Antenna Selection for MIMO Systems based on Information Theoretic Considerations *

Michael A. Jensen* and Jon W. Wallace

Department of Electrical and Computer Engineering,
459 CB, Brigham Young University, Provo, UT 84602-4099,
jensen@ee.byu.edu, wall@ieee.org

1 Introduction

Multiple-input multiple-output (MIMO) wireless systems have demonstrated the potential for increased capacity in rich multipath environments [1]. In traditional studies of MIMO systems, the system capacity depends on the transmit and receive array configurations used. More recently, however, we have developed the notion of *Intrinsic Capacity* which is the capacity of an electromagnetic propagation channel over all possible communication parameters (coding, signal processing, and antenna configuration) [2].

The intrinsic capacity formulation generates optimal transmit current and receive field sampling distributions that typically are impractical to realize physically and require complex transmit/receive hardware. A more practical scenario would be to deploy large, reconfigurable arrays and select an optimal or near-optimal subset of the antennas for connection to the (fewer available) transmit and receive hardware chains. This paper presents algorithms based upon mutual information quantities derived from the intrisic capacity computation that can efficiently and effectively identify good choices of antennas. While the algorithms do not guarantee optimal antenna selection, results obtained using realistic channel models reveal the excellent performance of the techniques.

2 Intrinsic Capacity Framework

Consider an arbitrary, narrow-band propagation scenario, where the transmit and receive antennas are confined to the volumes $\Delta V'$ and ΔV respectively. In the transmit space, the current is represented using a sum of discrete basis functions, where the ith basis function is weighted by the coefficient X_i. Similarly, the received signal Y_k represents the field in the receive space projected onto the kth receive basis function. Assuming that the generalized Green's function representing the electromagnetic propagation channel is known, then the signals are related by the equation

$$\overline{Y} = \overline{\overline{H}}\,\overline{X} + \overline{\eta} = \overline{S} + \overline{\eta} \tag{1}$$

where $\overline{\overline{H}}$ is a matrix representing the channel transfer function between each transmit and receive basis function.

In this work, we assume a single electromagnetic polarization with multipath propagation confined to the horizontal plane [3]. The transmit and receive volumes are rectangular parallelepipeds with dimensions Δx, Δy, and Δz in the x, y, and z directions, respectively. Because the field is constant in the z direction, the height Δz simply controls the power-collecting capability of the receive antennas and will be set to $\Delta z = \lambda/2$ to loosely represent physically practical half-wavelength dipoles. In the x and y dimensions, however,

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we assume that $\Delta x = \Delta y$ and divide the volume into N^2 equally-sized sub-squares of dimensions $\Delta x/N \times \Delta y/N$. Scalar transmit ($T_i$) and receive ($R_i$) basis functions appropriate for this configuration can be expressed as

$$\begin{matrix} R_i(\overline{r}) \\ T_i(\overline{r}) \end{matrix} = \begin{cases} \frac{N}{\sqrt{\Delta x \Delta y \Delta z}}, & \begin{matrix} x_i - \frac{\Delta x}{2N} < x < x_i + \frac{\Delta x}{2N} \\ y_i - \frac{\Delta y}{2N} < y < y_i + \frac{\Delta y}{2N} \\ -\frac{\Delta z}{2} < z < \frac{\Delta z}{2} \end{matrix} \\ 0, & \text{otherwise,} \end{cases} \tag{2}$$

where (x_i, y_i) defines the center of the support region for the ith basis function.

