MODELING THE TIME-VARIANT MIMO CHANNEL

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Keywords: MIMO systems, Time-varying channels, Information theory

Abstract

This paper studies two different modeling strategies for capturing the time-variant behavior of multiple-input multiple-output wireless channels. The first method generates a multi-variate random description of the channel matrix based on the measured space-time covariance. The second method describes the physical variation of the multipath propagation components with time. Information theoretic metrics are used to compare the performance of the models to measured data.

1 Introduction

The performance benefits of using multiple antennas for communicating over multipath propagation channels have been well-documented in the literature. However, the effectiveness of this multiple-input multiple-output (MIMO) communication technology is highly dependent on the availability of high quality channel state information (CSI) at the communicating nodes. In practice, CSI is obtained by periodic transmission of known training data from which the receiver may estimate the channel transfer matrix. However, when the nodes are highly mobile, this training must be frequently repeated, exhausting the available transmission bandwidth and therefore resulting in low effective channel capacities.

These observations suggest that accurate MIMO performance prediction requires an accurate representation of the channel temporal variation as well as the commonly studied spatial characteristics. While the understanding regarding true time-variant channel behavior must be obtained from experimental measurements or detailed electromagnetic simulations, such understanding only becomes useful when it is properly represented in models that can be used in performance simulations of the radio system.

This paper presents two approaches for modeling time-variant MIMO channels. Each strategy is based on measurements of indoor and outdoor MIMO channels at 2.55 and 5.2 GHz. The first method is based on generating multivariate complex normal matrices which possess the correct space-time statistics. These statistics are first obtained by obtaining a parameterized approximation to the function describing the space-time correlation of the channel coefficients from the measured data. This function is then used to computationally create complex normal matrices with the correct covariance structure. For the second method, a time varying model for the double-directional channel angular power spectrum is generated from the measured data. This spectrum is then represented as a set of clusters, with each cluster consisting of multiple plane waves or rays. This description allows stochastic generation of time-varying channels whose physical multipath structure matches that obtained from the measured data.

Once synthetic time-variant channels have been created, it is essential to test the fidelity with which they reproduce the characteristics of the actual (experimentally-measured) MIMO channel. Performing such a comparison requires the development of quantitative measures which capture the channel properties. In this work, information theoretic metrics which approximate the loss in capacity as a function of outdated CSI are used for this comparison [10]. Applying these metrics to both the measured and modelled channels shows that both proposed models provide a good physical representation of the actual channel temporal variation.

2 MIMO Channel Models

Our goal is to explore the extension of conventional MIMO channel modeling techniques to time-varying channels by extracting model parameters from measured data [10] and using information theoretic metrics derived in [10] to determine if the models capture the correct channel behavior. Although many models exist, we focus on (1) a random matrix model following the multivariate complex normal (MVCN) distribution and (2) a physical time-variant clustering (TVC) model. The MVCN model parameters can be directly estimated from collected data, but can be difficult to interpret physically. In contrast, the TVC model exhibits a compelling physical interpretation, although unique extraction of cluster parameters can be difficult.

2.1 MVCN Model

We represent the complex gain from the jth transmitter to the ith receiver at time index n for a single frequency bin as $H_{ij}^{(n)}$. If these gains follow a (possibly time-dependent) MVCN distribution in both time and space, the spatio-temporal variation of the MIMO channel is completely characterized by the multivariate mean ($\boldsymbol{\mu}$) and covariance ($\mathbf{R}$), or

$$\mu_{ij}^{(n)} = \mathrm{E}\left\{H_{ij}^{(n)}\right\} \tag{1}$$

$$R_{ij,k\ell}^{(n,m)} = \mathrm{E}\left\{(H_{ij}^{(n)} - \mu_{ij}^{(n)})(H_{k\ell}^{(n+m)} - \mu_{k\ell}^{(n+m)})^*\right\}, \tag{2}$$

where $\mathrm{E}\{\cdot\}$ is expectation. For a stationary distribution, $\boldsymbol{\mu}$ and $\mathbf{R}$ are not a function of n and can be obtained with sample averages. The difficulty of extracting these parameters from a non-stationary process depends on the severity of the non-stationarity, and may even be impossible for overspread processes [4]. Here, we consider a process characterized by a mean and covariance that vary slowly in time, allowing estimation by weighted sample averages, or

$$\hat{\mu}_{ij}^{(n)} = \sum_{s=-\infty}^{\infty} w_s H_{ij}^{(n+s)} \tag{3}$$

$$\hat{R}_{ij,k\ell}^{(n,m)} = \sum_{s=-\infty}^{\infty} w_{s+m/2} Z_{ij}^{(n+s)} Z_{k\ell}^{(n+s+m)*}, \tag{4}$$

where $Z_{ij}^{(n)} = H_{ij}^{(n)} - \hat{\mu}_{ij}^{(n)}$, and w_s is the weighting window shifted in (4) to apply a weight of w_0 when the points $n+s$ and $n+s+m$ are equidistant from the estimation point n.

