On Signal Strength and Multipath Richness in Multi-Input Multi-Output Systems*

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Abstract— **A common practice when analyzing systems using multiple antennas at both ends is to focus on the multipath properties and normalize the SNR out of the channel matrix. This paper studies the relationship between the signal strength and multipath richness using indoor measurements taken at the Brigham Young University campus. It is found that the SNR and the multipath richness can be strongly correlated. Hence, some caution is needed when normalizing out the SNR in system performance studies. Furthermore, for an unnormalized channel, the channel capacity usually rises when moving from NLOS into LOS since the loss in multipath is more than compensated for by an increase in SNR. A theoretical study also reveals that for moderate sized systems, the required SNR increase for a LOS channel to yield the same channel capacity as a NLOS channel is not very large.**

I. Introduction

The wireless communication industry has experienced a tremendous growth during the last decades. However, the available bandwidth has not grown at the same pace, resulting in an increased cost for bandwidth. Hence, techniques that try to increase the bandwidth efficiency have become even more interesting than before. A new promising way of utilizing the spatial dimension of the channel is to employ multiple antennas at both the transmitter and receiver. These Multiple-Input Multiple-Output (MIMO) systems have been shown [3, 4, 5, 10, 11] to support much higher data rates than traditional single antenna systems while using the same amount of bandwidth. This increase is achieved by exploiting (instead of suffering from) the multiple paths that most signals will take between the transmitter and the receiver.

Due to large gains in data rate and high bandwidth efficiency, MIMO systems have received much attention during the last years. Most of these papers have studied an upper bound on the achievable data rate, i.e. the channel capacity [1, 9]. Since the large capacity gain by using MIMO systems is mainly due to the multipath richness, most authors have focused on this property by normalizing the Signal to Noise Ratio (SNR) out of the channel matrix. However, Non Line Of Sight (NLOS) scenarios with very rich multipath that would result in a high capacity often experience low SNR. On the other hand, scenarios with Line Of Sight (LOS) usually have high SNR but low multipath richness. Thus, the multipath richness and the SNR are usually not independent and some care should be practiced when normalizing out the SNR in system performance studies.

The relationship between SNR and multipath richness will be investigated in this paper using indoor measurements taken at the Brigham Young University (BYU) campus. The same measurements will also be used to study the channel capacity when taking into account both the SNR and the multipath richness. Finally, a theoretical investigation is conducted that tries to answer how much higher SNR a LOS channel requires to yield the same capacity as a rich NLOS channel.

II. MIMO Channel

Consider a communication system with N_T antennas that transmits independent data streams which are received by N_R antennas. For a narrowband system, the N_R received baseband signals $\mathbf{x}$ are related to the N_T transmitted signals $\mathbf{s}$ as

$$\mathbf{x} = \mathbf{H}\mathbf{s} + \mathbf{n}, \tag{1}$$

where the $N_R \times 1$ vector $\mathbf{n}$ denotes additive noise and $\mathbf{H}$ is an $N_R \times N_T$ matrix that describes the channel. Here, the element $\mathbf{H}_{ij}$ represents the complex path gain from transmitter j to receiver i.

The channel capacity of a narrowband MIMO channel, assuming uniform power allocation and no channel state information at the transmitter, can be written as [1, 9]

$$C(\xi) = \log\det\left[\mathbf{I} + \frac{\xi}{N_T}\mathbf{H}\mathbf{H}^H\right], \tag{2}$$

where ξ denotes the SNR at each receive antenna. Most MIMO studies assume some kind of normalization of the $\mathbf{H}$ matrix, in order to focus on the multipath richness. Here, a normalized channel capacity will be calculated where each channel matrix is normalized so that $||\mathbf{H}||_F = \sqrt{\mathrm{Tr}(\mathbf{H}\mathbf{H}^H)} = \sqrt{N_R N_T}$. However, this normalization removes any power fluctuation along the measurement path. Therefore, the capacity is also calculated without normalizing over each $\mathbf{H}$ but normalized so that the average received SNR over the entire measurement path is the same as for the previous case. This capacity will be denoted unnormalized capacity to emphasize that the normalization is not performed for each $\mathbf{H}$.

