Communicating on MIMO channels with covariance information: antenna correlation and coupling

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***Abstract*— A recent study on correlated block-fading MIMO channels with transmit covariance information indicates guaranteed capacity growth with additional transmit elements, in contrast to previous results for uncorrelated channels. Also, for very rapidly fading channels, the results indicate the optimality of placing antenna elements as close together as possible. In this work, application of radiated power considerations indicates that transmit beamforming only increases capacity when this gain results from correlation inherent in the channel, not correlation that is simply offset by increased antenna coupling. The new analysis reveals that when multipath is directionally biased, antenna spacings of 0.3 to 0.6 wavelengths are optimal, and that when no such bias is present, antennas should be placed as far apart as possible. As expected, the electromagnetic analysis shows that at zero element separation, the array yields no capacity gain over a single antenna.**

I. Introduction

In rich multipath environments, node mobility limits the quality of attainable channel state information (CSI) due to wavelength-scale fast fading, leading to effectively lower channel capacities [1–3]. Analytical results for block fading i.i.d. Gaussian channels with block length T indicate a severe drop in capacity as T becomes small and that having more than T antennas does not increase capacity [4]. More recent work studies the capacity of block-fading Gaussian MIMO channels with Kronecker correlation. For this case, capacity always increases with additional transmit antennas beyond T, as long as the transmit antennas are correlated. For very rapidly fading channels (T=1) the analysis indicates that antennas should be placed as close together as possible for maximum capacity [5].

Instead of simply constraining the sum of the squares of the transmit signals as in previous analyses, this paper accounts for the true radiated power of the transmit array, indicating whether capacity growth results from increased radiated power or from the actual channel. The fundamental observation is that transmit correlation increases capacity through simple beamforming techniques, and that true capacity gain only comes from increased *channel* correlation, not *antenna* correlation whose gain is offset by increased antenna coupling. With these considerations, optimal antenna placement is found to be close to that of conventional phased arrays (0.3 to 0.6 wavelengths).

The remainder of the paper is organized as follows: Section II presents the block fading MIMO channel model. Section III introduces the concept of radiated power derived from electromagnetic considerations. Section IV analyzes capacity increase with correlation in terms of beamforming mechanisms and reveals new conditions for capacity growth. Section V discusses optimal antenna placement for rapidly fading channels. Section VI concludes the paper.

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II. Channel Model

As in [4, 5], we adopt the block-fading channel model

$$\mathbf{X} = \sqrt{(\rho/P)}\mathbf{S}\mathbf{H} + \mathbf{W}, \tag{1}$$

where $\mathbf{S}$ is the $T \times M$ matrix of complex baseband transmit signals, $\mathbf{H}$ is a single realization of the $M \times N$ channel transfer matrix, $\mathbf{X}$ is the $T \times N$ matrix of receive samples, T is the block length, and M and N are the number of transmit and receive antennas, respectively. The quantities P and ρ represent the average power generated per unit time by the transmit signal matrix $\mathbf{S}$ and the average signal to noise ratio (SNR), respectively. The $T \times N$ matrix $\mathbf{W}$ of noise samples consists of i.i.d. elements $W_{ij} \sim \mathcal{CN}(0,1)$, with $\mathcal{CN}(\mu, \sigma^2)$ denoting the univariate complex Gaussian distribution with mean μ and variance σ^2.

The channel $\mathbf{H}$ is assumed to be constant over blocks of length T, with elements given by the Kronecker model, or

$$\mathbf{H} = \mathbf{R}_T^{1/2}\mathbf{H}_w\mathbf{R}_R^{1/2}, \tag{2}$$

where $\mathbf{R}_T = (1/N)\mathrm{E}\left\{\mathbf{H}\mathbf{H}^H\right\}$ and $\mathbf{R}_R = (1/M)\mathrm{E}\left\{\mathbf{H}^H\mathbf{H}\right\}$ are the transmit and receive covariance matrices, and $H_{w,ij} \sim \mathcal{CN}(0,1)$. Covariance matrices are generated in this work with a directional channel model, where the probability density function (pdf) of departures or arrivals at angle ϕ for either transmit or receive in the azimuthal plane is $p(\phi)$. For a uniform linear array (ULA) of infinitesimal dipoles, the transmit or receive covariance matrix has elements

$$R_{ik} = \int_0^{2\pi} d\phi\ p(\phi)\exp[j2\pi(i-k)\Delta x\cos\phi], \tag{3}$$

where Δx is the inter-element spacing in wavelengths.

