

THE ACCURACY OF TENSOR AND DIRECTIONAL METHODS FOR MIMO CHANNEL MODELING

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ABSTRACT

The goal of this work is to identify MIMO modeling strategies with high accuracy and relatively few parameters. Candidates are existing correlation tensor models (Kronecker, Weichselberger, and maximum entropy), a new generalized tensor model based on the higher-order singular value decomposition (HOSVD), and an unstructured diffuse directional model. The performance of the models is investigated by applying them to 8×8 MIMO reference channels generated with a realistic double-directional cluster channel model. It is shown that all of the existing models exhibit high error in the reconstructed spatial spectra (10-30%) and even higher error in the reconstructed full covariance (20-35%). Analysis of a new sparse core tensor model indicates that this error decays slowly (logarithmically) as the number of parameters is increased. For the same number of parameters as the simple Kronecker model, the directional model is able to reconstruct the spectrum with improved accuracy (about 3% error). More surprisingly, the error in the reconstructed covariance matrices (about 16%) is also substantially lower than all of the reduced-order correlation tensor methods.

Key words: MIMO; model; tensor; kronecker; SVD.

1. INTRODUCTION

Characterizing the performance of multiple-input multiple-output (MIMO) systems is facilitated by accurate models that capture the true spatial behavior of MIMO channels. Although direct measurement provides a nearly exact characterization of site-specific scenarios, there are a number of drawbacks: measurement campaigns are costly, the resulting data is not always openly available, working with measured data can be cumbersome, and only relatively few scenarios can be measured. MIMO models, on the other hand, are relatively simple to work with, but the accuracy is often questionable, especially when only a few parameters describing the channel are available. An important question, therefore, is how to represent MIMO channels with high accuracy, yet with relatively few parameters.

MIMO modeling efforts have focused mainly on random matrix based methods assuming a zero-mean joint complex Gaussian distribution for the elements of the

channel transfer matrix. For narrowband MIMO channels, this distribution is completely specified by an $(N_T N_R) \times (N_T N_R)$ covariance matrix (referred to herein as the *full covariance*), where N_T and N_R are the number of transmit and receive antennas, respectively. Early theoretical studies assumed an i.i.d. distribution (full covariance equal to the identity matrix), which has the ideal property of linear capacity growth with additional transmit/receive antennas. Subsequent measurement campaigns indicated lower capacities than the ideal channel, due to inherent spatial correlation created by the limited multipath of the propagation channel.

The inclusion of correlation information in the random matrix models has been accomplished in a number of ways. An initial logical approach assumed that the covariance of channel transfer matrix elements could be represented by a product of two separate contributions (one due to the transmit environment and another due to the receive environment). This separable correlation structure results in the Kronecker model [10], approximating the full covariance as a separable product of transmit and receive covariances, and improving modeled capacity predictions. The so-called Weichselberger model [9] further improved the accuracy of the Kronecker model by only enforcing a separable structure for the covariance eigenvectors, not the eigenvalues.

A different modeling strategy known as the double-directional channel was developed in [6], employing information about wave propagation mechanisms combined with knowledge of the array configuration. Such models boast high accuracy, but usually require detailed measurements and data processing to identify directions of arrival and departure of the multipath. This same basic double-directional strategy was used in later work to extract clusters of arrivals from the full covariance, rather than individual multipath components [7]. Such techniques have the potential of representing the full covariance with high accuracy and relatively few parameters compared to the matrix based methods.

A widespread feature of past MIMO channel model validation efforts is the use of channel capacity as the only metric to judge model accuracy. Although capacity is perhaps the most important metric, intuition suggests that capacity will always be somewhat insensitive to the exact nature of the multipath, since capacity only depends on a weighted combination of the channel singular values, and not at all on the singular vectors, which contain most of the directional information. For example, use of the double-directional spatial spectrum indicates that the

Kronecker model in particular significantly distorts the directional channel response [4]. Such errors could lead to incorrect assessment of specific antenna array structures or directional transmission algorithms.