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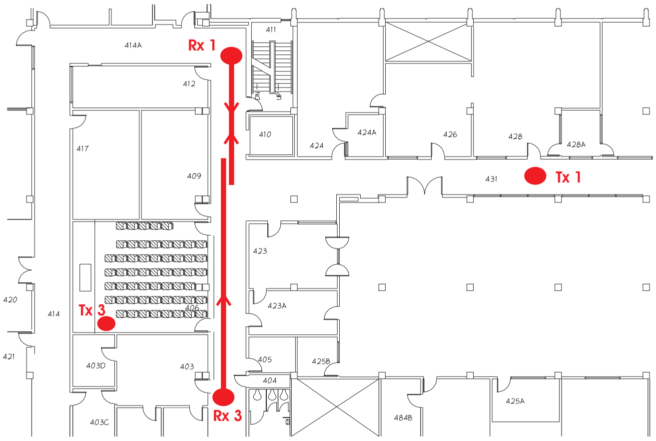

Fig. 1. Locations within the Clyde building where the first and third measurement sets were taken. The direction in which the measurements were taken are indicated by arrows and the starting point of each set by a circle.

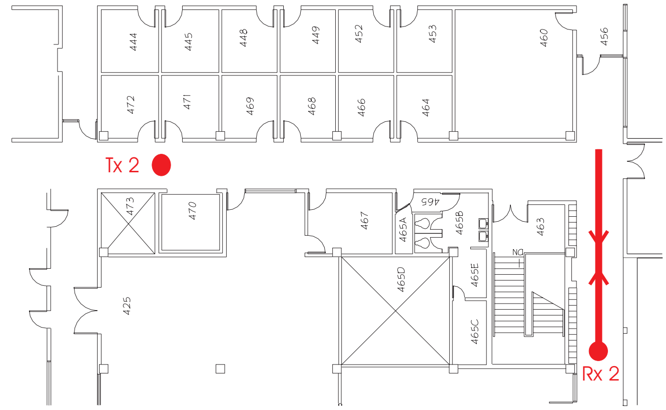

Fig. 2. Location within the Clyde building where the second measurement set was taken.

The notion of multipath richness is less formal than capacity and there are several potential measures that could be used. Here, the concept of Effective Degrees of Freedom (EDOF) [8] will be used. This measure is based on the fact that for an $N \times N$ channel with rich multipath (fully decorrelated channel), a capacity increase of N bits is obtained when doubling the transmitted power. A correlated channel, i.e. a channel with less multipath, will exhibit a smaller capacity increase. Hence, a convenient measure of the multipath richness is the slope of the capacity curve defined as

$$\text{EDOF} = \frac{\partial}{\partial \delta} C(2^{\delta}\xi)\Big|_{\delta=0}. \tag{3}$$

By rewriting the capacity expression in (2) [1] as

$$C(\xi) = \sum_{k=1}^{\min\{N_T,N_R\}} \log\left[1 + \frac{\xi}{N_T}\sigma_k^2\right], \tag{4}$$

where σ_k denotes the singular values of the normalized channel matrix it is straightforward to calculate the derivative in (3)

$$\frac{\partial}{\partial \delta} C(2^{\delta}\xi) = \sum_{k=1}^{\min\{N_T,N_R\}} \frac{1}{1 + \frac{N_T}{2^{\delta}\xi\sigma_k^2}}. \tag{5}$$

The EDOF is then obtained as

$$\text{EDOF} = \sum_{k=1}^{\min\{N_T,N_R\}} \frac{1}{1 + \frac{N_T}{\sigma_k^2\xi}}. \tag{6}$$

Note that the EDOF is a real number in $[0, \min\{N_T, N_R\}]$. A LOS channel with one dominant propagation path will yield an EDOF close to one while a rich NLOS channel will be close to $\min\{N_T, N_R\}$. For channels in between these, the EDOF will essentially be the minimum of the number of transmit and receive antennas or the number of propagation paths with non-negligible strength [7]. Unfortunately, the EDOF measure depends on the SNR since the number of independent transmission channels that rise above the noise floor depends on the SNR. The EDOF will be calculated assuming a medium SNR of 10dB.

III. Measurement Setup

A narrowband, custom-made MIMO channel probing system designed and built at Brigham Young University (BYU) in Utah was used to collect measurements. The system was equipped with ten monopoles forming a uniform circular array at each end. However, since the elements were mounted over a ground plane the monopoles behave as dipoles and essentially have the same radiation patterns as dipoles. Furthermore, the elements were positioned in a circle with radius 0.86 wavelengths that approximately gives an element separation of a half wavelength. The operating frequency was 2.43GHz and the MIMO channel was sampled every 80ms. For a detailed description of the measurement equipment, see [10].