To understand the effects of different multipath distributions, three forms of $p(\phi)$ at the transmitter are considered:

1 For full angular spread $p(\phi) = 1/(2\pi)$, and the covariance elements become $R_{T,ik} = J_0[2\pi\Delta x(i-k)]$, where $J_0(\cdot)$ is the zeroth order Bessel function.

2 For a set of L discrete paths with powers β_ℓ and arrival/departure angles ϕ_ℓ, we have

$$R_{T,ik} = \sum_{\ell=1}^{L}\beta_\ell\exp[j2\pi\Delta x(i-k)\cos\phi_\ell]. \tag{4}$$

3 For a single von Mises cluster [6],

$$R_{T,ik} = I_0\left(\sqrt{\kappa^2 - y^2 + j2\pi\kappa y\cos\overline{\phi}}\right)/I_0(\kappa), \tag{5}$$

where $y = 2\pi(i-k)\Delta x$, $\kappa \in [0,\infty)$ controls the cluster width, and $\overline{\phi}$ is the mean cluster departure angle.

III. Radiated Power

In MIMO analyses, the transmit signal is typically constrained to have unit average power for each antenna and symbol time [4, 5], or

$$P = P_{\rm tr} = (1/T){\rm ETr}\left\{\mathbf{S}\mathbf{S}^H\right\} = M, \tag{6}$$

where ${\rm E}\{\cdot\}$, ${\rm Tr}\{\cdot\}$, and $\{\cdot\}^H$ represent expectation, trace, and conjugate transpose, respectively. In a realistic system, this is equivalent to constraining the sum of the squared currents on the antenna elements, which for uncoupled antennas also constrains the radiated power. When the antenna elements are electromagnetically coupled, the power radiated during the ith symbol time becomes [7]

$$P_i = \mathbf{s}_i\mathbf{A}\mathbf{s}_i^H, \tag{7}$$

where $\mathbf{A}$ is an $M\times M$ coupling matrix, and $\mathbf{s}_i$ is the ith row of $\mathbf{S}$. The average radiated power per unit time is therefore expressed as

$$P = P_{\rm rad} = (1/T){\rm ETr}\left\{\mathbf{S}\mathbf{A}\mathbf{S}^H\right\}. \tag{8}$$

A ULA of infinitesimal dipoles oriented perpendicular to the azimuthal plane is assumed, giving $A_{ij} = J_0[2\pi\Delta x(i-j)]$.

In the analysis that follows, we assume that the signaling strategy $\mathbf{S}$ is chosen by letting $P = P_{\rm tr} = M$ as in (6) and then scaling $\mathbf{S}$ so that the radiated power computed in (8) achieves the desired value. This method is suboptimal, since the optimal solution would find $\mathbf{S}$ by directly constraining the radiated power [8]. However, the suboptimal scheme is convenient for this present analysis since it reveals the source of the gains in [5] and whether such gains are realistic.

IV. Beamforming and Capacity Growth

Understanding the behavior of capacity for correlated block-fading channels is facilitated by an eigenbeamforming interpretation. Substituting (2) into (1) and taking the eigenvalue decomposition (EVD) of the covariance matrices ($\mathbf{R}_P = \boldsymbol{\xi}_P\boldsymbol{\Lambda}_P\boldsymbol{\xi}_P^H$) yields

$$\mathbf{X} = \sqrt{\frac{\rho}{P}}\mathbf{S}\mathbf{R}_T^{1/2}\mathbf{H}_w\mathbf{R}_R^{1/2} + \mathbf{W}, \qquad \text{or} \tag{9}$$

$$\underbrace{\mathbf{X}\boldsymbol{\xi}_R}_{\mathbf{X}'} = \sqrt{\frac{\rho}{P}}\underbrace{\mathbf{S}\,\boldsymbol{\xi}_T}_{\mathbf{S}'}\boldsymbol{\Lambda}_T^{1/2}\underbrace{\boldsymbol{\xi}_T^H\mathbf{H}_w\boldsymbol{\xi}_R}_{\mathbf{H}_w'}\boldsymbol{\Lambda}_R^{1/2}\underbrace{\boldsymbol{\xi}_R^H\boldsymbol{\xi}_R}_{\mathbf{I}} + \underbrace{\mathbf{W}\boldsymbol{\xi}_R}_{\mathbf{W}'}. \tag{10}$$