The choice of the window is a tradeoff between the bias and variance of the estimator, and optimal windows may be specified if prior information about the distribution is available [11]. In this work, we apply an exponential window, of the form $w_s = \exp(-|s/\ell_c|)$, where ℓ_c is the correlation length. If the process is determined to be nearly stationary over N_s samples, faithful estimates can be obtained with $4\ell_c = N_s$.

2.1.1 Channel Stationarity

Determining the distance over which the channel can be considered a stationary process is non-trivial. One option is application of direct statistical tests for multivariate normality [8] applied to the channel data stacked into a vector. If either the data is non-normal or moments are time-variant, the tests should fail. Thus, N_s can be determined by increasing the size of the data window until these statistical tests begin to indicate non-conformance.

Because no single test is robust against all possible alternative distributions, several tests should be applied to assess normality [5]. We determine a suitable value for ℓ_c by applying three different tests for multivariate normality: (1) Mardia's tests for multivariate skewness and (2) kurtosis [3], and (3) the Henze-Zirkler test [2] with $\beta = 0.5$. One problem of applying these tests is that MIMO channels are apparently not strictly MVCN for large numbers of antennas [8]. Although this presents possible technical difficulties for the MVCN model, assuming MVCN statistics seems to be the only logical and practical starting point. Since we are mainly interested in testing the temporal stationarity of the process (whether constant first and second order statistics represent the data) and not the normality of the data, we apply the tests to all 2×2 antenna subsets of the data, rather than the full 8×8 data. Thus, the tests should indicate for what window size the data is temporally stationary, even if the full 8×8 spatial distributions are not MVCN.

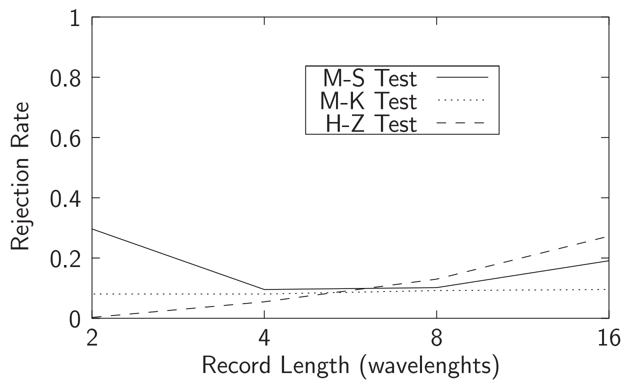

Figure 1: Average rejection rates for three multivariate normal tests for indoor measurements at 2.55 GHz

Figure 1 depicts the average rejection rates for a significance level of 5% and a varying record length for indoor measurements at 2.55 GHz. The results indicate that over distances of $4-8\lambda$ the rejection rates are near the optimal 5%, and we therefore let $\ell_c = 2\lambda/\Delta$, where Δ is the sample spacing ($4\ell_c = N_s$). Applying this test to 5.2 GHz indoor measurements leads to the same value of ℓ_c. For outdoor measurements, application of these tests results in rejection rates aacceptably close to 5% for a record length of $8 - 16\lambda$, implying $\ell_c = 4\lambda/\Delta$ for outdoor measurements.

2.1.2 Synthetic Channel Generation

Once the time-varying mean and covariance have been estimated from the data via (4), we require a way of generating simulated channels. This can be accomplished by forming the full space-time covariance matrix and using it to correlate the elements of i.i.d. complex normal vectors. This approach is numerically prohibitive, however, since for 8 transmitters and receivers and 500 time steps, the covariance matrix has dimensions $32,000\times32,000$. Another approach involves modeling the channel as the output of an autoregressive filter, with weights obtained from the block Yule-Walker equations, fed by spatially and temporally white noise. In practice, however, large arrays and time windows leads to an ill-conditioned system which can not be easily solved.