This work comprises three main contributions: First, we compare the relative accuracy of random matrix methods and a directional method in terms of their ability to match the full covariance, which is a much more sensitive indicator of accuracy than capacity. We also compare the models in terms of the number of parameters required. Second, we investigate fundamental limitations of separable transmit/receive models by providing a maximum entropy alternative to the Kronecker model, indicating whether or not joint transmit/receive information is required for high accuracy. Third, we generalize the concept of random-matrix modeling using the correlation tensor concept that was proposed in [1] for MIMO channels. Our contribution in this area is to use the higher-order singular value decomposition (HOSVD), which is an extension of the matrix SVD for tensors [2]. The different models are approximated by special cases of a general HOSVD model with different specifications of the core tensor. This analysis indicates a fundamental trade-off between the number of core tensor entries and model accuracy, and approximately logarithmic improvement in accuracy with the number of parameters.

2. CHANNEL MODELS

This section presents background material on existing modeling strategies as well as our new proposed models. Although we consider only narrowband stationary channels for simplicity, the tensor based framework easily accommodates wideband and non-stationary extensions.

2.1. General Tensor Model

First we present the general tensor model, since other random matrix models can be treated as special cases. Given an $N_R \times N_T$ channel matrix \mathbf{H} described by a multivariate zero mean complex Gaussian distribution, the channel behavior is completely characterized by the fourth-order covariance tensor

$$\mathcal{R}_{i_1 j_1 i_2 j_2} = \mathbb{E} \{ H_{i_1 j_1} H_{i_2 j_2}^* \}. \quad (1)$$

The information in the correlation tensor can be represented as a matrix by stacking the channel into a vector according to $\mathbf{h} = \text{Vec} \{ \mathbf{H} \}$, and letting $\mathbf{R} = \mathbb{E} \{ \mathbf{h} \mathbf{h}^H \}$. This has the same effect as stacking the i and j indices as $k_1 = i_1 + N_R(j_1 - 1)$ and $k_2 = i_2 + N_R(j_2 - 1)$ in (1) to obtain $R_{k_1 k_2}$. This operation can be represented symbolically as

$$R_{[i_1 j_1][i_2 j_2]} = \mathcal{R}_{i_1 j_1 i_2 j_2}, \quad (2)$$

which indicates that the dimensions in brackets have been stacked (or *unfolded* using tensor terminology). Such stacking is familiar to anyone who has resized multidimensional arrays in numerical matrix software such as MATLAB or Octave. The tensor and matrix representations are referred to as the *full covariance* tensor and matrix, respectively, and these are assumed to capture the channel spatial correlations exactly.

A useful method for considering reduced-order approximations of the full covariance is obtained through the HOSVD. As described in [2], any N th order tensor \mathcal{A} can be represented by the decomposition

$$\mathcal{A} = \mathcal{S} \times_1 \mathbf{U}^{(1)} \times_2 \mathbf{U}^{(2)} \cdots \times_N \mathbf{U}^{(N)}, \quad (3)$$

where \mathcal{S} is the N th order *core tensor*, $\mathbf{U}^{(n)}$ is the matrix of n -mode singular values, and \times_n is the n -mode product defined as

$$\begin{aligned} (\mathcal{A} \times_n \mathbf{U})_{i_1 i_2 \dots i_{n-1} j_n i_{n+1} \dots i_N} \\ = \sum_{i_n} \mathcal{A}_{i_1 i_2 \dots i_{n-1} i_n i_{n+1} \dots i_N} U_{j_n i_n}. \end{aligned} \quad (4)$$

The $\mathbf{U}^{(n)}$ are given by the left singular vectors of $\mathbf{A}^{(n)}$, with elements

$$\{ \mathbf{A}^{(n)} \}_{i_n [i_{n-1} i_{n-2} \dots i_1 i_N i_{N-1} \dots i_{n+1}]} = \mathcal{A}_{i_1 \dots i_N}, \quad (5)$$

which is the n th matrix unfolding of the tensor \mathcal{A} . Note that due to the Hermitian structure of the covariance, we have $\mathbf{U}^{(1)} = \mathbf{U}^{(3)}$ and $\mathbf{U}^{(2)} = \mathbf{U}^{(4)}$, and the HOSVD is really a higher-order eigenvalue decomposition (HO-EVD).

