect}\left(\frac{f+f_c}{W}\right)\right]. \quad (7)$$

The left hand side of (7) is the autocorrelation function of the bandpass noise, which is plotted in Fig. 1 where the x-axis is the time lag normalized by multiplying with W. Fig. 1 indicates that the envelope of the autocorrelation

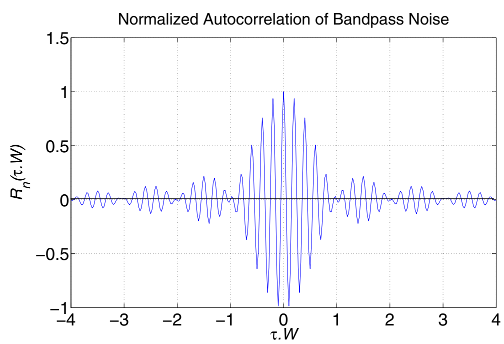

Fig. 1. The autocorrelation of bandpass noise with filter bandwidth 20 MHz

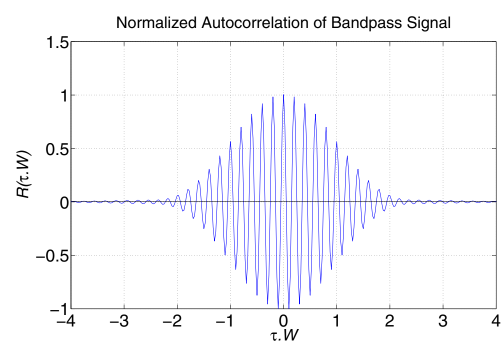

Fig. 2. The autocorrelation of the bandpass QPSK signal (10 MS/s) with filter bandwidth 20 MHz

$\operatorname{sinc}(Wt)$ has a first null at $\tau = 1/W$. The envelope of the signal autocorrelation deviates from this depending on the symbol rate, modulation, and pulse shape. As an example, Fig. 2 depicts the autocorrelation of a QPSK signal with symbol rate 10 MS/s occupying 20 MHz bandwidth. The CorrSum method exploits this difference by integrating the autocorrelation envelope from $\tau = 0$ to $\tau = 1/W$.

The autocorrelation function of the noise component $n_c(t)$ over duration T is defined as

$$R_c(\tau) = \int_0^{T-\tau} n_c(t)\, n_c(t+\tau)\; dt. \quad (8)$$

Note, the integration only extends to $T-\tau$ since $n_c(t)$ has a limited observation window of time T. In discrete time this is approximately

$$R_{ck} \approx \frac{1}{W}\sum_{i=1}^{TW-k} a_{ci}a_{c(i+k)}, \quad (9)$$

where k is the lag in samples which satisfies $k = W\tau$. The results are same for $n_s(t)$ by replacing 'c' with 's'. Since $n_c(t)$ and $n_s(t)$ are sampled at the rate of W samples/s, the samples are uncorrelated. Using the sampling theorem, as in the case of energy, the discrete envelope of the autocorrelation function of the bandpass noise can be represented as

$$\begin{aligned}\widehat{R}_k &= \frac{1}{2}\left[R_{ck}+R_{sk}\right] \\ &= \frac{1}{2W}\sum_{i=1}^{TW-k}\left[a_{ci}a_{c(i+k)}+a_{si}a_{s(i+k)}\right]. \quad (10)\end{aligned}$$

The CorrSum method exploits the autocorrelation at the time lag of $\tau = 1/W$ corresponding to $k = 1$ along with the energy which is the autocorrelation at $\tau = 0\,(k=0)$, which is done by integrating the envelope of the autocorrelation of the received signal to the first null of the autocorrelation of bandpass noise, or

$$\Xi = \int_0^{1/W} \widehat{R}(\tau)\; d\tau \approx \frac{1}{2W}\left[\widehat{R}_0+\widehat{R}_1\right]. \quad (11)$$

From (10) and (11), and simplification analogous to (2), we get

$$\Xi \approx \frac{N_0}{2W}\left(\sum_{i=1}^{TW}\left[b_{ci}^2+b_{si}^2\right]+\sum_{i=1}^{TW-1}\left[b_{ci}b_{c(i+1)}+b_{si}b_{s(i+1)}\right]\right), \quad (12)$$

where $b_{ci}, b_{si}, b_{c(i+1)}$ and $b_{s(i+1)}$ are all zero mean Gaussian random variables with unit variance.