Perhaps the simplest approach is to assume that the covariance is separable in the time and space or $R_{ij,k\ell}^{(n,m)} = R_{S,ij,k\ell}^{(n)} R_T^{(n,m)}$. Values for the separate space and time covariances are obtained by averaging the full covariance over all time steps and antennas, respectively. The synthetic channels are generated stepwise as

$$B_{ij}^{(n)} = \sum_{n'} X_{T,nn'} A_{ij}^{(n')} \quad (5)$$

$$H_{ij}^{(n)} = \sum_{i'j'} X_{S,ij,i'j'}^{(n)} B_{i'j'}^{(n)}, \quad (6)$$

where $\mathbf{X}_T = \mathbf{R'}_T^{1/2}$, $\mathbf{X}_S^{(n)} = \mathbf{R}_S^{(n)1/2}$, $R'_{T,nn'} = R_T^{(n,n'-n)}$, i and j are stacked when used as a covariance index, and $A_{ij}^{(n)}$ are i.i.d. complex normal random variables.

To reduce the number of model parameters, an average value for the temporal correlation is used at each time step, or

$$R_T^{(n,k)} = (1/N) \sum_{n'=1}^{N} \hat{R}_T^{(n',k)}, \quad (7)$$

where N is the number of time steps considered and $\hat{\mathbf{R}}_T^{(n,m)}$ is the raw estimate of the temporal correlation from the collected data. We refer to this model with a coherent average of the temporal correlations as MVCN(CE) where CE stands for complex envelope. On the other hand, the averaging can be performed incoherently as

$$R_T^{(n,k)} = (1/N) \sum_{n'=1}^{N} |\hat{R}_T^{(n',k)}|, \quad (8)$$

and this is referred to as MVCN(PE) for power envelope.

Forcing this space-time separability and averaging the temporal correlation reduce the accuracy of the MVCN model. However, such simplifications seem necessary to arrive at a model which is reasonable in terms of both computational burden and parametric complexity.

2.2 TVC Model

The double-directional channel concept [7] is a powerful technique for system-independent representation of spatial channels, and much research effort has been dedicated to extracting the parameters for individual multipath components from measured data [1, 6, 7]. Alternatively, we can treat the channel as an incoherent process described by a double-directional power spectrum [9]. This method groups multipath components into clusters of arrivals and departures and estimates only the cluster parameters.

2.2.1 Cluster Extraction

We first compute the double-directional Bartlett spectrum at time step n from the data according to [9]

$$P^{(n)}(\Omega) = \mathbf{b}^H(\Omega)\mathbf{R}_S^{(n)}\mathbf{b}(\Omega), \quad (9)$$

where $\Omega = (\phi_T, \phi_R)$, $b_{ik}(\Omega) = \psi_{R,i}(\phi_R)\ \psi_{T,k}(\phi_T)$ is the joint steering vector with $\psi_{S,i}(\phi_S) = \exp[j2\pi(x_{S,i}\cos\phi_S + y_{S,i}\sin\phi_S)]$, i and k become a single stacked index, S is either T or R for transmit or receive, ϕ_S is azimuth angle, and x_i and y_i are x and y coordinates of the ith antenna. Given a true incoherent arrival power spectrum of $A(\Omega)$, the covariance is

$$\mathbf{R} = \int d\Omega A(\Omega)\mathbf{\Psi}(\Omega), \quad (10)$$

where $\mathbf{\Psi}(\Omega) = \mathbf{\Psi}_T(\phi_T) \otimes \mathbf{\Psi}_R(\phi_R)$ ($\otimes$ is the Kronecker product) and $\mathbf{\Psi}_S(\phi_S) = \boldsymbol{\psi}_S(\phi_S)\boldsymbol{\psi}_S(\phi_S)^H$. Decomposing the true spectrum into basis functions (clusters) $A_p(\Omega)$, we have

$$A(\Omega) = \sum_p a_p A_p(\Omega), \quad (11)$$

and the Bartlett spectrum becomes

$$P^{(n)}(\Omega) = W[A] = \int d\Omega' A(\Omega')\mathbf{b}^H(\Omega)\mathbf{\Psi}(\Omega')\mathbf{b}(\Omega) \quad (12)$$

$$= \sum_p a_p W[A_p]. \quad (13)$$

The advantages of using the Bartlett spectrum instead of the covariance are (1) the resulting equations are real, and (2) covariance structure representing non-propagating modes is removed. By discretizing all functions of Ω and matching left and right hand sides at a number of discrete points, we obtain the matrix equation $\mathbf{p} = \mathbf{Wa}$, which can solved via linear programming for the positive real basis coefficients a_p. In practice, the linear-programming method returns a fairly sparse solution, consisting of only a small set of nonzero coefficients, which we refer to loosely as "clusters."