In the HOSVD, the core tensor \mathcal{S} is not necessarily diagonal as with the matrix SVD, but rather has the so-called ‘‘all orthogonality’’ property

$$\langle \mathcal{S}_{i_n=\alpha}, \mathcal{S}_{i_n=\beta} \rangle = 0 \text{ when } \alpha \neq \beta, \quad (6)$$

where the single subscript indicates that a specific index is held constant to produce a sub tensor with $N - 1$ dimensions, and the inner-product operator is defined as

$$\langle \mathcal{A}, \mathcal{B} \rangle = \sum_{i_1} \sum_{i_2} \cdots \sum_{i_N} b_{i_1 i_2 \dots i_N}^* a_{i_1 i_2 \dots i_N}. \quad (7)$$

Since \mathcal{S} is in general a full tensor, the specific elements cannot be ordered like with the matrix SVD. Instead, the ordering is

$$\| \mathcal{S}_{i_n=1} \| \geq \| \mathcal{S}_{i_n=2} \| \geq \cdots \geq \| \mathcal{S}_{i_n=N} \| \geq 0, \quad (8)$$

where the Frobenius norm for tensors is

$$\| \mathcal{A} \| = \sqrt{\langle \mathcal{A}, \mathcal{A} \rangle}. \quad (9)$$

Later we will also have need of the outer product operator, defined as

$$(\mathcal{A} \circ \mathcal{B})_{i_1 i_2 \dots i_N j_1 j_2 \dots j_N} = \mathcal{A}_{i_1 i_2 \dots i_N} \mathcal{B}_{j_1 j_2 \dots j_N}. \quad (10)$$

In this work, we define the M th order sparse core tensor model as the tensor $\hat{\mathcal{R}}$ formed by approximating the full core tensor with $\hat{\mathcal{S}}$, which is equal to \mathcal{S} for the M entries with largest magnitude and zero elsewhere. Note that $(\hat{\cdot})$ refer to assumed or approximate quantities in this paper. Although not guaranteed to be the best M th order representation (as would be the case with the matrix SVD), as argued in [2], this method should still be a very good reduced rank approximation, allowing the overall tradeoff of complexity and accuracy to be investigated. We also define the *principal hyperplane* approximation by specifying the approximate core tensor to be $\hat{\mathcal{S}}_{i_1 j_1 i_2 j_2} = \mathcal{S}_{i_1 j_1 i_1 j_1} \delta_{i_1 i_2} \delta_{j_1 j_2}$.

2.2. Kronecker Model

The Kronecker model assumes that the full covariance matrix can be expressed as the Kronecker product of the separate transmit and receive covariances according to

$$R_{T,ij} = \frac{1}{\alpha} \sum_{k=1}^{N_R} \mathcal{R}_{kikj} = \frac{1}{\alpha} \mathbb{E} \{ (\mathbf{H}^H \mathbf{H})^T \}_{ij} \quad (11)$$

$$R_{R,ij} = \frac{1}{\beta} \sum_{k=1}^{N_T} \mathcal{R}_{ikjk} = \frac{1}{\beta} \mathbb{E} \{ \mathbf{H} \mathbf{H}^H \}_{ij}, \quad (12)$$

respectively, where for consistency, the scale factors α and β satisfy

$$\alpha\beta = \sum_{i_1=1}^{N_R} \sum_{j_1=1}^{N_T} \mathcal{R}_{i_1 j_1 i_1 j_1} = \text{Tr} \{ \mathbf{R} \}. \quad (13)$$

For simplicity, we can let $\alpha = \beta = \sqrt{\text{Tr} \{ \mathbf{R} \}}$.

In the Kronecker model, the full covariance matrix is assumed to be

$$\hat{\mathbf{R}} = \mathbf{R}_T \otimes \mathbf{R}_R. \quad (14)$$

If the eigenvalue decompositions (EVD) of the transmit and receive covariances are $\mathbf{R}_T = \mathbf{U}_T \mathbf{\Lambda}_T \mathbf{U}_T^H$ and $\mathbf{R}_R = \mathbf{U}_R \mathbf{\Lambda}_R \mathbf{U}_R^H$, respectively, the EVD of the full covariance is

$$\hat{\mathbf{R}} = (\mathbf{U}_T \otimes \mathbf{U}_R) (\mathbf{\Lambda}_T \otimes \mathbf{\Lambda}_R) (\mathbf{U}_T \otimes \mathbf{U}_R)^H. \quad (15)$$