Measurements were taken within the Clyde building at the BYU campus, which is a building containing classrooms. Since the aim of the data collection was to address the joint properties of signal strength and multipath richness, several measurements were taken where the receiver started in NLOS, passed a LOS situation, and continued into NLOS again. This was achieved by putting the transmitter in a corridor and letting the receiver move along another corridor that intersects the first. At the intersection, there were LOS conditions while NLOS conditions reappeared as the receiver moved away from the intersection.

Several measurements of this type of NLOS/LOS scenario were taken and in Figure 1, the layout of the first measurement set is shown. The layout of a similar experiment, the second measurement set, is shown in Figure 2. A slightly different experiment was carried out in the third measurement, see Figure 1. In this case, the transmitter was placed in a classroom and the receiver moved along an adjacent corridor. There were two doorways where the conditions became LOS, but most of the measurement path was NLOS. Many other measurements were also taken, with similar results as these three sets.

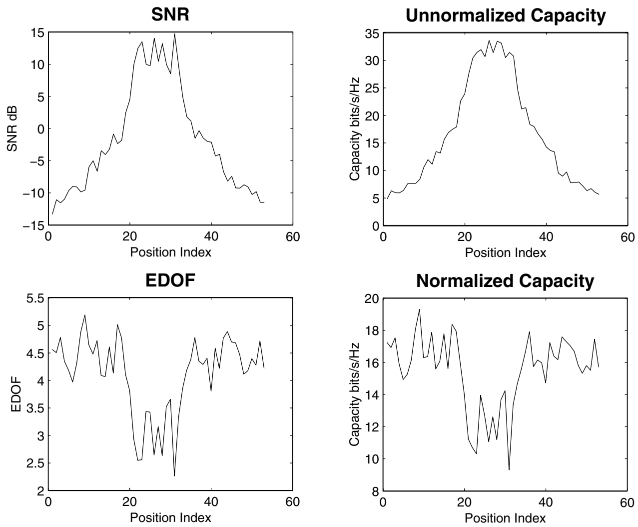

Fig. 3. Capacity, SNR, and EDOF of the first measurement set.

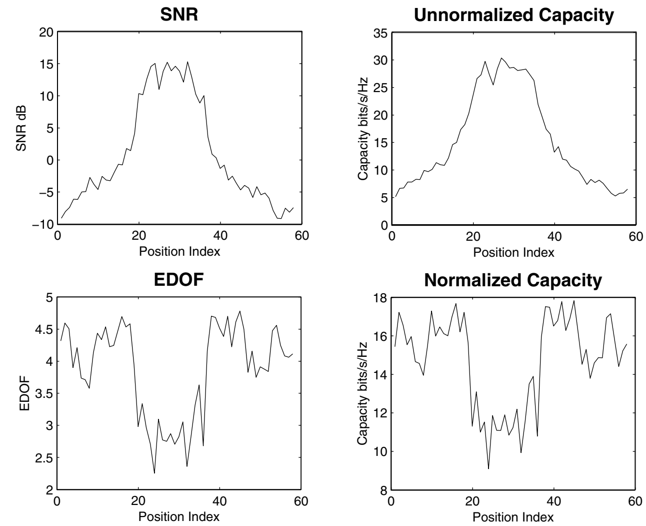

Fig. 5. Capacity, SNR, and EDOF of the second measurement set.

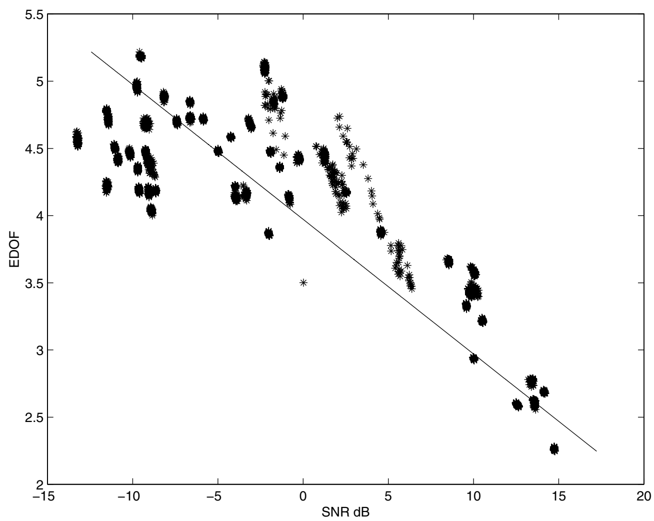

Fig. 4. Scatter plot of EDOF and SNR for the first measurement set.