The unitary transformations do not change the statistics of the channel and noise nor the capacity [4], resulting in the simplified model

$$\mathbf{X}' = \sqrt{\frac{\rho}{P}}\mathbf{S}'\boldsymbol{\Lambda}_T^{1/2}\mathbf{H}_w\boldsymbol{\Lambda}_R^{1/2} + \mathbf{W}. \tag{11}$$

Note that since $\mathbf{H}_w'$ and $\mathbf{W}'$ have the same distributions as $\mathbf{H}_w$ and $\mathbf{W}$, respectively, the primes have been dropped for convenience. The behavior of the channel is depicted graphically in Figure 1.

Equation (11) indicates that correlation ($\boldsymbol{\Lambda}_{\{T,R\}} \neq \mathbf{I}$) allows the transmitter and receiver to form beams that excite the spatial modes of the channel with the highest gain, referred to commonly as *dominant modes*. From a physical perspective, communicating on the dominant modes corresponds to transmitting and receiving power in directions of high multipath. Adding more antennas allows improved control of the radiation and reception patterns and therefore improved spatial filtering. Thus, the dominant modes of the covariance become stronger as antennas are added, yielding higher beamforming gain and leading to the guaranteed capacity growth in [5].

A logical observation that can be drawn from [5] is that mutual information only depends on the distribution of $\mathbf{S}\mathbf{R}_T\mathbf{S}^H = \mathbf{S}'\boldsymbol{\Lambda}_T\mathbf{S}'^H$. This allowed a scheme in [5] to be developed where mutual information remains constant with added antennas even though the transmit power is reduced, indicating capacity growth. Here we review this analysis and study the impact of including radiated power.

A. Effective Power Gain

From the perspective of Figure 1, mutual information only depends on the temporal correlation $\mathbf{S}''\mathbf{S}''^H$ of signals transmitted through the channel. Consider adding ΔM transmit antennas to a system consisting of M antennas. Transmit signals can be rearranged so that the first M columns of $\mathbf{S}''$ remain the same, but the additional ΔM columns are zero, meaning no change in the spatio-temporal excitation.

Given a system with M transmit antennas and an abstract quantity $(\cdot)$ (such as $\mathbf{R}_T$, $\mathbf{S}$, etc.), let $\hat{(\cdot)}$ represent the same quantity for $M+\Delta M$ transmit antennas. Assuming that radiation patterns do not change with additional elements, the transmit covariance for the $M+\Delta M$ antenna system has the form

$$\hat{\mathbf{R}}_T = \begin{bmatrix} \mathbf{R}_T & \mathbf{Q} \\ \mathbf{Q}^H & \mathbf{R} \end{bmatrix}, \tag{12}$$

with corresponding eigenvalues

$$\hat{\boldsymbol{\Lambda}}_T = \begin{bmatrix} \hat{\boldsymbol{\Lambda}}_{T,M} & 0 \\ 0 & \hat{\boldsymbol{\Lambda}}_{T,\Delta M} \end{bmatrix}, \tag{13}$$

where $\{\cdot\}_M$ and $\{\cdot\}_{\Delta M}$ represent the upper left $M\times M$ and lower right $\Delta M\times\Delta M$ sub-blocks of the matrix, respectively. Mutual information remains constant if the signaling matrix is rearranged as $\hat{\mathbf{S}}'' = [\mathbf{S}''\ \mathbf{0}_{T\times\Delta M}]$, which is equivalent to

$$\hat{\mathbf{S}}' = [\mathbf{S}'\boldsymbol{\Lambda}_T^{1/2}\hat{\boldsymbol{\Lambda}}_{T,M}^{-1/2}\ \mathbf{0}_{T\times\Delta M}]. \tag{14}$$

To assess the impact of signal reassignment on the radiated power, we define the effective gain of adding ΔM antennas using the ratio of radiated powers, or

$$G_{\rm eff} = \frac{{\rm ETr}\left\{\mathbf{S}\mathbf{A}\mathbf{S}^H\right\}}{{\rm ETr}\left\{\hat{\mathbf{S}}\hat{\mathbf{A}}\hat{\mathbf{S}}^H\right\}}. \tag{15}$$

Thus, if adding the ΔM antennas decreases radiated power, $G_{\rm eff}$ will be greater than unity, indicating capacity growth.