Compared with the HOSVD we have

$$\hat{\mathcal{R}} = \mathcal{A} \times_1 \mathbf{U}_R \times_2 \mathbf{U}_T \times_3 \mathbf{U}_R^* \times_4 \mathbf{U}_T^* \quad (16)$$

$$\mathcal{A}_{i_1 j_1 i_2 j_2} = \mathbf{\Lambda}_{R, i_1 i_2} \mathbf{\Lambda}_{T, j_1 j_2} = \lambda_{R, i_1} \lambda_{T, j_1} \delta_{i_1 i_2} \delta_{j_1 j_2}.$$

Thus the Kronecker model is basically a principal hyperplane approximation where the core tensor values have a separable structure, meaning that there are only $N_T + N_R$ unique parameters in the core tensor. The total number of unique real parameters for this model is found to be $N_R^2 + N_T^2$, or $N_R^2 + N_T^2 + N_R + N_T$ if the eigenvalues and eigenvectors are to be specified separately.

2.3. Separable Maximum Entropy Model

It has been argued that the Kronecker model enforces a structure on the data that may be artificial. Supposing that only the separate transmit and receive covariances are known, another approach to defining the full covariance is to use the principle of maximum entropy [3], thus avoiding "artificial structure" that cannot be deduced from the information at hand. In this case, we have constraints

$$\mathbb{E} \{ \mathbf{H} \mathbf{H}^H \} = \mathbf{R}_R \quad \mathbb{E} \left\{ \sum_k H_{ik} H_{jk}^* \right\} = R_{R,ij} \quad (17)$$

$$\mathbb{E} \{ \mathbf{H}^H \mathbf{H} \} = \mathbf{R}_T^T \quad \mathbb{E} \left\{ \sum_k H_{ki} H_{kj}^* \right\} = R_{T,ij} \quad (18)$$

$$p(\mathbf{H}) \geq 0, \quad \int p(\mathbf{H}) d\mathbf{H} = 1. \quad (19)$$

where $p(\mathbf{H})$ is the joint probability density function (pdf) of the elements of \mathbf{H} . The entropy

$$\int p(\mathbf{H}) \log p(\mathbf{H}) d\mathbf{H} \quad (20)$$

is maximized by forming the Lagrangian and setting the derivative equal to zero, resulting in the form of the solution

$$p(\mathbf{H}) = c_0 \exp \left[\sum_{ijkl} H_{ij} H_{kl}^* \underbrace{(\mu_{R,ik} \delta_{jl} + \mu_{T,jl} \delta_{ik})}_{-R_{[ij][kl]}^{-1}} \right], \quad (21)$$

which is a standard multivariate complex Gaussian pdf with covariance

$$\mathbf{R} = - \underbrace{(\mathbf{I}_T \otimes \mathbf{\mu}_R + \mathbf{\mu}_T \otimes \mathbf{I}_R)}_{\mathbf{A}}^{-1}. \quad (22)$$

Note that although expressed with Kronecker products, this form is not identical to the Kronecker model. Taking the eigenvalue decomposition (EVD) of $\mathbf{\mu}_T$ and $\mathbf{\mu}_R$, or

$$\mathbf{\mu}_T = \boldsymbol{\xi}_T \mathbf{\Lambda}_T \boldsymbol{\xi}_T^H, \quad \mathbf{\mu}_R = \boldsymbol{\xi}_R \mathbf{\Lambda}_R \boldsymbol{\xi}_R^H, \quad (23)$$

allows (22) to be expanded as

$$\mathbf{R}^{-1} = \underbrace{(\boldsymbol{\xi}_T \otimes \boldsymbol{\xi}_R)}_{\boldsymbol{\xi}} \underbrace{(\mathbf{I}_T \otimes \mathbf{\Lambda}_R + \mathbf{\Lambda}_T \otimes \mathbf{I}_R)}_{\mathbf{\Lambda}} \underbrace{(\boldsymbol{\xi}_T \otimes \boldsymbol{\xi}_R)^H}_{\boldsymbol{\xi}^H}, \quad (24)$$

such that $\mathbf{R} = -\boldsymbol{\xi} \mathbf{\Lambda}^{-1} \boldsymbol{\xi}^H$. The matrix $\mathbf{\Lambda}$ involves Kronecker products and sums of diagonal matrices, which will also be diagonal, or

$$\Lambda_{[ij][kl]} = \delta_{j\ell} \delta_{ik} \lambda_{R,i} + \delta_{j\ell} \delta_{ik} \lambda_{T,j} = \delta_{j\ell} \delta_{ik} [\lambda_{R,i} + \lambda_{T,j}] \quad (25)$$

$$\Lambda_{[ij][kl]}^{-1} = \delta_{j\ell} \delta_{ik} \frac{1}{\lambda_{R,i} + \lambda_{T,j}}. \quad (26)$$

