DIRECTIONAL CHANNEL MODELING WITH

SPARSE POWER AZIMUTH SPECTRUM ESTIMATION

J. W. Wallace[(1)]

[(1)]*School of Engineering and Science, Jacobs University Bremen,*
Campus Ring 1, 28759 Bremen, Germany, Email: wall@ieee.org

I. INTRODUCTION

For directional wireless channel modeling, a number of techniques exist to estimate the power azimuth spectrum (PAS) from multiple-antenna measurements. Beamforming is a typical method to estimate PAS directly from array data, but resolution limitations arise from the finite aperture of the array, which can be partially overcome by applying deconvolution [1, 2]. Another approach finds double-directional multipath components via CLEAN, ESPRIT, or SAGE and then extracts PAS from empirical directional probability density functions (pdfs) [3, 4], but potential difficulties arise due to calibration sensitivity [5] and the existence of dense multipath [6]. Another shortcoming of previous methods is that they do not necessarily represent PAS with few parameters, which is useful in many applications.

In this work, a novel technique for PAS estimation is presented, referred to as sparse power azimuth spectrum estimation (SPASE). Although some of the particulars have already been presented in previous work [7, 8], the purpose of this paper is to present the method in a complete, cohesive form. The method is based on minimum ℓ_1-norm representations of the PAS, which produce very sparse representations of signals and operators, in contrast to minimum ℓ_2-norm solutions.

II. SPARSE POWER AZIMUTH SPECTRUM ESTIMATION (SPASE)

Tensor notation is sometimes used in this work where $\mathcal{A}$ is an Nth order tensor with elements $a_{i_1 i_2 \ldots i_N}$ and $i_\ell \in \{1, \ldots, I_\ell\}$. The tensor $\mathcal{A}$ may be reshaped into a matrix $\mathbf{A}$ with elements $a_{[i_1 i_2 \ldots i_M][i_{M+1} \ldots i_N]} = a_{k_1 k_2}$ where the indices in brackets denote stacking (like in MATLAB). A repeated index not appearing on the left-hand side of an equation implies summation. The inner product of two tensors is denoted $< \mathcal{A}, \mathcal{B} >= a_{i_1, i_2, \ldots, i_N} b^*_{i_1, i_2, \ldots, i_N}$. Outer product of an Nth order tensor $\mathcal{A}$ and an Mth order tensor $\mathcal{B}$ is $\{\mathcal{A} \circ \mathcal{B}\}_{i_1 i_2 \ldots i_N j_1 j_2 \ldots j_M} = a_{i_1 \ldots i_N} b_{j_1 \ldots j_M}$.

A. Channel Statistics

Consider a communications system with a single transmit antenna and N_R receive antennas. Assuming a set of L discrete multipath arrivals (L can be arbitrarily large), the narrowband channel transfer function can be written as $h_i = 1/\sqrt{L} \sum_{\ell=1}^{L} \alpha_\ell g_i(\phi_\ell)$ where α_ℓ is the complex amplitude of the ℓth path, $g_i(\phi) = e_i(\phi) \exp[j\psi_i(\phi)]$ is the steering vector, $e_i(\phi)$ is the complex azimuthal far-field radiation pattern, $\psi_i(\phi) = k_0(x_i \cos\phi + y_i \sin\phi)$ is the array factor, x_i and y_i are the coordinates of the ith antenna, and k_0 is the wavenumber. The channel covariance matrix is computed as

$$r_{ik} = \mathrm{E}\{h_i h_k^*\} = \frac{1}{L} \sum_{\ell_1=1}^{L} \sum_{\ell_2=1}^{L} \mathrm{E}\{\alpha_{\ell_1} \alpha^*_{\ell_2}\} \mathrm{E}\{g_i(\phi_{\ell_1}) g_k^*(\phi_{\ell_2})\}, \tag{1}$$

where independence of the arrival amplitudes and directions is assumed. If the arrival amplitudes are i.i.d. and zero mean, we can define

$$\mathrm{E}\{\alpha_{\ell_1} \alpha^*_{\ell_2}\} = F(\phi_{\ell_1}) \delta_{\ell_1 \ell_2}, \tag{2}$$

where $F(\phi)$ is the expected power of an arrival in the ϕ direction, in which case

$$r_{ik} = \frac{1}{L} \sum_{\ell=1}^{L} \int_0^{2\pi} d\phi_\ell \; f(\phi_\ell) F(\phi_\ell) g_{ik}(\phi_\ell), \tag{3}$$

where $f(\phi)$ is the pdf of multipath arrivals. Defining $p(\phi) = f(\phi)F(\phi)$ as the *true PAS* and assuming i.i.d. arrivals, (3) becomes

$$r_{ik} = \int_0^{2\pi} d\phi \; p(\phi) g_{ik}(\phi), \qquad g_{ik}(\phi) = g_i(\phi) g_k^*(\phi) \exp\{j[\psi_i(\phi) - \psi_k(\phi)]\}. \tag{4}$$