1) Full Angular Spread: Consider the case of full angular spread with transmit covariance $R_{T,ij} = J_0[2\pi\Delta x(i-j)]$. Since in this circumstance $\mathbf{R}_T = \mathbf{A}$, the radiated power for M antennas is simply $P_{\rm rad} = (1/T){\rm ETr}\left\{\mathbf{S}'\boldsymbol{\Lambda}_T\mathbf{S}'^H\right\}$, meaning that beamforming gain is accompanied by a commensurate increase in radiated power, or $G_{\rm eff} = 1$ indicating no capacity increase. This case highlights one of the key features of systems with mutual coupling: changing the transmit antenna configuration can only enhance beamforming gain if the increase in correlation is not offset by increased coupling.

2) Single Departure: Next consider the case where propagation to the receiver occurs for only a single departure direction ϕ with $L = 1$ and $\beta_1 = 1$. In this case, a single spatial transmission mode exists given by the eigenvalue and eigenvector pair

$$\lambda = M, \qquad \text{and} \tag{16}$$

$$v_i = 1/\sqrt{M}\exp(j2\pi i\Delta x\cos\phi), \tag{17}$$

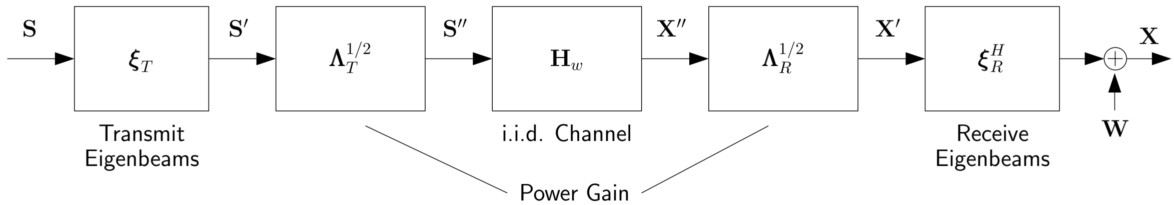

Fig. 1. Graphical representation of channels with separable (Kronecker) correlation

respectively. Optimal transmission involves exciting this mode with $\mathbf{S} = \mathbf{s}'\mathbf{v}^H$, where $\mathbf{s}'$ is the vector of time symbols for the current block, producing a radiated power of

$$P_{\text{rad}} = \underbrace{(1/T)\mathrm{E}\left\{\mathbf{s}'^H\mathbf{s}'\right\}}_{=1} \mathrm{Tr}\left\{\mathbf{v}^H\mathbf{A}\mathbf{v}\right\}\lambda^{-1}. \quad (18)$$

Assuming $A_{ij} = J_0[2\pi\Delta x(i-j)]$, we have

$$\mathrm{Tr}\left\{\mathbf{v}^H\mathbf{A}\mathbf{v}\right\} \quad (19)$$
$$= 1 + \frac{1}{M}\sum_{m=1}^{M-1}(M-m)\cos(2\pi m\Delta x\cos\phi)J_0(2\pi m\Delta x).$$

By placing antenna elements sufficiently far apart, a gain of M is obtained, which is expected from basic array gain concepts.

3) von Mises Cluster: Consider the case of a single continuous cluster of departures described by the von Mises distribution with covariance (5). Assuming communication on just the single dominant spatial mode, the effective gain is

$$G_{\text{eff}} = \frac{\mathbf{v}_1^H\mathbf{A}\mathbf{v}_1}{\lambda_1\hat{\lambda}_1^{-1}\hat{\mathbf{v}}_1^H\hat{\mathbf{A}}\hat{\mathbf{v}}_1}, \quad (20)$$

where λ_1 and $\mathbf{v}_1$ are the principal eigenvalue and corresponding eigenvector of $\mathbf{R}_T$, respectively.

Figures 2 and 3 plot the effective gain for uncoupled and coupled antennas, respectively, where a single cluster departs in the endfire direction ($\overline{\phi} = 0$), antenna spacing is $\Delta x = 0.5$, and various values of κ are considered. For uncoupled antennas, the capacity growth is inversely proportional to κ, and for $\kappa \to \infty$ effective gain approaches M just as in the case of a single departure. For coupled antennas, the case $\kappa = 0$ (equivalent to full angular spread) has no effective gain. Coupling also inhibits the effective gain for the other values of κ, mainly because the endfire excitation tends to increase radiated power relative to the uncoupled case for $\Delta x = 0.5$. On the other hand, for broadside excitation (not plotted), the coupling actually enhances the effective gain for $\Delta x = 0.5$.