Defining $\varpi = 2W\,(\Xi/N_0)$,

$$\varpi = \frac{2W}{N_0}\int_0^{1/W} \widehat{R}(\tau)\; d\tau \quad (13a)$$

$$\begin{aligned}&\approx \sum_{i=1}^{TW}\left[b_{ci}^2+b_{si}^2\right]+\sum_{i=1}^{TW-1}\left[b_{ci}b_{c(i+1)}+b_{si}b_{s(i+1)}\right] \\ &= \widehat{R}'_0+\widehat{R}'_1, \quad (13b)\end{aligned}$$

which is taken as the parameter for the decision statistic. Since it is difficult to extract $\widehat{R}'_1$ in (13b) from the received autocorrelation function (due to the unknown carrier), real time implementation should be done based on (13a) which does not require frequency or phase information of the carrier. The closed form analysis, however, is discussed based on (13b). The block diagram of CorrSum method is shown in Fig. 3.

A. Computation of Detection and False Alarm Probabilities for CorrSum Method

For two zero mean Gaussian distributed random variables X and Y with variances σ_x^2 and σ_y^2 the distribution of the product is given as [9]

$$P_{XY}(z) = \frac{\mathrm{K}_0\left(\dfrac{|z|}{\sigma_x\sigma_y}\right)}{\pi\sigma_x\sigma_y}, \quad (14)$$

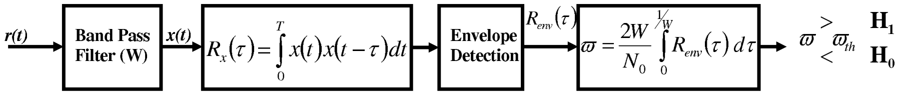

Fig. 3. Block diagram of CorrSum method

where $\mathrm{K}_0(\cdot)$ is the zeroth order modified Bessel function of the second kind. In our case the variances are both unity, so

$$P_{b_i b_{i+1}}(z)=\frac{\mathrm{K}_0(|z|)}{\pi}, \tag{15}$$

also having zero mean and unit variance.

To estimate $\widehat{R}_1'$ for sensing duration T, we need to add the $2(TW-1)$ products of the random noise samples. The distribution of $\widehat{R}_1'$ in (13b) is difficult to calculate in closed form. However, when $2(TW-1)$ is sufficiently large, the distribution is well approximated as Gaussian, requiring only mean and variance of the decision statistic.

For noise, we have $E[\widehat{R}_0']=E[U']=2TW$, and $E[\widehat{R}_1']=0$. So, under hypothesis H_0, from (13b)

$$E[\varpi]=E\left[\widehat{R}_0'+\widehat{R}_1'\right]=E\left[\widehat{R}_0'\right]+E\left[\widehat{R}_1'\right]=2TW. \tag{16}$$

Similarly, for variance in the noise only case, $Var[\widehat{R}_0']=Var[U']=4TW$ and $Var[\widehat{R}_1']=2(TW-1)$, which gives

$$\begin{aligned}Var[\varpi]&=Var\left[\widehat{R}_0'+\widehat{R}_1'\right]\\&=Var\left[\widehat{R}_0'\right]+Var\left[\widehat{R}_1'\right]+2Cov\left[\widehat{R}_0',\widehat{R}_1'\right] &\text{(17a)}\\&=4TW+2(TW-1), &\text{(17b)}\end{aligned}$$

where the last equality comes from the fact that the covariance $Cov[\widehat{R}_0',\widehat{R}_1']$ in (17a) is found to be zero.