For zero-mean complex Gaussian noise with covariance $\sigma^2\overline{\overline{I}}$ ($\overline{\overline{I}}$ representing the identity matrix), the intrinsic capacity computation yields the channel capacity as well as the covariance matrix $\overline{\overline{K}}_{XX} = \mathrm{E}\left\{\overline{X}\,\overline{X}^H\right\}$ which, assuming Gaussian signaling, indicates how the transmit data and power should be divided among the transmit basis functions. We can also immediately compute the covariance matrices $\overline{\overline{K}}_{SS} = \mathrm{E}\left\{\overline{S}\,\overline{S}^H\right\} = \overline{\overline{H}}\,\overline{\overline{K}}_{XX}\overline{\overline{H}}^H$ and $\overline{\overline{K}}_{YY} = \mathrm{E}\left\{\overline{Y}\,\overline{Y}^H\right\} = \overline{\overline{K}}_{SS} + \sigma^2\overline{\overline{I}}$.

3 Antenna Selection

Since the intrinsic capacity formulation returns current and field sampling distributions over the entire transmit and receive volumes, the goal for practical implementation is to determine which subset of the available elements (basis functions) will yield the highest capacity. The most straightforward approach involves an exhaustive search over the possibile combinations [4], a search that can quickly become computationally prohibitive as the array size becomes large. Instead, we utilize the covariance matrices obtained from the intrinsic capacity formulation to derive computationally efficient, sub-optimal yet high-performance algorithms for antenna selection. Two different basic approaches are considered.

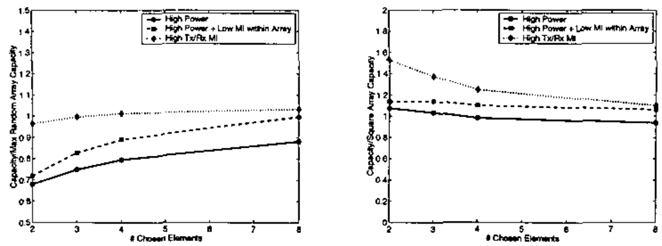
High Power and Low Mutual Information within Array

The first proposed metric for antenna selection involves choosing elements with high signal power, but where the mutual information (MI) between the signal (element) under investigation and the already selected signals (elements) is low. Let $\overline{\overline{K}}$ represent the covariance matrix $\overline{\overline{K}}_{XX}$ or $\overline{\overline{K}}_{SS}$, depending on whether we are applying the algorithm for transmit or receive antenna selection, respectively. Further let C represent the set of indices associated with the previously selected antennas. If $\log_2\{Q(X_i, \overline{X}_C)\}$ represents the MI between the signal X_i on the ith antenna and the signals $\overline{X}_C$ on the selected antennas, then a potential decision metric for high power and low MI can be constructed from the ratio

$$D_{iC} = \frac{K_{ii}}{Q(X_i, \overline{X}_C)} = \left| K_{ii} - \overline{\overline{K}}_{iC}\overline{\overline{K}}_{CC}^{-1}\overline{\overline{K}}_{Ci} \right| \tag{3}$$

where $\overline{\overline{K}}_{ab}$ denotes the block of $\overline{\overline{K}}$ corresponding to row and column indices contained in the sets a and b, respectively. Note that (3) is simply the variance of the signal on the ith element conditioned on the signals on the already selected elements.

The iterative selection algorithm proceeds as follows. The algorithm is initialized by selecting the element characterized by the highest average power, and the initial C therefore contains the index of this antenna. The metric in (3) is then computed for all $i \notin C$, and



(a) (b)

Figure 1: Average capacity as a function of the array size for 2λ apertures divided into 4 cells/wavelength. The capacity is normalized by (a) the maximum capacity achieved with 5000 randomly generated arrays and (b) the capacity achieved with an array around the aperture perimeter.

the antenna with the highest metric is selected. The set C is then augmented to include this index, and the process is repeated until the desired number of antennas has been selected.