The development above can naturally be extended to higher dimensions, in which case the tensor notation is convenient. For example, suppose we have arrays at both transmit and receive, the relationship becomes

$$r_{i_1 k_1 i_2 k_2} = \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_T \; p(\phi_R, \phi_T) g_{i_1 k_1 i_2 k_2}(\phi_R, \phi_T) \tag{5}$$

with $\mathcal{G} = \mathbf{a}_R(\phi_R) \circ \mathbf{a}_T(\phi_T) \circ \mathbf{a}_R^*(\phi_R) \circ \mathbf{a}_T^*(\phi_T)$, and $\mathbf{a}_P$ is the single-directional steering vector with $P = T$ (for transmit) or $P = R$ (for receive). For multiple independent polarizations, (4) can be extended to

$$r_{ik} = \int_0^{2\pi} d\phi \; p_\ell(\phi) g_{ik\ell}(\phi), \tag{6}$$

where $g_{ik\ell} = e_{i\ell}(\phi) e_{k\ell}^*(\phi) \exp\{jk_0[\psi_i(\phi) - \psi_k(\phi)]\}$, and ℓ indexes polarization. Similarly, multiple frequency bins and elevation can also be incorporated.

B. Element-space Solution

Given an estimate of the channel covariance $\hat{\mathbf{R}}$ and considering the relationship (3), a method similar to method-of-moments (MOM) can be used to find $p(\phi)$, by expanding in terms of a basis, or

$$p(\phi) = \sum_{n=1}^{N_B} a_n f_n(\phi), \tag{7}$$

where now $f_n(\phi)$ is the nth basis function (not a pdf), and N_B is the number of basis functions, yielding

$$r_{ik} = \sum_{n=1}^{N_B} a_n \underbrace{\int_0^{2\pi} d\phi \; g_{ik}(\phi) f_n(\phi),}_{q_{ikn}} \quad \text{or} \quad r_{[ik]} = \sum_{n=1}^{N_B} q_{[ik][n]} a_n, \tag{8}$$

where the latter stacked version can be written as $\mathbf{r} = \mathbf{Q}\mathbf{a}$. The obvious way to solve this relationship is to estimate the $\mathbf{a}$ directly using a pseudoinverse. One problem with this approach is that it is a minimum ℓ_2-norm solution that will not be sparse. Even worse, the pseudoinverse will not produce real, positive $\mathbf{a}$, which is necessary for the PAS.

These difficulties can be overcome by using linear programming (LP), which in standard form solves the real-valued minimization problem

$$\hat{\mathbf{x}} = \arg\min \mathbf{c}^T \mathbf{x} \quad \text{subject to} \quad \mathbf{y} = \mathbf{M}\mathbf{x}, \quad x_i \geq 0 \;\; \forall i. \tag{9}$$

In this work, LP solutions are obtained with the freely available PCx package. To transform the present problem into standard LP form, we can split real and imaginary parts of $\mathbf{r}$ and $\mathbf{Q}$ to obtain

$$\begin{bmatrix} \mathbf{r}_R \\ \mathbf{r}_I \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_R \\ \mathbf{Q}_I \end{bmatrix} \mathbf{a}, \tag{10}$$

which may be written as $\mathbf{r}' = \mathbf{Q}'\mathbf{a}$. To set equal cost for our basis coefficients, we let $\mathbf{c} = [1 \ldots 1]^T$. Our problem is now a standard LP problem where the solution minimizes the ℓ_1-norm of $\mathbf{a}$, favoring a sparse solution.