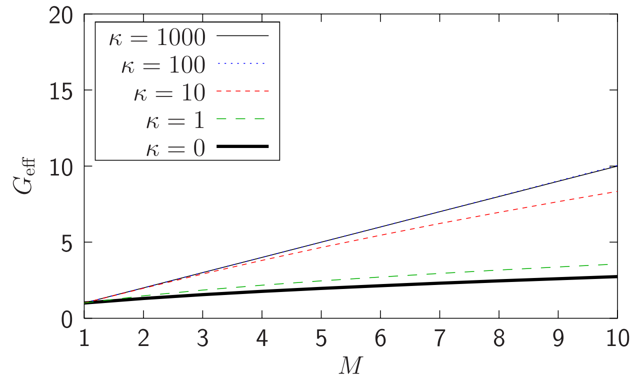

Fig. 2. Effective gain versus the number of antenna elements for a single endfire cluster for uncoupled antennas

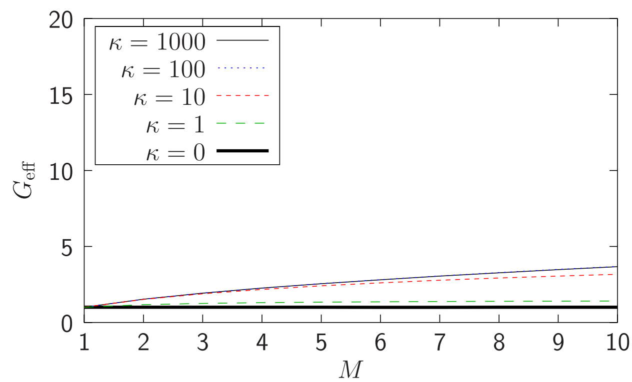

Fig. 3. Effective gain versus the number of antenna elements for a single endfire cluster for coupled antennas

V. Antenna Placement for Rapid Fading

For rapidly fading channels ($T = 1$), the results of [5] indicate that only one spatial mode (or beam) should be used and therefore antenna placement should be chosen to maximize the principal channel eigenvalue. Equation (3) indicates that this is accomplished by letting $\Delta x \to 0$ so that $R_{T,ij} \to 1$, producing a single nonzero eigenvalue of M. This concept of placing antennas as close together as possible is troubling from an electromagnetic perspective, since for $\Delta x = 0$ the antennas should function as a single element.

The apparent contradiction arises because the traditional power constraint (6) is not useful for close spacings, due to coherent addition of the fields [7]. For example, for $\Delta x = 0$, $A_{ij} = 1$, meaning that excitation along the principal eigenvector leads to $P_{\text{rad}} = M$ (which equals the channel eigenvalue). Since the increased signal gain is equal to the increased radiated power, there is no true performance increase. The following analysis studies this issue in more detail.

A. Effective Gain for $M = 2$

For $M = 2$ the transmit covariance matrix is of the form

$$\mathbf{R}_T = \begin{bmatrix} 1 & \gamma \\ \gamma^* & 1 \end{bmatrix}, \quad (21)$$

with eigenvalues $\lambda_{1,2} = 1 \pm |\gamma|$ and eigenvectors

$$\mathbf{v}_{1,2} = (1/\sqrt{2})[1 \quad \pm\exp(-j\angle\gamma)]^T, \quad (22)$$

where $\{\cdot\}^T$ denotes transpose. We consider the case identical to [5], where for $T = 1$ we use $\mathbf{S} = \mathbf{s}'\mathbf{v}_1^H$. For uncoupled antennas, the gain of two antennas over a single antenna is the eigenvalue $\lambda_1 = 1 + |\gamma|$. When considering radiated power, however, the transmit signals must be divided by the square