High Transmit/Receive Mutual Information

The second proposed metric for antenna selection involves choosing elements that maximize the MI between the signals on the transmit and receive arrays. Let the MI between the transmit signals and a subset of the receive signals or the MI between a subset of the transmit signals and the receive signals be denoted as $\log_2\{Q(\overline{Y}_C,\overline{X})\}$ and $\log_2\{Q(\overline{Y},\overline{X}_C)\}$, respectively, where

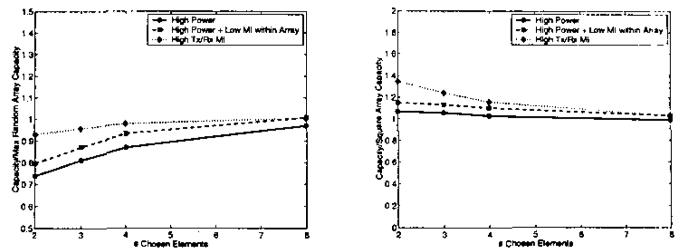
$$Q(\overline{Y}_C,\overline{X}) = \left|\left(\overline{\overline{H}}\,\overline{\overline{K}}_{XX}\overline{\overline{H}}^H + \sigma^2\overline{\overline{I}}\right)_{CC}\right|\left|\sigma^2\overline{\overline{I}}_{CC}\right|^{-1} \quad (4)$$

$$Q(\overline{Y},\overline{X}_C) = \left|\overline{\overline{K}}_{XX,CC}\right|\left|\left[\overline{\overline{K}}_{XX} - \overline{\overline{K}}_{XX}\overline{\overline{H}}^H\overline{\overline{K}}_{YY}^{-1}\overline{\overline{H}}\,\overline{\overline{K}}_{XX}\right]_{CC}\right|^{-1}. \quad (5)$$

Furthermore, let B_i represent the set of previously selected indices C plus the index i, where $i \notin C$. Initially, B_i contains only i. When selecting transmit or receive antennas, the value of i which maximizes the value of $Q(\overline{Y},\overline{X}_{B_i})$ or $Q(\overline{Y}_{B_i},\overline{X})$, respectively, is selected and added to the set C. This procedure is then repeated until the desired number of antennas has been selected. In this work, transmit antennas are chosen first, after which the required covariance matrices are recomputed using the columns of $\overline{\overline{H}}$ corresponding to the selected transmit antennas before selection of the receive antennas.

4 Results

To test the performance of these algorithms, multipath propagation channels were generated using a ray-based propagation model. Assuming transmit and receive apertures 2λ square in the x and y directions, the intrinsic capacity and resulting covariance matrices were then computed. A Monte Carlo simulation was then performed wherein 5000 N-element arrays were generated randomly – $N = 2, 3, 4$ or 8 – and the maximum capacity achieved for each value of N was recorded. Furthermore, the capacity for a "square" array of elements equally spaced around the aperture perimeter was computed. Finally, the capacity of the N-element array suggested by the two different antenna selection algorithms was evaluated. These capacity values were divided by the maximum capacity from the Monte Carlo arrays as



(a) (b)

Figure 2: Average capacity as a function of the array size for 2λ apertures divided into 2 cells/wavelength. The capacity is normalized by (a) the maximum capacity achieved with 5000 randomly generated arrays and (b) the capacity achieved with an array around the aperture perimeter.

well as the capacity of the square array. This process was repeated for 150 different random channel realizations, and the average of the normalized capacity values was computed.

Figure 1 illustrates the results of this computation when the aperture is discretized using 4 basis functions per wavelength. Also shown in this plot is the capacity that results from choosing the elements corresponding to the highest average power (diagonal elements of the covariance matrix). As can be seen, selecting the antennas based on the mutual information performs relatively well considering the low computational cost, with selection based upon High Transmit/Receive Mutual Information yielding the highest performance. Furthermore, selection based on mutual information is superior to selection based upon power alone. Figure 2 repeats these results when the aperture is discretized using 2 basis functions per wavelength. In this case, the benefit offered by the mutual information algorithms is slightly reduced.

5 Conclusion

We have presented algorithms for selecting a subset of available antennas for use in a MIMO communications system based upon mutual information quantities. Computational results obtained using a ray-based channel model in conjunction with the selection approaches has demonstrated that the algorithms are highly effective at providing a sub-optimal yet still high performance set of arrays at little computational cost.

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