DIRECTIONAL CHANNEL MODELING WITH

SPARSE POWER AZIMUTH SPECTRUM ESTIMATION

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I. Introduction

For directional wireless channel modeling, a number of techniques exist to estimate the power azimuth spectrum (PAS) from multiple-antenna measurements. Beamforming is a typical method to estimate PAS directly from array data, but resolution limitations arise from the finite aperture of the array, which can be partially overcome by applying deconvolution [1, 2]. Another approach finds double-directional multipath components via CLEAN, ESPRIT, or SAGE and then extracts PAS from empirical directional probability density functions (pdfs) [3, 4], but potential difficulties arise due to calibration sensitivity [5] and the existence of dense multipath [6]. Another shortcoming of previous methods is that they do not necessarily represent PAS with few parameters, which is useful in many applications.

In this work, a novel technique for PAS estimation is presented, referred to as sparse power azimuth spectrum estimation (SPASE). Although some of the particulars have already been presented in previous work [7, 8], the purpose of this paper is to present the method in a complete, cohesive form. The method is based on minimum $l_1$-norm representations of the PAS, which produce very sparse representations of signals and operators, in contrast to minimum $l_2$-norm solutions.

II. Sparse Power Azimuth Spectrum Estimation (SPASE)

Tensor notation is sometimes used in this work where $\mathcal{A}$ is an $N$th order tensor with elements $a_{i_1i_2...i_N}$ and $i_\ell \in \{1, \ldots, I_\ell\}$. The tensor $\mathcal{A}$ may be reshaped into a matrix $\mathbf{A}$ with elements $a_{[i_1i_2...i_M][i_M+1...i_N]} = a_{k_1k_2}$ where the indices in brackets denote stacking (like in MATLAB). A repeated index not appearing on the left-hand side of an equation implies summation. The inner product of two tensors is denoted $\langle \mathcal{A}, \mathcal{B} \rangle = a_{i_1,...,i_N}b_{i_1,...,i_N}$. Outer product of an $N$th order tensor $\mathcal{A}$ and an $M$th order tensor $\mathcal{B}$ is $\{\mathcal{A} \circ \mathcal{B}\}_{i_1i_2...i_Nj_1j_2...j_M} = a_{i_1...i_N}b_{j_1...j_M}$.

A. Channel Statistics

Consider a communications system with a single transmit antenna and $N_R$ receive antennas. Assuming a set of $L$ discrete multipath arrivals ($L$ can be arbitrarily large), the narrowband channel transfer function can be written as $h_i = 1/\sqrt{L} \sum_{\ell=1}^{L} \alpha_\ell g_\ell(\phi_\ell)$ where $\alpha_\ell$ is the complex amplitude of the $\ell$th path, $g_\ell(\phi) = e_\ell(\phi) \exp[j\psi_\ell(\phi)]$ is the steering vector, $e_\ell(\phi)$ is the complex azimuthal far-field radiation pattern, $\psi_\ell(\phi) = k_\ell x_\ell \cos \phi + y_\ell \sin \phi$ is the array factor, $x_\ell$ and $y_\ell$ are the coordinates of the $\ell$th antenna, and $k_\ell$ is the wavenumber. The channel covariance matrix is computed as

$$r_{ik} = E\{h_ih_k^*\} = \frac{1}{L} \sum_{\ell_1=1}^{L} \sum_{\ell_2=1}^{L} E\{\alpha_{\ell_1}\alpha_{\ell_2}^*\} E\{g_{\ell_1}(\phi_{\ell_1})g_{\ell_2}^*(\phi_{\ell_2})\},$$

(1)

where independence of the arrival amplitudes and directions is assumed. If the arrival amplitudes are i.i.d. and zero mean, we can define

$$E\{\alpha_{\ell_1}\alpha_{\ell_2}^*\} = F(\phi_{\ell_1}) \delta_{\ell_1\ell_2},$$

(2)

where $F(\phi)$ is the expected power of an arrival in the $\phi$ direction, in which case

$$r_{ik} = \frac{1}{L} \sum_{\ell=1}^{L} \int_{0}^{2\pi} d\phi f(\phi_{\ell}) F(\phi_{\ell}) g_{ik}(\phi_{\ell}),$$

(3)

where $f(\phi)$ is the pdf of multipath arrivals. Defining $p(\phi) = f(\phi)F(\phi)$ as the true PAS and assuming i.i.d. arrivals, (3) becomes

$$r_{ik} = \int_{0}^{2\pi} d\phi p(\phi) g_{ik}(\phi), \quad g_{ik}(\phi) = g_i(\phi)g_k^*(\phi) \exp[j\psi_i(\phi) - \psi_k(\phi)].$$

(4)
In some circumstances it may be useful to put a hard bound on the allowed error in the match of the covariance, or improvement in the fit. When aberrations are present in the data, forcing such a close match may actually make the model order much higher for only slight improvement in the fit. We have assumed that relation (8) holds exactly, which may not be possible due to imperfect estimates of $\hat{C}$ or imperfect calibration. Slack in the match is allowed with $\epsilon$, which may be advisable to make the match as close as possible. However, when aberrations are present in the data, forcing such a close match may actually make the model order much higher for only slight improvement in the fit.