We have assumed that relation (8) holds exactly, which may not be possible due to imperfect estimates of $\mathbf{r}$ or imperfect array calibration. Slack in the match is allowed with $\mathbf{r}' = \mathbf{Q}'\mathbf{a} + \boldsymbol{\epsilon}_P - \boldsymbol{\epsilon}_M$, where $\boldsymbol{\epsilon} = \boldsymbol{\epsilon}_P - \boldsymbol{\epsilon}_M$ is the error vector. The expanded LP problem becomes

$$\mathbf{y} = \mathbf{r}' \quad \mathbf{x} = \begin{bmatrix} \mathbf{a} \\ \boldsymbol{\epsilon}_P \\ \boldsymbol{\epsilon}_M \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} \mathbf{Q}' & \mathbf{I} & -\mathbf{I} \end{bmatrix}, \tag{11}$$

where $\mathbf{I}$ is the $M \times M$ identity matrix with $M = 2N_R^2$. The cost coefficients become $c_n = 1$ for $1 \leq n \leq N$ and $c_n = c_\epsilon$ otherwise, jointly minimizing the ℓ_1-norm and the absolute error. For very high quality data with careful calibration, large values of c_ϵ may be advisable to make the match as close as possible. However, when aberrations are present in the data, forcing such a close match may actually make the model order much higher for only slight improvement in the fit.

In some circumstances it may be useful to put a hard bound on the allowed error in the match of the covariance, or $0 \leq \epsilon_{Pi} \leq \epsilon_i^+$ and $0 \leq \epsilon_{Mi} \leq \epsilon_i^-$. The LP problem in standard form becomes

$$\mathbf{y} = \begin{bmatrix} \mathbf{r}' \\ \boldsymbol{\epsilon}^+ \\ \boldsymbol{\epsilon}^- \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \mathbf{a} \\ \boldsymbol{\epsilon}_P \\ \boldsymbol{\epsilon}_M \\ \mathbf{p} \\ \mathbf{m} \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} \mathbf{Q}' & \mathbf{I} & -\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} & \mathbf{I} \end{bmatrix}, \tag{12}$$

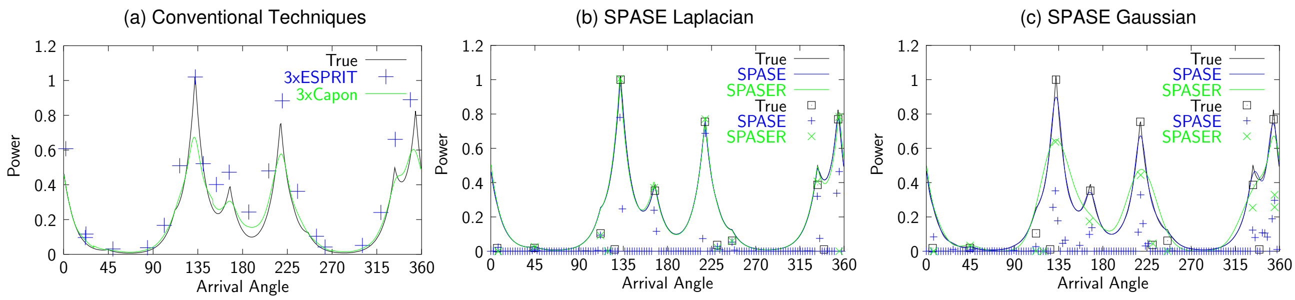

Fig. 1. Example PAS estimation with (a) conventional techniques and SPASE with either the (b) correct or (c) incorrect basis

where $\mathbf{p}$ and $\mathbf{m}$ are vectors of slack variables and $\mathbf{0}$ is a zero matrix. The cost vector is identical to the previous case, except it is padded with zeros (slack variables have zero cost). Hard limiting the error appears to have the benefit of making the solution less sensitive to the exact choice of c_ϵ.

We may not wish to include all of the equations in (11) or (12) due to either redundancy (repeated elements in the covariance matrix) or uncertainty (high error or missing covariance terms). Pruning the vector $\mathbf{r}$ back by removing elements that are either redundant or uncertain and discarding the corresponding elements of $\mathbf{y}$ and $\mathbf{M}$ reduces the number of equations in (11) or (12).

C. Beamspace Solution

In the case that we have a large number of sensors (antennas and frequency bins), it may be more efficient to solve the problem in beamspace rather than element space. Any linear transformation of (3) will remain a linear system, where only the values for $\mathbf{Q}$ are transformed. Consider applying a simple Bartlett beamformer to (3), or

$$B(\phi) \;=\; <\mathbf{G}(\phi), \mathbf{R}> \;=\; <\mathbf{G}(\phi), \sum_{n=1}^{N_B} a_n \int_0^{2\pi} d\phi' \; \mathbf{G}(\phi') f_n(\phi')> = \sum_{n=1}^{N_B} a_n \int_0^{2\pi} d\phi' \; f_n(\phi') <\mathbf{G}(\phi), \mathbf{G}(\phi')>. \quad (13)$$

As with MOM, we can project both sides of (13) onto a set of test functions. Using point matching, we sample both sides at K angles ϕ_k to obtain

$$b_k = B(\phi_k) = \sum_{n=1}^{N_B} a_n \underbrace{\int_0^{2\pi} d\phi' \; f_n(\phi') < \mathbf{G}(\phi_k), \mathbf{G}(\phi') >}_{q_{kn}}. \quad (14)$$

This equation can be solved just like the element-space equation, but in this case $\mathbf{Q}$ is already purely real (and non-negative). Also, the number of elements of $\mathbf{Q}$ can be greatly reduced compared to the case when all covariance elements are retained.

