# The Capacity of MIMO Wireless Systems with Mutual Coupling \*

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#### Abstract

MIMO wireless systems employing antenna arrays with close inter-element spacings exhibit a high degree of mutual coupling. In this case, the channel transfer matrix is a function of the matching network employed at transmit and receive. Existing capacity expressions are inappropriate, since the maximum mutual information will depend on antenna element loading. This paper develops a new expression for capacity that includes the effect of mutual coupling and antenna matching. Two simple noise models for realistic high-frequency circuits are introduced. Capacity is computed by maximizing the mutual information in these models subject to a new radiated power constraint. Numerical simulations of realistic two-element arrays demonstrate the basic technique.

## 1 Introduction

Multiple-input multiple-output (MIMO) wireless systems have demonstrated the potential for increased capacity in rich multipath environments [3, 6]. Due to their small size, compact arrays are attractive for personal communications devices. However, close antenna element spacing inevitably leads to mutual coupling [1]. Intuition suggests that high inter-element coupling leads to a higher correlation in channel fading coefficients. Surprisingly, however, studies have demonstrated that two closely-spaced coupled dipoles exhibit a *lower* correlation coefficient than identically spaced uncoupled dipoles [7].

While prior studies have presented important findings concerning the effect of array mutual coupling on MIMO system performance, they have not presented a true definition of capacity that accounts for coherent interaction at the transmitter and antenna loading at the receiver. We examine these issues by applying an exact network theory framework to account for mutual coupling in MIMO systems. This framework includes a new power constraint that limits the actual radiated power when mutual coupling is present. New expressions for capacity are derived that maximize mutual information of transmit and receive signals over all possible loading networks, providing a true upper bound on system performance for systems with mutual coupling.

## 2 Narrowband MIMO Channel Capacity

In previous studies, a system with  $N_R$  receive antennas and  $N_T$  transmit antennas has been characterized by the equation

$$\overline{Y} = \overline{H} \,\overline{X} + \overline{N},\tag{1}$$

where  $\overline{X}$  is the vector of transmit signals,  $\overline{\overline{H}}$  is the  $N_R \times N_T$  complex narrowband channel matrix,  $\overline{Y}$  is the vector of receive signals, and  $\overline{N}$  is a noise vector of i.i.d. complex Gaussian elements with variance  $\sigma^2$ . For the optimal case of Gaussian transmit signaling, the mutual information of the vectors  $\overline{Y}$  and  $\overline{X}$  is

$$I(\overline{Y};\overline{X}) = \log_2 \left| \frac{\overline{\overline{H}} \ \overline{\overline{K}}_X \overline{\overline{H}}^H}{\sigma^2} + \overline{\overline{I}} \right|, \tag{2}$$

where  $\overline{\overline{K}}_X = E\{\overline{X}\ \overline{X}^H\}$  and  $\{\cdot\}^H$  is the Hermitian or conjugate transpose operator. The Shannon capacity is the maximum of (2) over all possible  $\overline{\overline{K}}_X$  subject to a transmit power constraint [2]. Traditionally, a power constraint of the form  $P_T = \text{Tr}(\overline{\overline{K}}_X) \leq P_{\text{max}}$  has been assumed. In this case, if the transmitter has no knowledge of the channel, mutual information is maximized by dividing transmit power equally among the  $N_T$  transmit antennas in uncorrelated streams [3]. If the transmitter has knowledge of the channel, the water-filling solution in [6] is appropriate.

When coherent interactions exist among the transmit antennas, the expected (or average, for ergodic signaling) radiated power is  $P_T = \text{Tr}(\overline{\overline{K}}_X \overline{\overline{A}})$ . In this case, a "radiated

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Figure 1. Receive subsystem model

power constraint" of the form  $P_T = \text{Tr}(\overline{K}_X \overline{\overline{A}}) \leq P_{\text{max}}$  is appropriate. When the transmitter has full knowledge of the mutual coupling and the channel matrix, a modified waterfilling solution is obtained for the capacity [8].

## **3** Network Analysis

The scattering parameter (or S-parameter) representation is convenient for high-frequency circuits [5]. This Sparameter description can be generally expressed as  $\overline{b} = \overline{\overline{S}} \ \overline{a}$ , where the vectors  $\overline{a}$  and  $\overline{b}$  denote the complex envelopes of inward and outward propagating waves, respectively, and  $\overline{\overline{S}}$  is the S-parameter matrix. The total voltage and current on the *n*th port are given as  $v_n = Z_0^{1/2}(a_n + b_n)$ and  $i_n = Z_0^{-1/2}(a_n - b_n)$ , where  $Z_0$  is a chosen reference impedance used for computing the S-parameters. In this representation, the net power flowing into the *n*th port is simply  $|a_n|^2 - |b_n|^2$ .

### 3.1 Receive Subsystem: Matching Networks

We first consider the network model depicted in Figure 1 for the receive subsystem. This model treats the antenna as a source with  $N_R$  ports that creates the source wave vector  $\overline{b}_0$  due to the received electromagnetic wave. If a load of characteristic impedance  $Z_0$  is placed on each source port, the total power collected in the loads is equal to  $\|\overline{b}_0\|^2$ . The source is further characterized by a (full) S-parameter matrix  $\overline{\overline{S}}_{RR}$  such that  $\overline{b}_R = \overline{b}_0 + \overline{\overline{S}}_{RR}\overline{a}_R$ . A matching network with S-parameter matrix  $\overline{\overline{S}}_M$  is used to maximize the power transfer from the source to the  $N_R$  loads of resistance  $Z_0$ . We partition this matrix as

$$\overline{\overline{S}}_{M} = \begin{bmatrix} \overline{\overline{S}}_{11} & \overline{\overline{S}}_{12} \\ \overline{\overline{S}}_{21} & \overline{\overline{S}}_{22} \end{bmatrix},$$
(3)

where the subscripts 1 and 2 refer to input and output ports, respectively.