These difficulties can be overcome by using linear programming (LP), which in standard form solves the real-valued minimization problem

$$\hat{x} = \arg \min \mathbf{c}^T \mathbf{x} \quad \text{subject to} \quad \mathbf{y} = \mathbf{Mx}, \quad x_i \geq 0 \quad \forall i.$$  

In this work, LP solutions are obtained with the freely available PCx package. To transform the present problem into standard LP form, we can split real and imaginary parts of $\mathbf{r}$ and $\mathbf{Q}$ to obtain

$$\begin{bmatrix} \mathbf{r}_R \\ \mathbf{r}_I \end{bmatrix} = \begin{bmatrix} \mathbf{Q}_R \\ \mathbf{Q}_I \end{bmatrix} \mathbf{a},$$

which may be written as $\mathbf{r}' = \mathbf{Q}' \mathbf{a}$. To set equal cost for our basis coefficients, we let $\mathbf{c} = [1 \ldots 1]^T$. Our problem is now a standard LP problem where the solution minimizes the $\ell_1$-norm of $\mathbf{a}$, favoring a sparse solution.

We have assumed that relation (8) holds exactly, which may not be possible due to imperfect estimates of $\mathbf{r}$ or imperfect array calibration. Slack in the match is allowed with $\epsilon' = \mathbf{Q}' \mathbf{a} + \epsilon_p - \epsilon_m$, where $\epsilon = \epsilon_p - \epsilon_m$ is the error vector. The expanded LP problem becomes

$$\mathbf{y} = \mathbf{r}' \quad \mathbf{x} = \begin{bmatrix} \mathbf{a} \\ \epsilon_p \\ \epsilon_m \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} \mathbf{Q}' & \mathbf{I} & -\mathbf{I} \end{bmatrix},$$

where $\mathbf{I}$ is the $M \times M$ identity matrix with $M = 2N_R^2$. The cost coefficients become $c_n = 1$ for $1 \leq n \leq N$ and $c_n = c_e$ otherwise, jointly minimizing the $\ell_1$-norm and the absolute error. For very high quality data with careful calibration, large values of $c_e$ may be advisable to make the match as close as possible. However, when aberrations are present in the data, forcing such a close match may actually make the model order much higher for only slight improvement in the fit.

In some circumstances it may be useful to put a hard bound on the allowed error in the match of the covariance, or $0 \leq \epsilon_p \leq \epsilon_p^+$ and $0 \leq \epsilon_m \leq \epsilon_m^+$. The LP problem in standard form becomes

$$\mathbf{y} = \begin{bmatrix} \mathbf{r}' \\ \epsilon_p \\ \epsilon_m \end{bmatrix} \quad \mathbf{x} = \begin{bmatrix} \mathbf{a} \\ \epsilon_p \\ \epsilon_m \\ \mathbf{p} \end{bmatrix} \quad \mathbf{M} = \begin{bmatrix} \mathbf{Q}' & \mathbf{I} & -\mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{I} & \mathbf{0} \end{bmatrix},$$
where \( p \) and \( m \) are vectors of slack variables and \( 0 \) is a zero matrix. The cost vector is identical to the previous case, except it is padded with zeros (slack variables have zero cost). Hard limiting the error appears to have the benefit of making the solution less sensitive to the exact choice of \( \epsilon \).

We may not wish to include all of the equations in (11) or (12) due to either redundancy (repeated elements in the covariance matrix) or uncertainty (high error or missing covariance terms). Pruning the vector \( r \) back by removing elements that are either redundant or uncertain and discarding the corresponding elements of \( y \) and \( M \) reduces the number of equations in (11) or (12).

C. Beamspace Solution

In the case that we have a large number of sensors (antennas and frequency bins), it may be more efficient to solve the problem in beamspace rather than element space. Any linear transformation of (3) will remain a linear system, where only the values for \( Q \) are transformed. Consider applying a simple Bartlett beamformer to (3), or

\[
B(\phi) = \langle G(\phi), R \rangle = \langle G(\phi), \sum_{n=1}^{N_B} a_n \int_0^{2\pi} d\phi' \; G(\phi') f_n(\phi') \rangle = \sum_{n=1}^{N_B} a_n \int_0^{2\pi} d\phi' \; f_n(\phi') \langle G(\phi), G(\phi') \rangle.
\]

(13)

As with MOM, we can project both sides of (13) onto a set of test functions. Using point matching, we sample both sides at \( K \) angles \( \phi_k \) to obtain

\[
b_k = B(\phi_k) = \sum_{n=1}^{N_B} a_n \int_0^{2\pi} d\phi' \; f_n(\phi') \langle G(\phi_k), G(\phi') \rangle.
\]

(14)

This equation can be solved just like the element-space equation, but in this case \( Q \) is already purely real (and non-negative). Also, the number of elements of \( Q \) can be greatly reduced compared to the case when all covariance elements are retained.