D. Choice of Basis Functions

The PAS can be represented with an unstructured basis (pulse functions, triangle functions, etc.) when no a-priori information about the PAS is available. A structured basis may be chosen that corresponds to PAS shapes that are known to fit well from previous analysis with unstructured techniques. Also, there is no restriction that the basis be orthogonal, and an "overcomplete" super basis can also be formed, containing Laplacian, Gaussian, von Mises, Dirac delta functions, etc., together, and the LP solution then chooses the sparsest representation.

III. Example Applications

Due to space limitations, just two examples will be provided. First we consider single-directional estimation. The true PAS is a sum of Laplacian-shaped clusters with $\sigma = 15°$ angular spread. The simulated channel is probed with a ULA at the receiver consisting of 0.4λ-separated directional antennas, each having a 3 dB beamwidth of $120°$ and $\sin\phi$ azimuthal gain pattern. The array is rotated to 3 different orientations to get $360°$ of view. Figure 1(a) depicts the results for two conventional PAS estimation techniques (ESPRIT and the Capon beamformer). ESPRIT cannot resolve all arrivals, since the number of multipath components is larger than the number of antennas. The Capon beamformer gives meaningful results, but there is some smoothing of the peaks, smaller arrivals are lost, and the solution is not sparse. Figure 1(b) depicts the solution with SPASE, showing true clusters (angle/power) and PAS with boxes and solid

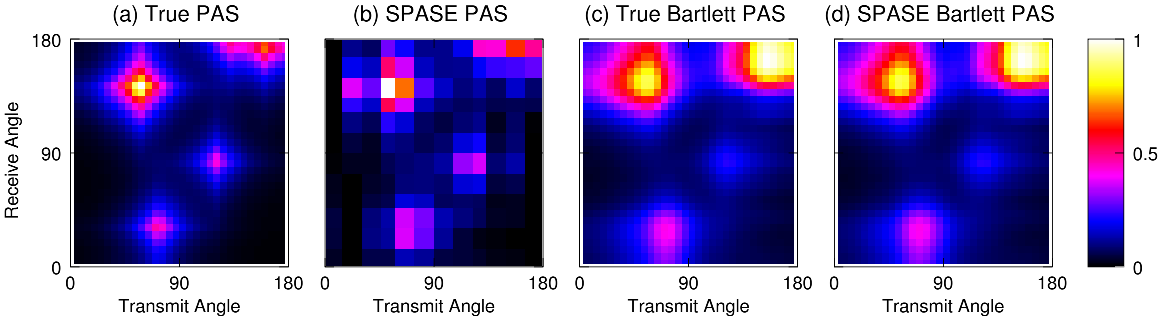

Fig. 2. Comparison of joint double-directional PAS: (a) true PAS generated with the SVA model, (b) piecewise-constant SPASE estimate, (c) true Bartlett spectrum, (d) SPASE estimated Bartlett spectrum

lines, respectively, along with the SPASE estimate where a Laplacian basis with angular spreads of $5°, 10°, ..., 35°$ and arrival angles of $0°, 2.5°, ..., 360°$ is assumed. The SPASE solution is nearly identical to the true PAS, but there is discrepancy in the cluster coefficients. SPASE with a reduced basis (SPASER) gives much better results, where only the strongest basis coefficient in each "cluster" of coefficients is kept. Figure 1(c) shows the result when the assumed basis shape is not correct (Gaussian). SPASE still gives meaningful results, but the peaks are clipped and the representation is not as sparse.