Let us now consider the case when signal is present, which is represented in a similar form as the noise in (1), or

$$g(t)=g_c(t)\cos 2\pi f_c t-g_s(t)\sin 2\pi f_c t. \tag{18}$$

We can assume that the in-phase and quadrature components $g_c(t)$ and $g_s(t)$ have variances equal to the signal power, which is generally true in most of the modulations. Let the ith sample of $g_c(t)$ and $g_s(t)$ be denoted as v_i. If we scale the sample as $u_i=v_i/\sigma_i$, then u_i and u_{i+1} have average power P_u equal to the signal-to-noise ratio γ. In this case, ϖ becomes

$$\begin{aligned}\varpi=&\sum_{i=1}^{TW}\left([b_{ci}+u_{ci}]^2+[b_{si}+u_{si}]^2\right)+\\&\sum_{i=1}^{TW-1}\Big([b_{ci}+u_{ci}]\left[b_{c(i+1)}+u_{c(i+1)}\right]+\\&\qquad [b_{si}+u_{si}]\left[b_{s(i+1)}+u_{s(i+1)}\right]\Big)\\=&\widehat{R}_0'+\widehat{R}_1'.\end{aligned} \tag{19}$$

We can easily find that

$$E\left[\widehat{R}_0'\right]=E[U']=2TW(1+\gamma) \tag{20}$$

and

$$\begin{aligned}E\left[\widehat{R}_1'\right]&=E\Bigg(\sum_{i=1}^{TW-1}\Big([b_{ci}+u_{ci}]\left[b_{c(i+1)}+u_{c(i+1)}\right]+\\&\qquad [b_{si}+u_{si}]\left[b_{s(i+1)}+u_{s(i+1)}\right]\Big)\Bigg)\\&=2(TW-1)\rho_1\gamma,\end{aligned} \tag{21}$$

where ρ_1 is the autocorrelation coefficient of the signal for a shift (lag) of one sample. Now, for the hypothesis H_1

$$E[\varpi]=2TW(1+\gamma)+2(TW-1)\rho_1\gamma. \tag{22}$$

The variance of $\widehat{R'}(0)$ when signal is present is given as [5]

$$Var\left[\widehat{R}_0'\right]=4TW(1+2\gamma). \tag{23}$$

The variance of $\widehat{R}_1'$ when signal is present, or

$$\begin{aligned}Var\left[\widehat{R}_1'\right]=Var\Bigg(&\sum_{i=1}^{TW-1}\Big([b_{ci}+u_{ci}]\left[b_{c(i+1)}+u_{c(i+1)}\right]+\\&[b_{si}+u_{si}]\left[b_{s(i+1)}+u_{s(i+1)}\right]\Big)\Bigg)\end{aligned} \tag{24}$$

is somewhat difficult to estimate due to the dependence of the variables involved. It is exactly this dependence we are trying to exploit with the method. Since in-phase and quadrature components are independent with equal variances, we can write (24) as

$$Var\left[\widehat{R}_1'\right]=2\,Var\left(\sum_{i=1}^{TW-1}[b_{ci}+u_{ci}]\left[b_{c(i+1)}+u_{c(i+1)}\right]\right). \tag{25}$$