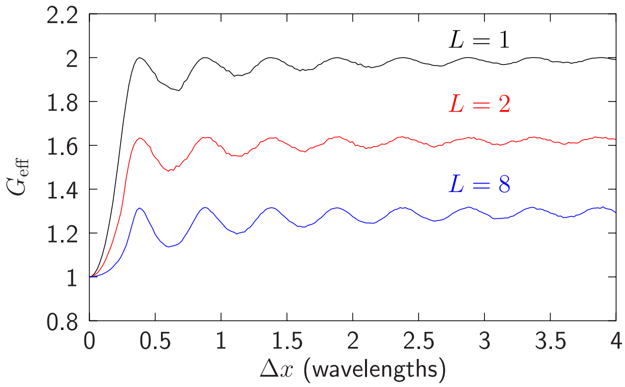

Fig. 4. Effective gain for the L-path model from Monte Carlo simulations

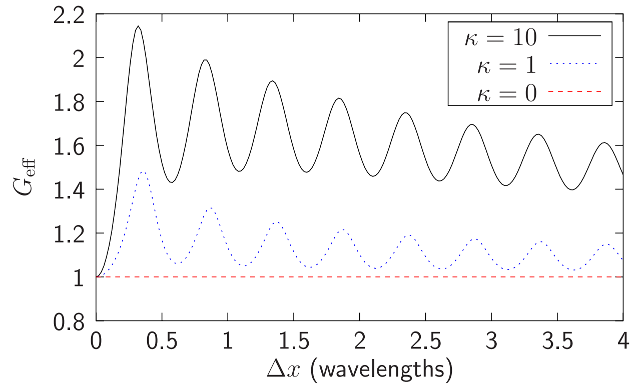

Fig. 5. Effective gain as a function of antenna spacing assuming a single departing cluster distributed according to the von Mises distribution for three values of κ with $\overline{\phi} = 0$

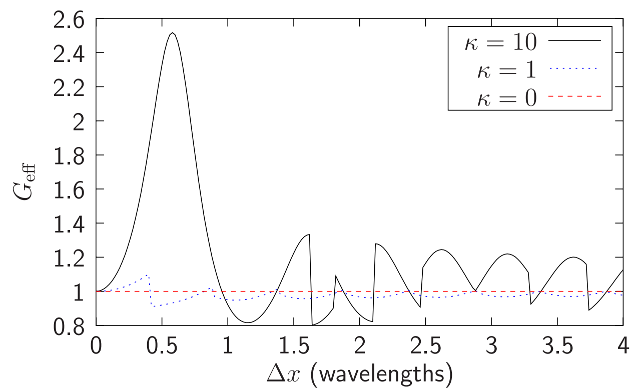

Fig. 6. Effective gain as a function of antenna spacing assuming a single departing cluster distributed according to the von Mises distribution for three values of κ with $\overline{\phi} = \pi/2$

root of the factor

$$\begin{aligned} P_{\text{rad}}/P_{\text{tr}} &= (1/M)P_{\text{rad}} = (1/M)\text{ETr}\left\{\mathbf{SAS}^H\right\} \\ &= \underbrace{(1/M)\text{E}\left\{\mathbf{s}'^H\mathbf{s}'\right\}}_{=1}\mathbf{v}_1^H\mathbf{A}\mathbf{v}_1, \end{aligned} \tag{23}$$

leading to $G_{\text{eff}} = \lambda_1/(\mathbf{v}_1^H\mathbf{A}\mathbf{v}_1)$. For infinitesimal dipoles

$$G_{\text{eff}} = \frac{1+|\gamma|}{1+\cos(\angle\gamma)J_0(2\pi\Delta x)}. \tag{24}$$

Thus, we are left with finding the antenna spacing that maximizes the effective gain.

1) Full Angular Spread: The case of full angular spread is trivial, since $\gamma = J_0(2\pi\Delta x)$, and (24) gives $G_{\text{eff}} = 1$, regardless of the antenna spacing. Thus, any increase in correlation due to reduced spacing is exactly offset by an increase in radiated power. To avoid difficulties with element coupling, antenna spacing should be as large as possible.

2) L-path Model: Next we consider the case of L discrete paths, each having a mean power of $1/L$. The path directions ϕ_ℓ are assumed to be i.i.d. uniform on $[0, 2\pi]$. Figure 4 plots the mean effective gain computed by averaging G_{eff} over 10^4 channel realizations as a function of spacing. As expected, the effective gain decreases with increasing multipath. Also, antennas should be placed no closer than about 0.4 wavelengths since coupling begins to counteract the benefits of correlation leading to a reduction in G_{eff}.