The second example uses an unstructured version of SPASE for double-directional PAS estimation, which may be useful for interference avoidance and suppression in ad-hoc networking. The beamspace formulation in Section II-C can be extended to the joint MIMO case where

$$p(\phi_R, \phi_T) = \sum_{m=1}^{N_{BR}} \sum_{n=1}^{N_{BT}} a_{mn} f_{mn}(\phi_R, \phi_T) \quad \text{and} \quad B(\phi_R, \phi_T) = < \mathcal{R}, \mathcal{G}(\phi_R, \phi_T) > \tag{15}$$

are the basis expansion (pulse functions are used for the f_{mn}) and Bartlett spectrum, respectively, $r_{i_1 k_1 i_2 k_2} = \mathrm{E}\left\{h_{i_1 k_1} h^*_{i_2 k_2}\right\}$, $\mathcal{G}(\phi_R, \phi_T) = \mathbf{a}_R(\phi_R) \circ \mathbf{a}_T(\phi_T) \circ \mathbf{a}^*_R(\phi_R) \circ \mathbf{a}^*_T(\phi_T)$, and $\mathbf{H}$ is the channel transfer matrix. Using point matching on a grid of $K_R \times K_T$ points denoted $\overline{\phi}_{R,k}$ and $\overline{\phi}_{T,k}$ to obtain $b_{k\ell} = B(\overline{\phi}_{R,k}, \overline{\phi}_{T,\ell})$, we have

$$b_{k\ell} = \sum_{m=1}^{N_{BR}} \sum_{n=1}^{N_{BT}} a_{mn} \underbrace{\int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_T f_{mn}(\phi_R, \phi_T) < \mathcal{G}(\phi_R, \phi_T), \mathcal{G}(\overline{\phi}_{R,k}, \overline{\phi}_{T,\ell}) >}_{q_{k\ell mn}}.$$

Stacking the dimensions appropriately, $b_{[k\ell]} = \sum_{[mn]} q_{[k\ell][mn]} a_{[mn]}$, which can be solved using the LP techniques described previously.

Figures 2(a) and (b) plot a single realization of the true spectrum from the SVA model ($\sigma = 26°$, $\Gamma = 2$) and the SPASE estimate, respectively, using 12×12 basis functions point-matched at 32×32 points. Although the basic shape of the spectra is similar, some of the detail is lost due to the stairstep approximation. Figures 2(c) and (d) compare the joint Bartlett spectra, indicating nearly an exact match. Averaging the performance over 100 random realizations gives an average error of 0.73% and 25% for the Bartlett spectrum and true PAS, respectively. The average number of basis coefficients used was 69, which is small compared to the full covariance tensor (4096 real parameters).

REFERENCES

[1] J. Tsao and B. D. Steinberg, "Reduction of sidelobe and speckle artifacts in microwave imaging: the CLEAN technique," *IEEE Trans. Antennas Propag.*, vol. 36, pp. 543–556, Apr. 1988.

[2] M. J. Gans, R. A. Valenzuela, Y.-S. Yeh, and N. Amitay, "Antenna pattern deconvolution for precise incident power density pattern measurement," in *Proc. 2000 IEEE 51st Veh. Technol. Conf.*, vol. 3, Tokyo, Japan, 15-18 May 2000, pp. 2532–2535.

[3] B. H. Fleury, M. Tschudin, R. Heddergott, D. Dahlhaus, and K. Ingeman Pedersen, "Channel parameter estimation in mobile radio environments using the SAGE algorithm," *IEEE J. Selected Areas Commun.*, vol. 17, pp. 434–450, Mar. 1999.

[4] Q. H. Spencer, B. D. Jeffs, M. A. Jensen, and A. L. Swindlehurst, "Modeling the statistical time and angle of arrival characteristics of an indoor multipath channel," *IEEE J. Selected Areas Commun.*, vol. 18, pp. 347–360, Mar. 2000.

[5] M. Landmann, W. A. Kotterman, and R. S. Thomä, "On the influence of data models on estimated angular distributions in channel characterisation," in *Proc. 2007 European Antennas Propag. Conf.*, Edinburgh, Scotland, 11-16 Nov. 2007.

[6] M. Nilsson, B. Volcker, and B. Ottersten, "A cluster approach to spatio-temporal channel estimation," in *Proc. 2000 IEEE Intl. Conf. Acoustics, Speech, and Signal Processing*, vol. 5, Istanbul, Turkey, 5-9 June 2000, pp. 2757–2760.

[7] J. Wallace, H. Özcelik, M. Herdin, E. Bonek, and M. Jensen, "A diffuse multipath spectrum estimation technique for directional channel modeling," in *Proc. 2004 IEEE Intl. Conf. Commun.*, vol. 6, Paris, France, Jun. 20-24 2004, pp. 3183–3187.

[8] J. Wallace and B. Maharaj, "Accurate MIMO channel modeling: Correlation tensor vs. directional approaches," in *Proc. 2007 IEEE Global Telecomm. Conf.*, Washington, D.C., 26-30 Nov. 2007.