A NON-SEPARABLE DIRECTIONAL CHANNEL MODEL AND OBSERVATIONS FOR FUTURE MOBILE AIR INTERFACES

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I. INTRODUCTION

Communication nodes with multiple antennas are likely candidates for 4th generation (4G) wireless systems and beyond, providing increased spectral efficiency as well as improved spectrum reuse. Two important questions regarding the use of multiple antennas in a multi-user scenario are (1) what channel state information (CSI) should be made available to the communication nodes and (2) under what conditions should multiple antennas be used for spatial multiplexing and/or beamforming for simultaneous transmission. This paper addresses these two questions by analyzing a simple yet realistic channel model.

The multi-user MIMO channel can best be exploited when all users have perfect knowledge of the matrix channels connecting all transmitters to all receivers, allowing methods such as channel inversion [1] or known-interference coding to be used [2]. However, the overhead to distribute CSI among users may be large, and even worse, outdated CSI is not useful after rapid wavelength-scale fading [3, 4]. These difficulties has sparked interest in using channel covariance information (CCI) instead of perfect CSI. Intuition, measurements, and models suggest that CCI is stable and useful for many fade durations, allowing less frequent distribution of CCI to nodes. To date, most analyses of CCI-based transmission assume that the channel covariance has a separable (Kronecker) structure, which is usually a requirement to simplify analytical manipulations. However, when using CCI in a multi-user scenario, the full benefit of multiple antennas is likely not to be realized with separable channels, since a transmitter cannot know the spatial interference pattern incurred at other receiving nodes, but rather only the total interference level.

This work proposes a simplified directional model, referred to herein as the multibeam angular power (MAP) model. The model is sound from the standpoint of previous channel measurements and modeling strategies, captures the non-separable behavior of real channels, and can be represented very compactly to reduce transmission overhead. Subsequent simulations with the MAP model indicate where MIMO-TDMA and MAP-beamforming are more advantageous, suggesting that future air interfaces should support both MIMO and beamforming modes for optimal performance. It is also expected that information provided by modeling methods like MAP are simple and stable enough to be exploited by the medium access control (MAC) layer, allowing effective cross-layer PHY-MAC optimization in future wireless systems.

II. MULTIBEAM ANGULAR POWER MODEL

In this treatment, a narrowband (flat-fading) channel is assumed for simplicity, and extension to the wideband case is also possible. For a transmit and receive node equipped with multiple antennas, the input-output relation is given by $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbf{y}$, $\mathbf{x}$, and $\mathbf{n}$ are vectors of complex baseband output samples, input samples, and noise samples, respectively, and $\mathbf{H}$ is the channel transfer matrix. Assuming transmit and receive basis transformation matrices $\mathbf{B}_T$ and $\mathbf{B}_R$, respectively, such that $\mathbf{x} = \mathbf{B}_T\mathbf{x}'$ and $\mathbf{y}' = \mathbf{B}_R\mathbf{y}$, we have

$$\mathbf{y}' = \mathbf{B}_R\mathbf{H}\mathbf{B}_T\mathbf{x}' + \mathbf{n}' = \mathbf{H}'\mathbf{x}' + \mathbf{n}', \tag{1}$$

which is similar to the method developed in [5], where the $\mathbf{H}'$ are called the "virtual matrix" coefficients. In the present case, however, the basis transformation matrices do not have to be unitary, but should be chosen so that the H'_{ij} are approximately uncorrelated or

$$\mathrm{E}\left\{H'_{ij}{H'_{k\ell}}^*\right\} \approx \delta_{ik}\delta_{j\ell}M_{ij}, \tag{2}$$

where M_{ij} are called the MAP coefficients, capturing the average non-separable power gain from the jth transmit beam to the ith receive beam. This independence could approximately be obtained by judiciously sampling the transmit and receive steering vectors at N_T and N_R points, respectively, using the eigenvectors of the separable transmit and receive covariances [6], etc.

For simulations in this work, transmission maps $\mathbf{M}$ are generated randomly with varying levels of sparseness, by setting

N_{path} entries in $\mathbf{M}$ to a uniform random variable on $(0,1)$ and leaving the other elements equal to zero. The case of *full* multipath is also considered, where all elements of $\mathbf{M}$ are uniform on $(0,1)$.