3) Von Mises Cluster: Consider a single departing cluster described with a von Mises angular distribution, where $\overline{\phi}$ and κ are fixed. For a specific array orientation, γ is computed from (5) with $y = -2\pi\Delta x$. Figures 5 and 6 plot G_{eff} versus Δx for three values of κ for endfire ($\overline{\phi} = 0$) and broadside ($\overline{\phi} = \pi/2$) mean departure angle, respectively.

The results reveal that increased multipath causes a gain reduction. However, in contrast to the results observed for the discrete path model, very large spacings are now less desirable. This behavior likely stems from the fixed mean departure angle, as the effective gain averaged over a uniformly distributed sequence of mean departure angles looks similar to the curves for the discrete path model. The key observation from this result is that if array orientation relative to the multipath can be controlled, close spacings may be advantageous. When the arrival angles are more random, very wide spacings appear to be nearly as optimal as narrow spacings.

For a realistic cluster size of 14° ($\kappa = 10$), the optimal spacing for endfire and broadside departures is approximately 0.3 and 0.6 wavelengths, respectively. The optimality of these spacings can be understood by phasing the two antennas such that the main beam is steered in direction $\overline{\phi}$, or $\mathbf{v}^H = (1/\sqrt{2})[1 \;\; \exp(j2\pi\Delta x\cos\overline{\phi})]$. The resulting radiation pattern of the array is

$$P(\phi) = \cos^2[\pi\Delta x(\cos\phi - \cos\overline{\phi})]. \tag{25}$$

At spacings of 0.3 and 0.6 wavelengths this radiation pattern has a single main lobe (with no side lobes) in the endfire and broadside directions, respectively.

4) von Mises Cluster with Input Power Constraint: One of the strange aspects of the effective gain curves for broadside excitation in Figure 6 is the presence of discontinuities. These artifacts appear as γ (which is purely real for $\overline{\phi} = \pi/2$) changes sign abruptly as the spacing increases, meaning that signaling changes between even and odd-mode array excitation. This observation highlights the suboptimality of communicating on the principal eigenvector of $\mathbf{R}_T$ and ignoring the effect of antenna coupling in signal construction.

To explore the idea of optimal signaling with covariance information, consider only constraining the signal matrix $\mathbf{S}$ such that $P = P_{\text{rad}} = M$. This constraint can be transformed into the traditional power constraint by making the substitution $\mathbf{S} = \mathbf{S}'\mathbf{A}^{-1/2}$, and the relationship for the channel becomes

$$\mathbf{X} = \sqrt{\frac{\rho}{M}}\mathbf{S}'\underbrace{\mathbf{A}^{-1/2}\mathbf{R}_T^{1/2}}_{\mathbf{R}_T'^{1/2}}\mathbf{H}_w\mathbf{R}_R^{1/2} + \mathbf{W}, \tag{26}$$

thus creating the effective transmit covariance

$$\mathbf{R}_T' = \mathbf{A}^{-1/2}\mathbf{R}_T\mathbf{A}^{-(1/2)H}, \tag{27}$$

which includes both the effects of antenna correlation and coupling. For $T = 1$, the optimal strategy now directs power along the principal eigenvector of $\mathbf{R}'_T$ rather than $\mathbf{R}_T$.

As $\Delta x \to 0$, this strategy is problematic, since the supergain effect becomes significant [8]. To avoid the appearance of impractical supergain solutions, we assume modified coupling and covariance matrices of the form [9]

$$\mathbf{A} = \eta \mathbf{A}_0 + (1-\eta)\mathbf{I}, \quad \mathbf{R}_T = \eta \mathbf{R}_{T,0} \tag{28}$$

where $\mathbf{A}_0$ and $\mathbf{R}_{T,0}$ are the radiation-only coupling and covariance matrices, respectively, defined in Section III, and η is the antenna efficiency. The first and second terms of $\mathbf{A}$ represent radiation and ohmic loss, respectively. The ohmic loss regularizes the inverse of $\mathbf{A}$, which corresponds to removal of supergain solutions. Since $\mathbf{A}$ now contains loss, the formulation actually constrains the system *input power*.