Defining

$$X_i=[b_{ci}+u_{ci}]\left[b_{c(i+1)}+u_{c(i+1)}\right], \tag{26}$$

the variance of the single product X_i for any i is found, using the formula in [10], to be

$$Var[X_i]=1+2\gamma+\left(\rho_1^2+1\right)\gamma^2. \tag{27}$$

Now, $Var\left[\widehat{R}_1'\right]$ is derived, using (27) as

$$\begin{aligned}Var\left[\widehat{R}_1'\right]&=2\;Var\left[\sum_{i=1}^{TW-1}X_i\right]\\&=2\;(TW-1)\left[1+2\gamma+\left(\rho_1^2+1\right)\gamma^2\right]+\\&\quad 4\sum_{j=1}^{TW-2}(TW-1-j)\,Cov_j,\end{aligned} \tag{28}$$

where the covariances are given as

$$Cov_1 = \left(\rho_2 + \rho_1^2\right)\gamma^2 + \rho_2\gamma, \tag{29}$$

$$Cov_j = \left(\rho_{j-1}\rho_{j+1} + \rho_j^2\right)\gamma^2, \quad 2 \le j \le TW-2, \tag{30}$$

with ρ_j being the autocorrelation coefficient of u_{ci} or u_{si} for a shift (lag) of j samples. For BPSK or QPSK modulated primary signal with rectangular pulses, ρ_j is derived as

$$\rho_j = \frac{1+j\eta}{\pi}\ \mathrm{Si}\left\{\left(\frac{1}{\eta}+j\right)\pi\right\} + \frac{1-j\eta}{\pi}\ \mathrm{Si}\left\{\left(\frac{1}{\eta}-j\right)\pi\right\} - \frac{2j\eta}{\pi}\ \mathrm{Si}\left(\pi j\right) + \frac{\eta}{\pi^2}\left(\cos\left\{\left(\frac{1}{\eta}-j\right)\pi\right\} + \cos\left\{\left(\frac{1}{\eta}+j\right)\pi\right\}\right) - \frac{2\eta}{\pi^2}\cos\left(\pi j\right), \tag{31}$$

where $\eta = R_s/W$ is the ratio of the primary transmission symbol rate R_s and sensing bandwidth W, and $\mathrm{Si}\left(\cdot\right)$ is the sine integral defined as $\mathrm{Si}\left(u\right) = \int_0^u \frac{\sin x}{x}\,dx$.

Finally, U' and $\widehat{R}_1'$ are to be combined to get the parameter for decision statistic. So, for hypothesis H_1

$$Var\left[\varpi\right] = Var\left[U'\right] + Var\left[\widehat{R}_1'\right] + Cov_{cc}, \tag{32}$$

where $Var[U']$ and $Var[\widehat{R}_1']$ are defined in (23) and (28), respectively, and $Cov_{cc} = 2Cov[U', \widehat{R}_1']$ is the correction component due to covariance between U' and $\widehat{R}_1'$. Its value is calculated to be

$$Cov_{cc} = 8\left(2TW-3\right)\rho_1\gamma^2 + 8\left(2TW-3\right)\rho_1\gamma + 2\sum_{j=1}^{TW-2}\left[4\left(2TW-3\right) - 8j\right]\rho_j\rho_{j+1}\ \gamma^2. \tag{33}$$

The decision statistic can be described as

$$\varpi \sim \begin{cases} N\left(\mu_n, \sigma_n^2\right) & \mathsf{H}_0 \\ N\left(\mu_{sn}, \sigma_{sn}^2\right) & \mathsf{H}_1, \end{cases} \tag{34}$$

where, from (16), (17), (22) and (32)

$$\mu_n = 2TW,$$

$$\sigma_n^2 = 4TW + 2\left(TW-1\right),$$

$$\mu_{sn} = 2TW\left(1+\gamma\right) + 2\left(TW-1\right)\rho_1\gamma,$$

and

$$\sigma_{sn}^2 = 4TW(1+2\gamma) + (2TW-2)\left[1 + 2\gamma + \left(\rho_1^2+1\right)\gamma^2\right] + 8\left(2TW-3\right)\rho_1\gamma^2 + 8\left(2TW-3\right)\rho_1\gamma + 4\sum_{j=1}^{TW-2}\left((TW-1-j)\,Cov_j + (4TW-4j-6)\,\rho_j\rho_{j+1}\,\gamma^2\right), \tag{35}$$

respectively.