Therefore, we have the form of the full covariance matrix. The eigenvectors are just the Kronecker product of the separate transmit and receive eigenvectors. The eigenvalues, however, must be found by substituting \mathbf{R} back into the original constraints. Although the constraints (17) and (18) depend on expectations of \mathbf{H} , they can be rewritten in terms of the full covariance as

$$R_{R,ik} = \sum_j R_{[ij][kj]}, \quad R_{T,j\ell} = \sum_i R_{[ij][i\ell]}. \quad (27)$$

The covariance matrix from (24) can be written in component form as

$$R_{[ij][kl]} = \{ -\mathbf{A}^{-1} \}_{[ij][kl]} = - \sum_{mn} \xi_{[ij][mn]} \Lambda_{[mn][mn]}^{-1} \xi_{[kl][mn]}^*. \quad (28)$$

Substituting (28) into (27) results in

$$\{-\mathbf{R}_T\}_{jl} = \sum_{mn} \sum_i \xi_{[ij][mn]} \Lambda_{[mn][mn]}^{-1} \xi_{[i\ell][mn]}^* \quad (29)$$

$$= \sum_{mn} \xi_{T,jn} \xi_{T,\ell n}^* \Lambda_{[mn][mn]}^{-1} \underbrace{\sum_i \xi_{R,im} \xi_{R,im}^*}_{=1} \quad (30)$$

$$= \sum_n \xi_{T,jn} \xi_{T,\ell n}^* \underbrace{\sum_m \Lambda_{[mn][mn]}^{-1}}_{-D_{T,nn}}, \quad (31)$$

Similarly, one can expand the receive covariance constraint as

$$\{-\mathbf{R}_R\}_{ik} = \sum_m \xi_{R,im} \xi_{R,km}^* \underbrace{\sum_n \Lambda_{[mn][mn]}^{-1}}_{-D_{R,mm}}. \quad (32)$$

We recognize (31) and (32) as just the eigenvalue decompositions of our transmit and receive covariances,

$$\mathbf{R}_T = \boldsymbol{\xi}_T \mathbf{D}_T \boldsymbol{\xi}_T^H \quad (33)$$

$$\mathbf{R}_R = \boldsymbol{\xi}_R \mathbf{D}_R \boldsymbol{\xi}_R^H. \quad (34)$$

To solve for the full covariance, the system of equations

$$D_{T,nn} = d_{T,n} = - \sum_m \frac{1}{\lambda_{R,m} + \lambda_{T,n}} \quad (35)$$

$$D_{R,mm} = d_{R,m} = - \sum_n \frac{1}{\lambda_{R,m} + \lambda_{T,n}} \quad (36)$$

must be solved, but a direct solution can be problematic since we have to find the roots of multivariate large order polynomials. An indirect approach is possible by noticing that since \mathbf{H} is a Gaussian process, maximum entropy maximizes $\det(\mathbf{R})$. Since we already know the eigenvectors, we only need find Λ , such that $\det(-\Lambda^{-1})$ is maximized. Letting

$$f_{mn} = -\Lambda_{[mn][mn]}^{-1}, \quad (37)$$

we must find the maximum of $\det(\mathbf{R}) = \prod_{ij} f_{ij}$, subject to the constraints

$$d_{T,j} = \sum_i f_{ij}, \quad d_{R,i} = \sum_j f_{ij}, \quad f_{ij} \geq 0. \quad (38)$$

Since the constraints are linear and $-\prod_{ij} f_{ij}$ is convex, we have a simple convex optimization problem that can be solved with conventional techniques. In this paper, the solution is obtained by finding an admissible initial guess for the f_{ij} using linear programming and then moving to the optimum with gradient descent.

This maximum entropy method generates a model with only the separate transmit/receive information at hand, which conceptually is the ‘‘best one can do’’ when no joint transmit/receive information is available, since for any other model, additional information must be incorporated. This model has the same basic structure as the Kronecker model, except that the core tensor is generated from the $N_T + N_R$ unique parameters in a slightly different way.