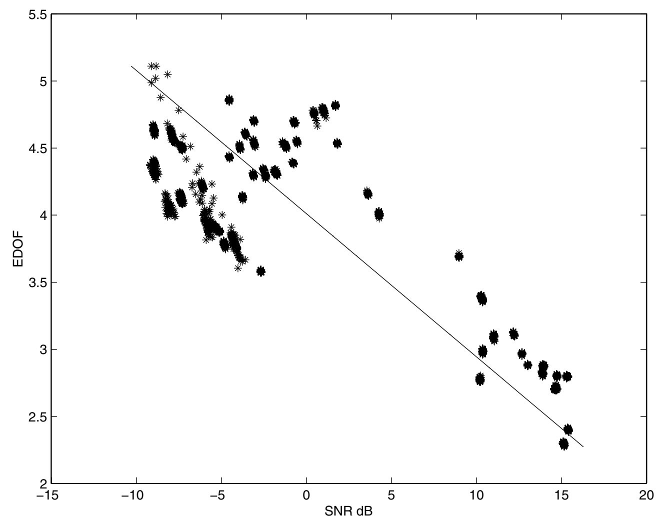

Fig. 6. Scatter plot of EDOF and SNR for the second measurement set.

IV. Measurement Results

In this section the channel capacity, SNR[1], and EDOF will be shown for the measured scenarios. The normalized channel capacity and the EDOF will both be calculated for an SNR of 10dB. For the unnormalized channel capacity, the mean SNR over the entire measurement set will be set to 10dB to yield similar capacities as the normalized capacity where each matrix is normalized.

In Figure 3, the SNR, EDOF, and capacity are shown for the first measurement set. The channel was measured at 53 positions (separated by a foot each) along the corridor, see Figure 1. At each step, about 5s of data were recorded and the average SNR, capacity, and EDOF were calculated. As expected, a sharp increase in SNR is observed as the receiver moves into LOS. At the same time, the EDOF drops significantly and a drop in the normalized capacity is observed. The unnormalized capacity, on the other hand, experiences an increase since the drop in EDOF is compensated for and outweighed by an increased SNR. This simple measurement illustrates the importance of channel normalization. The fact that the SNR increase in a LOS environment may compensate for the loss of multipath has previously been reported in [6, 10].

A scatter plot of the SNR and the EDOF is shown in Figure 4. Each measurement during the 5s measurement interval is marked with a star and a least-squares fitted line is also shown. A strong dependency between SNR and EDOF is visible, as might have been expected. In fact, the correlation coefficient for the SNR and EDOF in this scenario is as high as 0.84. It is also interesting to note that at most positions along the measurement path, the result does not vary with time since the stars are quite focused. However, for three positions the stars are spread out indicating

[1]In Figures 3-8, the squared Frobenius norm of the unnormalized channel matrix $\mathbf{H}$ is used as SNR since the measurements were all collected at a high SNR value.

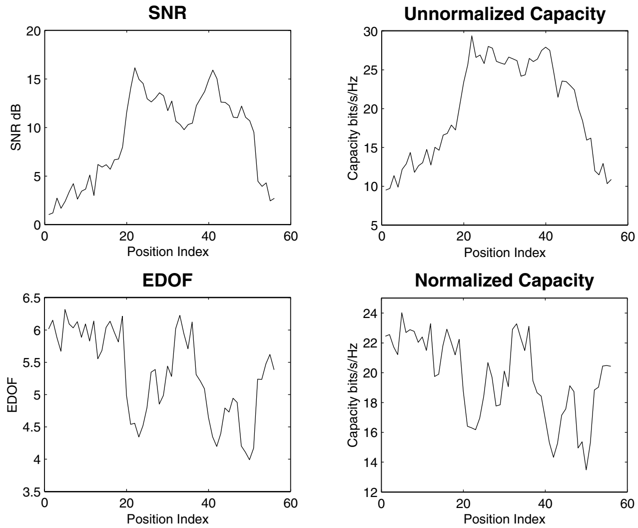

Fig. 7. Capacity, SNR, and EDOF of the third measurement set.