Extending to the case of N_U transmit and receive nodes, where the nth transmitter and receiver have $N_T^{(n)}$ transmit and $N_R^{(n)}$ receive beam patterns (elements) respectively, the MAP coefficients linking the mth transmitter to the nth receiver are given by $\mathbf{M}^{(n,m)}$. Assuming communication between the nth transmitter and receiver, with power p_j applied to the jth transmit element, the signal power at the ith receive element is

$$p_{S,i}^{(n)} = \sum_j M_{ij}^{(n,n)} p_j^{(n)}, \tag{3}$$

where $\sum_j p_j^{(n)} \leq P_T^{(n)}$. Linear combination of the signals at the receive elements gives a total signal and inteference power of

$$p_S^{(n)} = \sum_i \sum_j \alpha_i^{(n)} M_{ij}^{(n,n)} p_j^{(n)} \quad \text{and} \quad p_I^{(n)} = \sum_i \sum_{m \neq n} \sum_j \alpha_i^{(n)} M_{ij}^{(n,m)} p_j^{(m)}, \tag{4}$$

respectively, where $\sum_i \alpha_i^{(n)} = 1$. When each user has a single transmit stream, the sum capacity is assumed to be

$$C = \sum_n \log_2 \left(1 + \frac{p_S^{(n)}}{p_I^{(n)} + \sigma^{(n)2}} \right). \tag{5}$$

Note that this expression for capacity is only approximate, since the product of the transmission symbols and channel is not strictly Gaussian. The maximization of this expression is handled in Section III.

The multiuser beamforming capacity is compared to the case where TDMA is used to eliminate interference and each user performs water-filling to maximize single-user capacity. The ergodic capacity for this case is estimated by averaging the capacity of 100 random realizations generated with $\mathbf{H}'^{(n)} = \mathbf{G}^{(n)} \odot \sqrt{\mathbf{M}^{(n)}}$, where $\mathbf{G}$ is a matrix of i.i.d. unit-variance complex normal random variables, and $\odot$ and $\sqrt{\cdot}$ are elementwise product and square root, respectively.

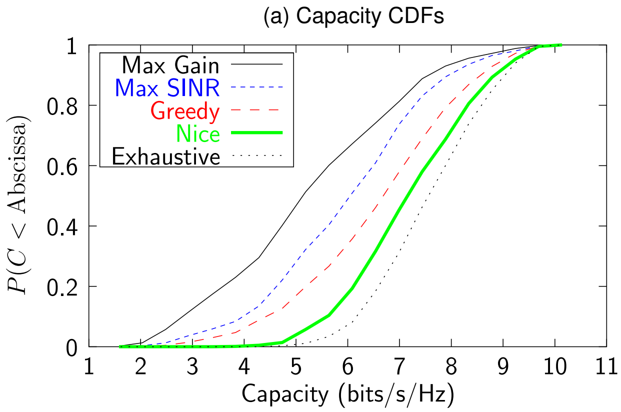
III. Beamforming Optimization Techniques

For the MAP model, optimal beamforming reduces to finding the optimal transmit and receive direction (element) for each user that maximizes the sum capacity in (5). It may be optimal for some users not to transmit at all if they cause too much interference in the network. The transmit/receive state of users is specified by letting $K_T^{(n)}$ be the index of the transmit element for user n that is "on," i.e.

$$K_T^{(n)} = j \;\Leftrightarrow\; p_j^{(n)} = P_T^{(n)}, \quad p_i^{(n)} = 0 \;\; \forall\, i \neq j. \tag{6}$$

A value of $K_T^{(n)} = 0$ indicates that all transmit antennas are off. Similarly, $K_R^{(n)}$ is the index of the receive element that is used. A number of search strategies have been considered:

1) *Exhaustive Search* In this case, all combinations of $K_T^{(n)} \in \{0, 1, \ldots, N_T^{(n)}\}$ and $K_R^{(n)} \in \{1, \ldots, N_R^{(n)}\}$ for all users are checked. This requires comparing $\prod_n N_R^{(n)}(N_T^{(n)} + 1)$ configurations, which can become very large for many users and elements. The exhaustive search is assumed to find the optimal configuration.
2) *Maximum Gain* In this case, for the nth transmit/receive pair, $K_T^{(n)}$ and $K_R^{(n)}$ are chosen to maximize
$$M_{ij}^{(n,n)}, \text{ where } j = K_T^{(n)},\; i = K_R^{(n)}. \tag{7}$$
3) *Interference Suppression/Avoidance* In this case, the desire for maximum gain is tempered by potential interference received and generated, quantified as
$$p_{IR,i}^{(n)} = \sum_j \sum_{m \neq n} M_{ij}^{(n,m)} \quad \text{and} \quad p_{IT,j}^{(n)} = \sum_i \sum_{m \neq n} M_{ij}^{(m,n)}, \tag{8}$$
and each user finds the K_T and K_R that maximize
$$M_{ij}^{(n,n)} \Big/ \sqrt{p_{IR,i}^{(n)} p_{IT,j}^{(n)}}, \quad \text{with } j = K_T^{(n)},\; i = K_R^{(n)}. \tag{9}$$