2.4. Weichselberger Model

This model also has the same basic form as the Kronecker model, except that one requirement is relaxed: the eigenvalues of the full covariance matrix are no longer required to be the separable product of the transmit and receive eigenvalues. In this case, the full covariance matrix is

$$\hat{\mathbf{R}} = (\mathbf{U}_T \otimes \mathbf{U}_R) \boldsymbol{\Lambda} (\mathbf{U}_T \otimes \mathbf{U}_R)^H, \quad (39)$$

where $\boldsymbol{\Lambda}$ is a diagonal $N_T N_R \times N_T N_R$ matrix with general entries along the diagonal. In tensor notation,

$$\hat{\mathcal{R}} = \mathcal{A} \times_1 \mathbf{U}_R \times_2 \mathbf{U}_T \times_3 \mathbf{U}_R^* \times_4 \mathbf{U}_T^* \\ \mathcal{A}_{i_1 j_1 i_2 j_2} = \Lambda_{[i_1 j_1][i_1 j_1]} \delta_{i_1 i_2} \delta_{j_1 j_2}. \quad (40)$$

The eigenvalues (which have also been called coupling coefficients) are found by projecting the eigenvectors onto the covariance matrix according to

$$\Lambda_{[ij][ij]} = \mathcal{R} \times_1 \mathbf{U}_R^{(i)} \times_2 \mathbf{U}_T^{(j)} \times_3 \mathbf{U}_R^{(i)*} \times_4 \mathbf{U}_T^{(j)*}, \quad (41)$$

where $\mathbf{U}^{(k)}$ here denotes the k th column. Note that this is precisely the same way the core tensor values along the principal hyperplane \mathcal{S}_{ijij} are computed in the general HOSVD. Thus, we expect the accuracy of the Weichselberger model to be similar to the principal hyperplane model already presented. The total number of real parameters is $N_R^2 + N_T^2 + N_R N_T$.

2.5. Directional Model

The initial formulation of the double-directional modeling concept used super-resolution methods to estimate the exact angles of arrival and departure of multipath components, extracted from measured channel data. Another possibility is to take a more statistical approach and model the covariance as a diffuse process. In this work, we consider the case of an unstructured diffuse model, where the ideal joint transmit/receive spatial spectrum is divided into $L_R \times L_T$ sectors of equal angular extent. The ideal joint transmit/receive spatial spectrum is assumed to be piecewise constant, or

$$V(\phi_R, \phi_T) = \sum_{m=1}^{L_R} \sum_{n=1}^{L_T} V_{mn} f_{mn}(\phi_R, \phi_T), \quad (42)$$

where the two-dimensional (2D) pulse functions are

$$f_{mn}(\phi_R, \phi_T) = \begin{cases} 1, & |\phi_R - \phi_{R,m}| \leq \frac{\Delta\phi_R}{2} \\ & |\phi_T - \phi_{T,n}| \leq \frac{\Delta\phi_T}{2}, \\ 0, & \text{otherwise.} \end{cases} \quad (43)$$

where $\Delta\phi_P$ is the angular extent of the pulse function. The full covariance tensor is found as

$$\mathcal{R} = \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_T V(\phi_R, \phi_T) \mathcal{L}(\phi_R, \phi_T), \\ \mathcal{L}(\phi_R, \phi_T) = \mathbf{a}_R(\phi_T) \circ \mathbf{a}_T(\phi_T) \circ \mathbf{a}_R^*(\phi_R) \circ \mathbf{a}_T^*(\phi_T), \quad (44)$$

where the steering vector $\{\mathbf{a}_P(\phi)\}_i = \exp[jk(x_i^P \cos \phi + y_i^P \sin \phi)]$, k is the wavenumber, and x_i^P and y_i^P are coordinates of the i th antenna for $P = T$ for transmit or $P = R$ for receive. The joint 2D Bartlett spatial spectrum is formed as

$$B(\phi_R, \phi_T) = \langle \mathcal{R}, \mathcal{L}(\phi_R, \phi_T) \rangle. \quad (45)$$