In this work, we assume a set of Gaussian-shaped basis functions (clusters) with possible arrival angles of $\{0°, 5°, 10°, ..., 355°\}$ and angular spreads of $\{5°, 10°, 20°, 40°\}$. The time-variant nature of the clusters is obtained by the following steps:

1. The average Bartlett spectrum is computed for all time steps in a data record.
2. A set of clusters is estimated from the average Bartlett spectrum, and the dominant clusters, representing 90% of the channel power, are retained.
3. The time-variant Bartlett spectrum is computed for each time step.
4. Using linear programming, best-fit values for a_p are estimated at each time step for the reduced set of clusters found in step 2.

Since the full joint double-directional estimation problem results in a very large coefficient matrix $\mathbf{W}$, we simplify the method by first estimating one-dimensional clusters for single-directional transmit and receive to determine which basis functions are significant. Then, only the significant clusters are used in the joint two-dimensional estimation.

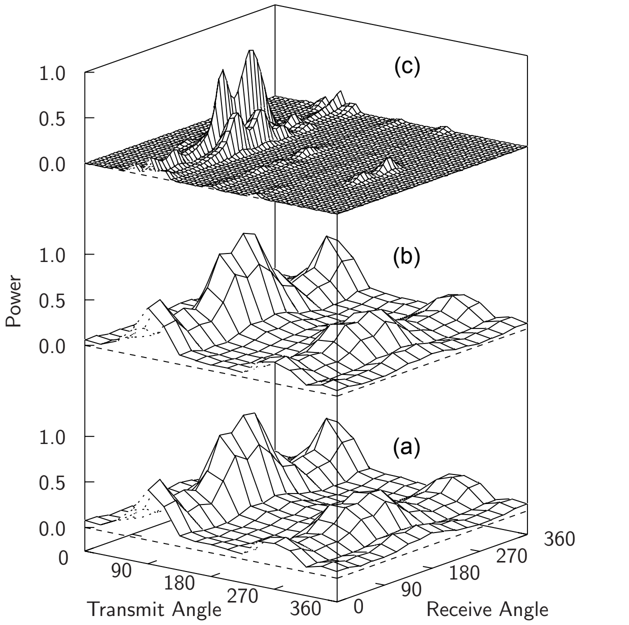

Figure 2: Example time-average spatial spectrum estimate for indoor data at 2.55 GHz: (a) measured and (b) modeled Bartlett spatial spectra and (c) estimated true spectrum

2.2.2 Synthetic Channel Generation

Synthetic channels are generated by assuming L rays per cluster and computing the channel response as

$$H_{ij}^{(n)} = L^{-1/2} \sum_{p,\ell} a_p^{(n)1/2} \beta_{p\ell}\, \psi_{R,i}(\phi_{R,p\ell})\, \psi_{T,j}(\phi_{T,p\ell}), \quad (14)$$

where $\phi_{S,p\ell} \sim \mathcal{N}(\overline{\phi}_{S,p}, \sigma_{S,p}^2)$, $\beta_{p\ell} \sim \mathcal{CN}(0,1)$, $\overline{\phi}_{S,p}$ and $\sigma_{S,p}^2$ are the mean and variance of the departures/arrivals for cluster p, and $\mathcal{N}(\mu, \sigma^2)$ and $\mathcal{CN}(\mu, \sigma^2)$ are the real and complex normal distributions with mean μ and variance σ^2, respectively. Also, note that ψ_S depends implicitly on n, since antenna position changes in time. Extensions to the model include allowing a different number of rays (richness) in each cluster, rays that dynamically appear or disappear in time, a time-variant set of clusters, etc. For this work, a fixed set of $L = 50$ rays per cluster is assumed for each realization of the model. Also, note that this current model only attempts to fit the channel covariance, and has no provision for including non-fading components (channel mean).