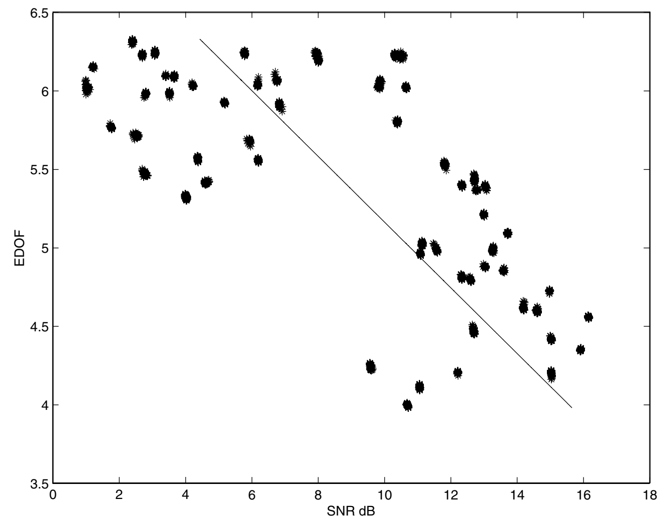

Fig. 8. Scatter plot of EDOF and SNR for the third measurement set.

changing conditions over time. In these cases, there were people walking in the corridor between the transmitter and the receiver.

Very similar results were obtained for the second measurement set, see Figure 5 and 6. Again the SNR increase more than compensates for the drop in multipath richness and a high capacity is obtained in the unnormalized case. On the other hand, the normalized capacity experiences a significant drop due to the loss of multipath. Similar results were also obtained in other similar measurements at other locations within the Clyde building. Hence, it seems that when using the same transmitting power it is usually better to be in LOS with less multipath than in NLOS and more multipath. However, it should be noted that different buildings experience different conditions and the results presented here only represent this particular building.

The first and second measurement sets represents scenarios where the transition between LOS and NLOS is well defined. A slightly less defined transition from LOS to NLOS is the case of the third measurement set, see Figure 1, 7 and 8. Here, the transmitter was placed in a classroom and the receiver was moved along an adjacent corridor. There are two doorways that provide LOS conditions that give rise to the two peaks in the SNR and corresponding drops in EDOF. Again, the SNR increase more than compensates for the loss in multipath richness. However, the differences are less obvious in this case since the doorways are narrow and there is a significant amount of multipath even in the LOS instances. It should also be noted that in between the doorways there is only one wall that separates the transmitter and receiver which offers only about 5dB of attenuation.

V. Required SNR Increase

In all the indoor environments discussed in the previous section, a capacity increase resulted when entering LOS. The drop in multipath was more than compensated for by a significant increase in SNR. However, in these indoor environments there were still several propagation paths even in the LOS case (EDOF > 1). In other environments, such as outdoor environments, the LOS path may be more dominating than in the indoor cases studied here. In those cases the SNR increase may not be enough to compensate for the drop in EDOF. Hence, it is interesting to examine how much the SNR must be increased in LOS in order to yield the same capacity as a completely diffuse NLOS scenario.

The channel capacity for a LOS channel with only one propagation path, i.e. a completely specular channel, can be written in a much simpler form than the general form given in (2). By exploiting the fact that the channel matrix $\mathbf{H}$ in this case is of rank one, the expression for the channel capacity can be reduced to [9]

$$C_s(\xi_s) = \log\left[1 + \xi_s N_R\right], \tag{7}$$

where the subscript in ξ_s denotes the SNR for this specular case. With this simple closed form expression, it is straightforward to calculate the required SNR for a specular channel to give the same average capacity as a diffuse channel which is denoted as $E[C_d]$. Here, $E[\cdot]$ represents expectation. By solving $C_s(\xi_s) = E[C_d]$ for ξ_s, the necessary SNR becomes

$$\xi_s = \frac{2^{E[C_d]} - 1}{N_R}, \tag{8}$$

where it is assumed that the capacity is calculated using the base two logarithm (bits). Unfortunately, no closed form expressions for the capacity exists in the general case. However, an upper bound on the SNR can be obtained by using an upper bound on the capacity that exploits Jensen's inequality and the fact that log det is a concave function [2]

$$E[C_d] \leq \log\det\left[\mathbf{I} + \frac{\xi_d E[\mathbf{H}\mathbf{H}^H]}{N_T}\right]. \tag{9}$$