Sampling this function at K_R and K_T discrete angles at receive and transmit, denoted as $\bar{\phi}_{R,k}$ and $\bar{\phi}_{T,\ell}$ to obtain $B_{k\ell} = B(\bar{\phi}_{R,k}, \bar{\phi}_{T,\ell})$ and making the required substitutions

$$B_{k\ell} = \sum_{m=1}^{L_R} \sum_{n=1}^{L_T} V_{mn} \int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_T \quad (46)$$

$$\times f_{mn}(\phi_R, \phi_T) \langle \mathcal{L}(\phi_R, \phi_T), \mathcal{L}(\bar{\phi}_{R,k}, \bar{\phi}_{T,\ell}) \rangle,$$

where the double integral can be computed from known information and can be represented as the tensor \mathcal{Q}_{klmn} . Stacking the dimensions appropriately

$$b_{[k\ell]} = \sum_{[mn]} \mathcal{Q}_{[k\ell][mn]} v_{[mn]}, \quad (47)$$

which can be solved using linear programming for \mathbf{v} when \mathbf{b} is known. In practice \mathbf{b} is computed from (45) using the sample full covariance for \mathcal{R} . Note that although the intermediate step of using the Bartlett beamformer is not necessary, it simplifies the solution by making all the elements of \mathcal{Q} real and positive and by removing any contributions from non-plane wave propagation. The resulting directional model only requires $L_R L_T$ parameters, each of which is a positive real value. Also, the linear programming solution tends to favor a sparse representation, so many of the $v_{[mn]}$ are identically zero in practice.

3. PERFORMANCE COMPARISON

The different models are compared by computing the fractional error in the modeled covariance matrix, or

$$\epsilon = \frac{\|\hat{\mathcal{R}} - \mathcal{R}\|}{\|\mathcal{R}\|}. \quad (48)$$

Additionally, we consider the error in the actual and modeled Bartlett spectra as

$$\epsilon_B = \frac{\int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_T |\hat{B}(\phi_R, \phi_T) - B(\phi_R, \phi_T)|}{\int_0^{2\pi} d\phi_R \int_0^{2\pi} d\phi_T |B(\phi_R, \phi_T)|}. \quad (49)$$

In the simulations that follow, full covariance matrices are generated using the narrowband Saleh-Valenzuela angular (SVA) model [8], assuming Laplacian shaped clusters with angular spread $\sigma = 26^\circ$ at both transmit and receive, unit arrival rate, and decay rate $\Gamma = 2$. These parameters are consistent with measurements of indoor MIMO channels [5]. Arrays at transmit and receive are 8-element uniform linear arrays (ULAs) of ideal dipoles with $\lambda/2$ interelement spacing.

For the directional channel model, $L_R = L_T = L = 12$ with corresponding angles $\phi_k = \pi(k - 0.5)/L$.

Note that for the ULA, only angles on one side of the symmetric array need be considered. The Bartlett spectrum is matched at $K_R = K_T = K = 32$ discrete angles given by $\bar{\phi}_k = \pi(k - 0.5)/K$.

Figure 1 plots the Bartlett spatial spectrum for the various models for a single random channel realization, indicating that the Kronecker model (b) significantly distorts the directional response of the true full covariance (a). The Weichselberger model (c) offers only a small improvement in the match of the spectrum over the Kronecker model. The general tensor models begin to approximate the true spectrum much better for the principal hyperplane (d) and general sparse core tensor (e), both of which have $N_T N_R$ nonzero core tensor entries. Finally, the directional model matches the Bartlett spectrum very well, which is not surprising since the method fits this directly. However, this fitting is accomplished with only as many parameters as required by the Kronecker model.

Next we look at the statistical behavior averaged over 100 random realizations of the channel model, and consider the accuracy of the sparse core tensor model, where only M of the largest core tensor entries are retained. Figure 2 plots the average fractional error in the covariance and Bartlett spectrum with respect to M . The result indicates that the error drops approximately linearly with the logarithm of the number of core entries until a knee at $N_T N_R$ entries, indicating that models with fewer than $N_T N_R$ eigenvalues will always suffer low accuracy.

Table 1 lists the number of parameters in the various models and the average accuracy of the modeled covariance and spatial spectra for the 100 realizations. First, we see that the maximum entropy method does not offer any improvement over the Kronecker model, suggesting that the Kronecker model is basically optimal when nothing more than the separable transmit/receive information is available. Second, we see that the Weichselberger model and the principal hyperplane core tensor model are similar, as expected. Third, we see that the directional model has superior accuracy to all of the random matrix models as well as relatively few parameters. Although seemingly remarkable, the parameters of the directional model focus on *joint* transmit/receive information, whereas the parameters of the tensor models are mostly *separate* transmit/receive information. It also seems reasonable that by matching the Bartlett spectrum, rather than the covariance matrix directly, the directional model is able to avoid wasting parameters on eigenvector pairs that have little contribution to the channel response.