For the CorrSum method, P_f and P_d can be calculated as

$$P_f = Pr\left(\varpi > \varpi_{th} | \mathsf{H}_0\right) = \frac{1}{2}\mathrm{erfc}\left[\frac{\left(\varpi_{th} - \mu_n\right)}{\sigma_n\sqrt{2}}\right] \tag{36}$$

and

$$P_d = Pr\left(\varpi > \varpi_{th} | \mathsf{H}_1\right) = \frac{1}{2}\mathrm{erfc}\left[\frac{\left(\varpi_{th} - \mu_{sn}\right)}{\sigma_{sn}\sqrt{2}}\right], \tag{37}$$

where ϖ_{th} is the decision threshold.

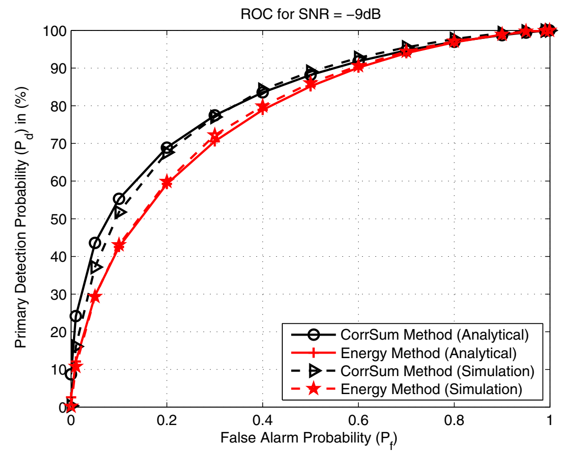

Fig. 4. ROC curves comparing the performance of CorrSum method and energy detection method for QPSK transmission ($2TW = 160$)

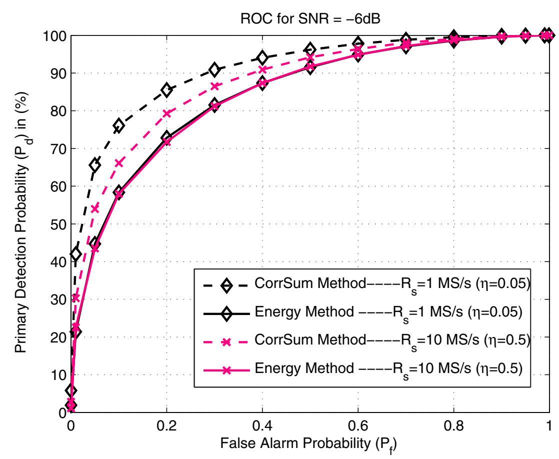

Fig. 5. Performance comparison of CorrSum method and energy detection in different symbol rates of BPSK transmission ($2TW = 80$)

IV. Results

The performance of CorrSum method is compared with the energy detection method according to analytical expressions and Monte Carlo simulations for different symbol rates and modulations. BPSK and QPSK signals (rectangular pulses) are considered with a symbol rate of 10 MS/s and sensing bandwidth of 20 MHz (η= 0.5), the maximum symbol rate that can effectively be transmitted. The results of the CorrSum method and energy detection method for sensing time 4 μs ($2TW = 160$) are shown in Fig. 4 in the form of receiver operating characteristics (ROC) curves. From the results it is apparent that the CorrSum method performs better than the energy detection method in terms of signal detection probability. It is also seen that for useful levels of P_d, the analytical model faithfully approximates the simulation results. The results for BPSK are virtually identical to Fig. 4 and are not plotted here. The simulation is carried out for a BPSK signal (rectangular pulses) with sensing bandwidth of 20 MHz and a symbol rate of 1 MS/s (η=0.05) and 10 MS/s (η=0.5). The results of the CorrSum method and energy detection method for sensing time 2 μs ($2TW = 80$) are shown in

Fig. 5. As expected, the performance of the CorrSum method degrades with increasing symbol rate (decreasing correlations) and approaches that of the energy detector for the diminishing correlation.