(b) Parameters

N_U	N_T	N_R	N_{path}	N_{real}	P_T	P_N
3	3	3	5	1000	1	0.1

(c) Performance / Complexity

	Max Gain	Interf Supp/Avoid	Greedy Search	Nice Search	Exhaust Search
$\%C$	70	80	87	95	100
N_{comp}	27	27	69	86	1728

Fig. 1. Simulations comparing capacity of different beam selection techniques: (a) CDFs of sum capacity (b) assumed parameters (c) percent capacity captured and total number of comparisons required

4) *Greedy Search* For this case, an iterative search is performed, where for each iteration, each user adjusts its transmit and receive index to maximize SINR without considering other users. Beginning with $K_T = 1$ and $K_R = 1$ for all users, for a given iteration, user n finds new K_T and K_R to maximize

$$C^{(n)} = \log_2 \left[1 + p_S^{(n)}/(p_I^{(n)} + \sigma^{(n)2})\right], \tag{10}$$

where

$$p_S^{(n)} = M^{(n,n)}_{K_R^{(n)},K_T^{(n)}} P_T^{(n)} \quad \text{and} \quad p_I^{(n)} = \sum_{m \neq n} M^{(n,m)}_{K_R^{(n)},K_T^{(m)}} P_T^{(m)}. \tag{11}$$

This is the same as just maximizing the SINR term inside the log in (10). The iteration stops when the new set of K_T and K_R values is equal to one that has already been tried, which not only stops when a stationary point is found, but also avoids infinite oscillations.

5) *Nice Search* This case is similar to the greedy search, except for each iteration, the nth user checks all possible combinations of its $K_T^{(n)}$ and $K_R^{(n)}$ and picks the one that gives the highest sum capacity given by (5). Each user also can turn "off" ($K_T = 0$) if this improves the sum capacity. The values for signal and interference power are identical to those in (11).

Figure 1(a) shows CDFs of the different methods for 1000 random realizations assuming the parameters in Figure 1(b). Figure 1(c) indicates the computational complexity of the different methods by tabulating the number of comparisons required. For this (and other cases tried), the "Nice Search" seems to come closest to the optimal capacity found with the exhaustive search (within 95%) with modest computational complexity. For the rest of the work, the "Nice Search" is used for beam selection.

IV. Capacity Comparison

Finally, it is of interest to see the performance of beamforming with MAP channels for varying numbers of users and antennas, and levels of channel sparseness and noise. For this comparison, 100 random sets of MAP channels were generated and the capacity for the "Nice Search" computed. Capacity for a TDMA-MIMO system was also computed by generating 100 random channels for each of the 100 realizations above and averaging the water-filling MIMO capacity for each user. The sum capacity for the TDMA-MIMO system is then given by the average capacity of the N_U users.

In the following tabulated results, sum capacity is divided by N_U to give capacity per user. Table I lists the beamforming capacity for the different cases, showing that capacity is highest for high SNR, sparse channels, many elements, and few users. Table II lists TDMA-MIMO ergodic capacity for the same cases. Now, decreasing sparseness (increasing richness) gives higher capacity, and the order of this legend was changed to highlight this trend. Table III takes the ratio of the beamforming capacity to the TDMA-MIMO capacity for the various cases, indicating that beamforming has equal or better performance than TDMA-MIMO in most cases. Beamforming has the most advantage for sparse channels, low SNR, few antennas, and many users. Note that at the extremes, there can be as much as 5x improvement when switching between beamforming and TDMA-MIMO.