If it is further assumed that $E[\mathbf{H}\mathbf{H}^H] = N_T\mathbf{I}$, an upper bound is obtained as

$$E[C_d] \leq N_R \log\left[1 + \xi_d\right]. \tag{10}$$

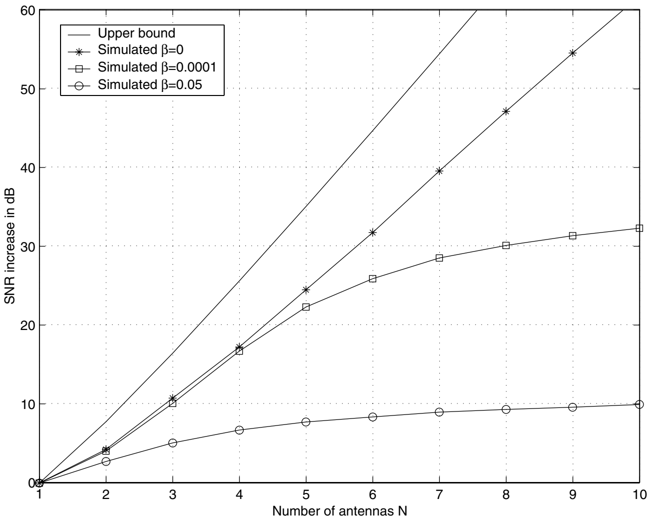

Fig. 9. Required SNR increase for a completely specular and partly diffuse channel to reach the capacity of a completely diffuse channel with $\xi_d = 10$dB.

The above assumption holds for diffuse channels where the elements are uncorrelated but also in the limit for large arrays. Using (8) and (9), an upper bound on the SNR needed for a completely specular channel to yield the same average capacity as a diffuse channel becomes

$$\xi_s = \frac{(1+\xi_d)^{N_R} - 1}{N_R}. \qquad (11)$$

The required SNR increase ξ_s/ξ_d and the corresponding upper bound are shown for different number of antennas $N_T = N_R = N$ in Figure 9. Also shown is the SNR increase required in order for channels that are a combination of specular and diffuse to yield the same capacity as a completely diffuse channel

$$\mathbf{H}_{sd} = \sqrt{1-\beta}\mathbf{H}_s + \sqrt{\beta}\mathbf{H}_d. \qquad (12)$$

Here, the completely specular channel is denoted $\mathbf{H}_s$, the completely diffuse channel $\mathbf{H}_d$, and β is a mixing parameter[2] $\beta \in [0,1]$. A value of β equal to zero corresponds to a fully specular channel while a value of one corresponds to a completely diffuse channel. For the completely diffuse channel, the elements of $\mathbf{H}_d$ are i.i.d complex Gaussian with zero mean and unit variance, i.e. $\mathbf{H}_{n_R,n_T} \sim \mathcal{CN}(0,1)$ for $n_R = 1,2,\ldots,N_R, n_T = 1,2,\ldots,N_T$. For the fully specular channel

$$\mathbf{H}_s = \mathbf{h}_R\mathbf{h}_T^H, \qquad (13)$$

where the vector of length N_R $\mathbf{h}_R$ and the vector of length N_T $\mathbf{h}_R$ both are i.i.d Gaussian with zero mean and unit variance. All calculations were done using an SNR of the completely diffuse channel of 10dB. Surprisingly, the required SNR increase for moderate sized systems is not very large. For a 3×3, an SNR increase of only 10dB is required even for the completely specular channel $\beta = 0$. However, it should be mentioned that for higher SNR levels of the completely diffuse channel, the required SNR increase becomes larger. The upper bound is not tight but was used as a starting point for the numerical search required for the other curves that were calculated using Monte-Carlo simulations. It is also interesting to note that adding only a small fraction of diffuseness reduces the required SNR significantly. For example, adding 5% diffuseness to a completely specular channel corresponds to a 17dB lower required SNR increase for a 5×5 system.

[2]The parameter β is related to the Ricean K-factor as $\beta^{-1} = 1 + K$

VI. Conclusions

Analyzing measurements of the MIMO channel taken at the BYU campus, it was found that the multipath richness and the signal strength usually not are independent. In fact, correlation coefficients as high as 0.84 were obtained. Thus, some caution is needed when normalizing out the SNR in system performance studies. Furthermore, for an unnormalized channel, the channel capacity usually rose when moving from NLOS into LOS since the loss in multipath was more than compensated for by an increase in SNR. Finally, an expression for the required SNR increase for a specular channel to yield the same capacity as a completely diffuse channel was derived. Surprisingly, the required SNR increase for moderate sized systems is not very large.

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