D. Choice of Basis Functions

The PAS can be represented with an unstructured basis (pulse functions, triangle functions, etc.) when no a-priori information about the PAS is available. A structured basis may be chosen that corresponds to PAS shapes that are known to fit well from previous analysis with unstructured techniques. Also, there is no restriction that the basis be orthogonal, and an “overcomplete” super basis can also be formed, containing Laplacian, Gaussian, von Mises, Dirac delta functions, etc., together, and the LP solution then chooses the sparsest representation.

III. Example Applications

Due to space limitations, just two examples will be provided. First we consider single-directional estimation. The true PAS is a sum of Laplacian-shaped clusters with \( \sigma = 15^\circ \) angular spread. The simulated channel is probed with a ULA at the receiver consisting of 0.4\( \lambda \)-separated directional antennas, each having a 3 dB beamwidth of 120\( ^\circ \) and \( \sin \phi \) azimuthal gain pattern. The array is rotated to 3 different orientations to get 360\( ^\circ \) of view. Figure 1(a) depicts the results for two conventional PAS estimation techniques (ESPRIT and the Capon beamformer). ESPRIT cannot resolve all arrivals, since the number of multipath components is larger than the number of antennas. The Capon beamformer gives meaningful results, but there is some smoothing of the peaks, smaller arrivals are lost, and the solution is not sparse. Figure 1(b) depicts the solution with SPASE, showing true clusters (angle/power) and PAS with boxes and solid
Fig. 2. Comparison of joint double-directional PAS: (a) true PAS generated with the SVA model, (b) piecewise-constant SPASE estimate, (c) true Bartlett PAS, (d) SPASE estimated Bartlett PAS

lines, respectively, along with the SPASE estimate where a Laplacian basis with angular spreads of $5^\circ$, $10^\circ$, ..., $35^\circ$ and arrival angles of $0^\circ$, $2.5^\circ$, ..., $360^\circ$ is assumed. The SPASE solution is nearly identical to the true PAS, but there is discrepancy in the cluster coefficients. SPASE with a reduced basis (SPASER) gives much better results, where only the strongest basis coefficient in each “cluster” of coefficients is kept. Figure 1(c) shows the result when the assumed basis shape is not correct (Gaussian). SPASE still gives meaningful results, but the peaks are clipped and the representation is not as sparse.

The second example uses an unstructured version of SPASE for double-directional PAS estimation, which may be useful for interference avoidance and suppression in ad-hoc networking. The beamspace formulation in Section II-C can be extended to the joint MIMO case where

\[ p(\phi_R, \phi_T) = \sum_{m=1}^{N_R} \sum_{n=1}^{N_T} a_{mn} f_{mn}(\phi_R, \phi_T) \quad \text{and} \quad B(\phi_R, \phi_T) = \langle R, \mathcal{G}(\phi_R, \phi_T) \rangle \]

are the basis expansion (pulse functions are used for the $f_{mn}$) and Bartlett spectrum, respectively. $r_{i_1k_1i_2k_2} = E \{ h_{i_1k_1} h_{i_2k_2} \}$. $\mathcal{G}(\phi_R, \phi_T) = a_R(\phi_R) \circ a_T(\phi_T) \circ a_R^*(\phi_R) \circ a_T^*(\phi_T)$, and $H$ is the channel transfer matrix. Using point matching on a grid of $K_E \times K_F$ points denoted $\phi_{R,k}$ and $\phi_{T,k}$ to obtain $b_{k\ell} = B(\phi_{R,k}, \phi_{T,\ell})$, we have

\[ b_{k\ell} = \sum_{m=1}^{N_R} \sum_{n=1}^{N_T} a_{mn} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_T f_{mn}(\phi_R, \phi_T) \langle \mathcal{G}(\phi_R, \phi_T), \mathcal{G}(\phi_{R,k}, \phi_{T,\ell}) \rangle. \]

Stacking the dimensions appropriately, $b_{k\ell} = \sum_{m=1}^{N_R} \sum_{n=1}^{N_T} q_{k\ell}[mn] a_{mn}$, which can be solved using the LP techniques described previously.

Figures 2(a) and (b) plot a single realization of the true PAS from the SVA model ($\sigma = 26^\circ$, $\Gamma = 2$) and the SPASE estimate, respectively, using 12x12 basis functions point-matched at 32x32 points. Although the basic shape of the spectra is similar, some of the detail is lost due to the stairstep approximation. Figures 2(c) and (d) compare the joint Bartlett spectra, indicating nearly an exact match. Averaging the performance over 100 random realizations gives of the spectra is similar, some of the detail is lost due to the stairstep approximation. Figures 2(c) and (d) compare the joint Bartlett spectra, indicating nearly an exact match. Averaging the performance over 100 random realizations gives

**References**