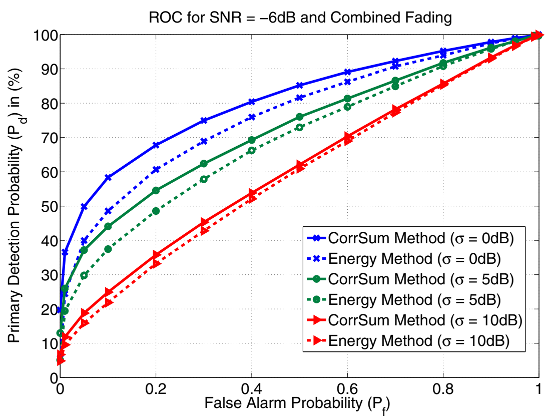

Fig. 6. Performance comparison of CorrSum method and energy detection in combined fading with different shadowing levels

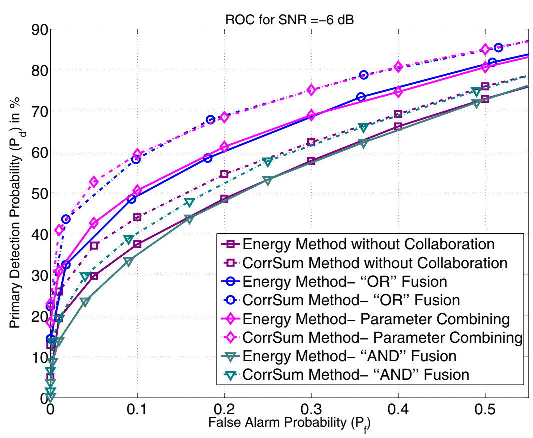

Fig. 7. Performance comparison in combined fading ($\sigma = 5$ dB) with and without collaborative sensing

A. Performance in Fading Environments

The sensing problem involves both small-scale (multipath) and large-scale (shadow) fading which are commonly treated as independent processes that combine to produce the overall fading effect. The performance of the CorrSum and energy detection method in a fading environment with three different values of standard deviation (σ) of log-normal shadowing is investigated for η= 0.05 and $(2TW = 80)$ using Monte Carlo simulations and the results are shown in Fig. 6. For severe combined fading P_d approaches P_f, indicating the performance degradation. However, the performance of the CorrSum method is better than energy detection in all fading cases.

B. Performance in Collaborative Sensing

Collaborative sensing is a technique to avoid the "hidden node" problem, which arises due to fading [4], [8]. Simulation results are shown in Fig. 7, where two nodes use collaborative decision schemes like "OR" fusion, "AND" fusion and "Parameter Combining" in a combined Rayleigh and log-normal shadowing ($\sigma = 5$ dB) environment. Here, we assume uncorrelated shadowing for simplicity, although a separation of few meters (2 m to 4 m in indoor environment) is needed for shadowing correlation to drop to a significantly low level. We find that the superiority of "OR" and "AND" fusion depends upon the probability distributions under both hypotheses and desired global false alarm probability. In our case of combined fading, the "AND" fusion does not help, but is rather destructive for both energy and CorrSum method which is verified by both simulation and analytical method. The "OR" fusion and "Parameter Combining" improve the performance giving almost the same results, but "OR" fusion should be preferred due to its simplicity.

V. Conclusion

This work proposed a new autocorrelation-based method (CorrSum method) and its performance was compared with standard energy detection for both closed form analysis and simulation. The results indicated that the new method has better performance than energy detection for different types of modulation. We also investigated the performance of the new method for fading channels as well as the performance in collaborative sensing with different fusion and combining rules and CorrSum method outperformed the energy detection in all environments. In future work, we plan to investigate extending the method to matrix correlation (i.e. multi-antenna systems) for improved performance. We also plan to study the performance of the two methods with more advanced decision rules with more sensors and more realistic channel models.

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