2.2.3 Example Application

Figure 2 plots an example of the average double-directional spectrum estimate for indoor data. In nearly all cases, only a few (<10) clusters are required to represent the specified 90% of the time-average power spectrum. Figure 3 depicts the time variation of the cluster coefficients for the three dominant clusters. Also shown is the amount of residual fractional error in the fit, which is close to the ideal 10%. A common feature to nearly all indoor and outdoor data considered is that the same set of clusters remains important over the complete 4.5 m path,

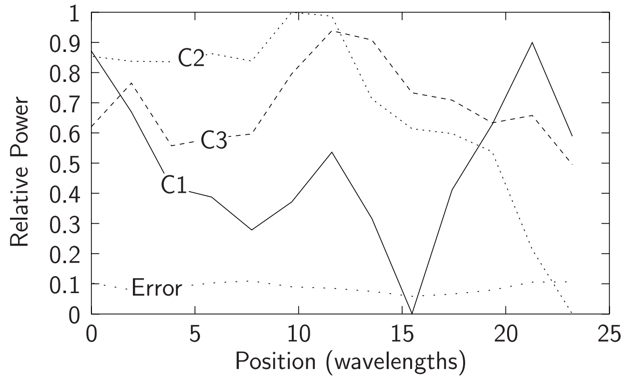

Figure 3: Variation of the cluster power coefficients versus movement distance for indoor data at 2.55 GHz for the dominant three clusters (C1-C3) as well as the fractional error of the fit

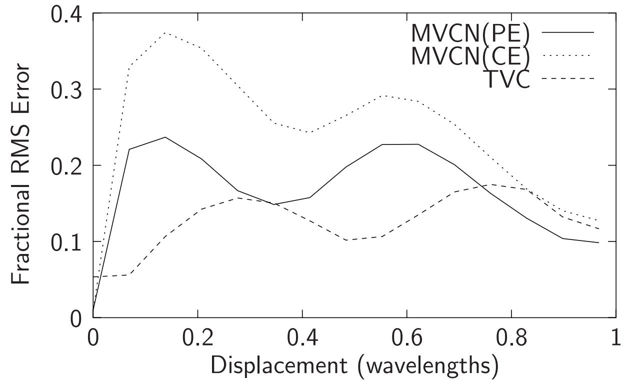

Figure 4: RMS error in the RCD capacity metric for the TVC model and MVCN model with power envelope (PE) and complex envelope (CE) temporal correlation

although the overall power can change dramatically over this distance.

3 Model Comparisons

We now compare the results of applying the TCD and RCD capacity metrics to the models and measured data. We show only the results for indoor data at 2.55 GHz, since all other results lead to similar conclusions.

Figure 4 plots the fractional RMS error in the RCD metric as a function of distance for indoor data for three different models. The discrepancy between the MVCN(CE) and MVCN(PE) results may stem from the fact that the coherent averaging in (7) will tend to underestimate the temporal correlation if the process is not stationary over the entire data window. Figure 5 plots the fractional RMS error of the TCD metric for the same set of data. In this case, the MVCN(PE) and MVCN(CE) models give nearly identical results, so only the results for MVCN(PE) are plotted.

These results suggest that the MVCN model works well for

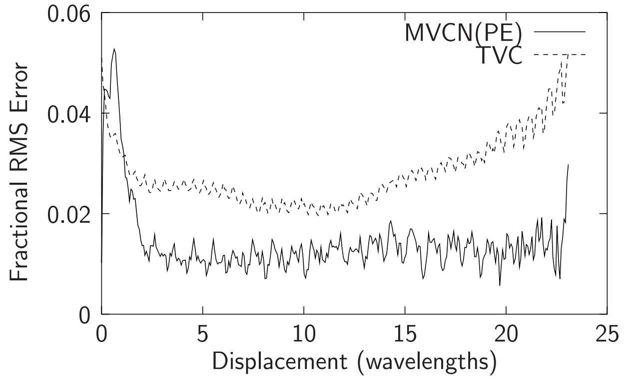

Figure 5: RMS error in the TCD capacity metric for the TVC model and MVCN model with power envelope (PE) temporal correlation

long-term channel variations. This is intuitive, since at large displacements the temporal statistics are independent and only the spatial covariance, which is properly represented at each time step, impacts the results. The failure of the model to adequately match the metrics for short displacements implies that the separable time-space assumption is rather poor. The good performance of the TVC model is somewhat remarkable, considering we only consider a small set of clusters and generate a fixed set of 50 rays for each cluster. This accuracy suggests that the random combination of a constant set of rays properly captures the short-term spatio-temporal covariance.

4 Conclusion

This work introduced two modeling strategies for time-varying MIMO channels: one based on the multivariate complex normal distribution and another physical model based on an incoherent cluster modeling strategy. The model parameters have been extracted from experimentally obtained data, and the resulting model performance has been compared to the same data based on information theoretic metrics. The results show that both models hold promise for accurate modeling of channel temporal variation.

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