Assuming $M = 2$, closed-form solutions for G_{eff} with the input power constraint are possible. $\mathbf{A}$ is of the form

$$\mathbf{A} = \begin{bmatrix} 1 & a \\ a^* & 1 \end{bmatrix}, \tag{29}$$

with eigenvalues $\lambda_{1,2} = 1 \pm |a|$ and eigenvectors $\mathbf{v}_{1,2} = (1/\sqrt{2})[1 \quad \pm e^{-j\angle a}]^T$. Thus, having the EVD of $\mathbf{A}$ allows (27) to be computed as

$$\mathbf{R}'_T = \eta \begin{bmatrix} c_1 & c_2 \\ c_2^* & c_1 \end{bmatrix}, \tag{30}$$

with

$$c_1 = b_1^2 + |b_2|^2 + 2\mathrm{Re}\{\gamma b_1 b_2^*\}, \tag{31}$$
$$c_2 = 2b_1 b_2 + \gamma b_1^2 + \gamma^* b_2^2, \tag{32}$$
$$b_1 = [(1+|a|)^{-1/2} + (1-|a|)^{-1/2}]/2, \tag{33}$$
$$b_2 = [(1+|a|)^{-1/2} - (1-|a|)^{-1/2}]\exp(j\angle a)/2, \tag{34}$$

where γ is from (21). The eigenvalues of $\mathbf{R}'_T$ are $\eta(c_1 \pm |c_2|)$. For unit input power, a single antenna has gain η, leading to the effective gain of $c_1 + c_2$ for two elements over a single element.

Figure 7 plots the effective gain versus antenna spacing for a single endfire von Mises cluster with $\kappa = 10$ and different antenna efficiencies (η). The effective gain for the traditional power constraint scaled according to radiated power is also plotted for comparison. The result shows that for lower antenna efficiencies the optimal and suboptimal solutions are almost equivalent. As the antenna efficiency increases, however, the optimal spacing approaches zero, indicating the existence of supergain solutions.

Figure 8 plots a similar result for a single broadside cluster with a fixed efficiency of $\eta = 0.99$. As can be seen, the discontinuities in the effective gain have been removed by the input-power constraint. For narrow spacings, however, the two solutions are nearly identical.

VI. Conclusion

This paper has explored the effect of applying radiated power considerations to the analysis of correlated block-fading MIMO channels with covariance information. The analysis indicated that capacity gain results from simple beamforming mechanisms. Also, different conditions were found for capacity growth and optimal antenna placement compared with previous analyses. Gains are only possible when increased correlation is not commensurate with increased coupling. Further, for rapidly fading channels ($T = 1$) radiated power dictates that placing antennas arbitrarily close is equivalent to a single antenna system.

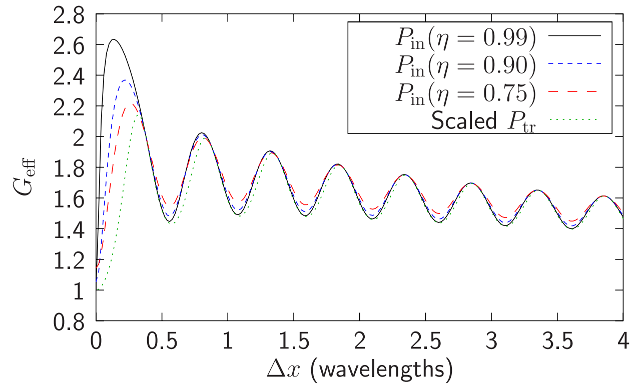

Fig. 7. Effective gain for the input-power constraint as a function of antenna spacing assuming a single departing cluster distributed according to the von Mises distribution for $\kappa = 10$ and $\bar{\phi} = 0$

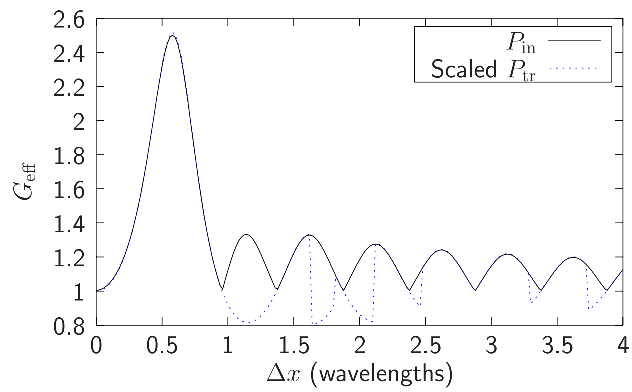

Fig. 8. Effective gain for the input-power constraint as a function of antenna spacing assuming a single departing cluster distributed according to the von Mises distribution for $\kappa = 10$ and $\bar{\phi} = \pi/2$

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