TABLE I
BEAMFORMING CAPACITY (PER USER)

SNR	N_{path}	$N_U = 8$ $N_A = 2$	$N_U = 8$ $N_A = 4$	$N_U = 8$ $N_A = 8$	$N_U = 4$ $N_A = 2$	$N_U = 4$ $N_A = 4$	$N_U = 4$ $N_A = 8$	$N_U = 2$ $N_A = 2$	$N_U = 2$ $N_A = 4$	$N_U = 2$ $N_A = 8$
3 dB	Full	0.4	0.4	0.5	0.6	0.8	0.9	1.0	1.2	1.3
	$2N_A$	0.5	0.9	1.3	0.7	1.2	1.4	1.0	1.3	1.5
	$1N_A$	0.6	1.0	1.3	0.8	1.2	1.4	1.0	1.3	1.4
10 dB	Full	0.6	0.7	0.8	1.1	1.3	1.4	1.8	2.2	2.6
	$2N_A$	0.9	1.7	2.8	1.5	2.6	3.2	2.1	3.0	3.3
	$1N_A$	1.2	2.1	3.0	1.6	2.7	3.2	2.2	2.9	3.3
20 dB	Full	0.9	1.0	1.1	1.7	1.9	2.2	3.2	3.5	4.0
	$2N_A$	1.7	3.3	5.5	2.9	5.0	6.4	4.4	6.2	6.5
	$1N_A$	2.3	4.1	6.0	3.5	5.4	6.3	4.7	6.0	6.4

TABLE II
TDMA-MIMO CAPACITY (PER USER)

SNR	N_{path}	$N_U = 8$ $N_A = 2$	$N_U = 8$ $N_A = 4$	$N_U = 8$ $N_A = 8$	$N_U = 4$ $N_A = 2$	$N_U = 4$ $N_A = 4$	$N_U = 4$ $N_A = 8$	$N_U = 2$ $N_A = 2$	$N_U = 2$ $N_A = 4$	$N_U = 2$ $N_A = 8$
3 dB	$1N_A$	0.1	0.2	0.3	0.2	0.3	0.5	0.4	0.7	1.0
	$2N_A$	0.2	0.2	0.4	0.3	0.5	0.7	0.6	1.0	1.4
	Full	0.2	0.4	0.8	0.4	0.8	1.7	0.8	1.7	3.4
10 dB	$1N_A$	0.3	0.4	0.6	0.5	0.8	1.3	1.0	1.7	2.5
	$2N_A$	0.4	0.6	0.9	0.7	1.1	1.7	1.4	2.3	3.5
	Full	0.4	0.9	1.9	0.9	1.8	3.7	1.7	3.6	7.4
20 dB	$1N_A$	0.6	1.0	1.6	1.3	2.0	3.2	2.5	4.1	6.3
	$2N_A$	0.8	1.3	2.2	1.6	2.7	4.5	3.2	5.5	8.8
	Full	1.0	2.0	4.1	2.0	4.0	8.2	3.8	8.1	16.4

TABLE III
RATIO OF BEAMFORMING TO TDMA-MIMO CAPACITY

N_{path}	SNR	$N_U = 2$ $N_A = 8$	$N_U = 2$ $N_A = 4$	$N_U = 2$ $N_A = 2$	$N_U = 4$ $N_A = 8$	$N_U = 4$ $N_A = 4$	$N_U = 4$ $N_A = 2$	$N_U = 8$ $N_A = 8$	$N_U = 8$ $N_A = 4$	$N_U = 8$ $N_A = 2$
Full	20 dB	0.2	0.4	0.8	0.3	0.5	0.9	0.3	0.5	1.0
	10 dB	0.3	0.6	1.1	0.4	0.7	1.2	0.4	0.8	1.4
	3 dB	0.4	0.7	1.2	0.5	0.9	1.5	0.6	1.0	1.8
$2N_A$	20 dB	0.7	1.1	1.4	1.4	1.8	1.8	2.5	2.4	2.2
	10 dB	1.0	1.3	1.5	1.8	2.3	2.2	3.2	3.1	2.7
	3 dB	1.0	1.4	1.7	2.0	2.4	2.5	3.6	3.6	3.2
$1N_A$	20 dB	1.0	1.5	1.9	2.0	2.7	2.7	3.7	4.2	3.6
	10 dB	1.3	1.8	2.2	2.5	3.2	3.2	4.8	5.1	4.5
	3 dB	1.4	1.8	2.3	2.7	3.4	3.6	5.2	5.8	5.5

V. CONCLUSION

This paper has proposed a simple directional channel model that captures the non-separable behavior of realistic propagation channels for multi-user communications. Subsequent analysis with the model indicates regimes where either beamforming or TDMA-MIMO is advantageous, and up to 5x change in the capacity per user could be seen when switching between the two modes. This work suggests the need for future air interfaces to incorporate channel covariance information that can be shared among users as well as the ability to shift between spatial multiplexing and beamforming modes. The use of spatial power information, such as that inherent in the MAP model, is not only a compact way to represent non-separable channels, but also should be sufficient to allow users to decide on a near network-optimal transmission strategy.

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