4. CONCLUSION

This paper has compared the ability of random matrix and directional channel models to capture the behavior of realistic MIMO channels. The performance of the models was compared in terms of the error in the estimated full covariance matrices, error in the Bartlett spectra, and the number of parameters required. The results suggest that models without sufficient joint transmit/receive information suffer from inherently low accuracy. The importance of joint transmit/receive information was further validated by the superior performance of the directional model compared to all of the tensor-based methods.

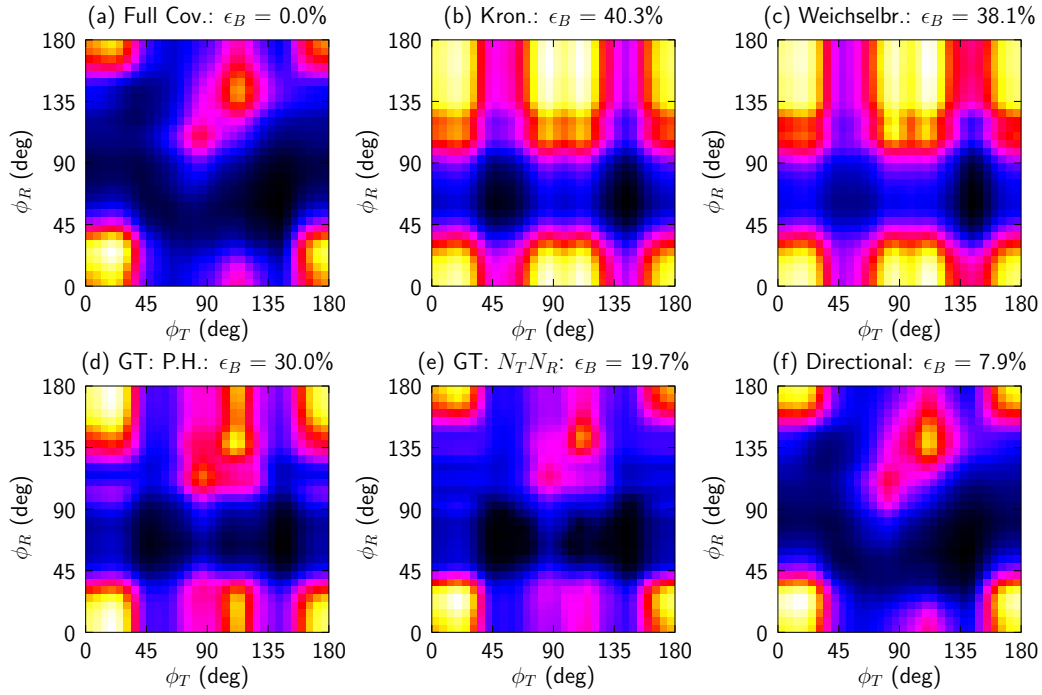


Figure 1. Bartlett spatial spectra of a single realization of the reference channel model for the various approximations to the true covariance

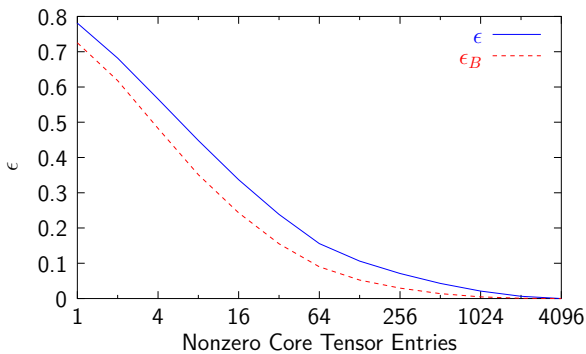


Figure 2. Average error as a function of the number of nonzero core tensor entries

Table 1. Average Model Error

Model	Parameters	ϵ	ϵ_B
Kronecker	144	0.32	0.25
Maximum Entropy	144	0.35	0.27
Weichselberger	192	0.24	0.16
P. Hyperplane	192	0.21	0.13
Directional	144	0.16	0.03

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