Ideally, the matching network is formed with passive, reactive elements so that it is lossless and reciprocal. If the network is lossless,  $\overline{\overline{S}}_M^H \overline{\overline{S}}_M = \overline{\overline{I}}$ . If it is reciprocal,  $(\overline{\overline{S}}_M = \overline{\overline{S}}_M^t)$ , where  $\{\cdot\}^t$  is the transpose operator) we also have  $\overline{\overline{S}}_M \overline{\overline{S}}_M^H = \overline{\overline{I}}$ . It can be shown that the singular values of each  $\overline{\overline{S}}_{ij}$  matrix lie on the range [0, 1]. Also, if  $\overline{\overline{S}}_{11}$  is set to be any symmetric matrix with singular values on the range [0, 1], a lossless, reciprocal network may always be formed by letting

$$\overline{\overline{S}}_{11} = \overline{\overline{V}}_{11}^* \overline{\overline{\Lambda}}_{11}^{1/2} \overline{\overline{V}}_{11}^H \qquad \overline{\overline{S}}_{12} = j \overline{\overline{V}}_{11}^* (\overline{\overline{I}} - \overline{\overline{\Lambda}}_{11})^{1/2} \overline{\overline{V}}_{22}^H 
\overline{\overline{S}}_{22} = \overline{\overline{V}}_{22}^* \overline{\overline{\Lambda}}_{11}^{1/2} \overline{\overline{V}}_{22}^H \qquad \overline{\overline{S}}_{21} = j \overline{\overline{V}}_{22}^* (\overline{\overline{I}} - \overline{\overline{\Lambda}}_{11})^{1/2} \overline{\overline{V}}_{11}^H$$
(4)

where  $\overline{\overline{V}}_{11}$  and  $\overline{\overline{\Lambda}}_{11}$  are the matrix of right singular vectors and diagonal matrix of singular values of  $\overline{\overline{S}}_{11}$ , respectively,  $\overline{\overline{V}}_{22}$  is an arbitrary unitary matrix, and  $\{\cdot\}^*$  is an elementwise conjugation operator.

Insertion of a lossless matching network between the source and the loads can increase the power collection if  $\overline{\overline{S}}_{RR} \neq \overline{\overline{0}}$ . In this case, the forward wave into the loads is

$$\overline{b}_2 = \overline{\overline{S}}_{21} (\overline{\overline{I}} - \overline{\overline{S}}_{RR} \overline{\overline{S}}_{11})^{-1} \overline{b}_0, \tag{5}$$

(7)

and the total power collected is proportional to

$$P(\overline{\overline{S}}) = \|\overline{b}_2\|^2 = \overline{b}_0^H (\overline{\overline{I}} - \overline{\overline{S}}_{RR} \overline{\overline{S}}_{11})^{(-1)H} \overline{\overline{S}}_{21}^H \\ \times \overline{\overline{S}}_{21} (\overline{\overline{I}} - \overline{\overline{S}}_{RR} \overline{\overline{S}}_{11})^{-1} \overline{b}_0.$$
(6)

For a lossless network, we have the condition that  $\overline{\overline{S}}_{11}^H \overline{\overline{S}}_{11} + \overline{\overline{S}}_{21}^H \overline{\overline{S}}_{21} = \overline{\overline{I}}$ , and the expression becomes

 $P(\overline{\overline{S}}) = \overline{b}_0^H \overline{\overline{W}}(\overline{\overline{S}}_{11}) \overline{b}_0,$ 

where

$$\overline{\overline{W}}(\overline{\overline{S}}_{11}) = (\overline{\overline{I}} - \overline{\overline{S}}_{RR}\overline{\overline{S}}_{11})^{(-1)H}(\overline{\overline{I}} - \overline{\overline{S}}_{11}^H\overline{\overline{S}}_{11}) \times (\overline{\overline{I}} - \overline{\overline{S}}_{RR}\overline{\overline{S}}_{11})^{-1}.$$
(8)

One may show that for a fixed (but arbitrary)  $\overline{b}_0$ , (7) is maximized when  $\overline{\overline{S}}_{11} = \overline{\overline{S}}_{RR}^H$ , termed the *Hermitian* match. This condition is analogous to the *conjugate* match condition that maximizes power transfer for a single port. Interestingly, the Hermitian match not only maximizes receive power, but also maximizes mutual information, as will be shown in Section 4.

## 3.2 Transmit Subsystem: Constrained Radiated Power

Traditional analyses of MIMO wireless systems have generally ignored the effect of mutual coupling on radiated power. Consider a transmit antenna array with  $N_T$  elements and network S-parameters  $\overline{\overline{S}}_{TT}$ . The net power flowing into the network is  $\|\overline{a}_T\|^2 - \|\overline{b}_T\|^2$ , which, for lossless antennas, equals the instantaneous radiated transmit power  $P_T^{\text{inst.}}$ . Since  $\overline{b}_T = \overline{\overline{S}}_{TT} \overline{a}_T$ , we have

$$P_T^{\text{inst}} = \|\overline{a}_T\|^2 - \|\overline{\overline{S}}_{TT}\overline{a}_T\|^2$$
$$= \overline{a}_T^H \overline{a}_T - (\overline{\overline{S}}_{TT}\overline{a}_T)^H \overline{\overline{S}}_{TT} \overline{a}_T$$
$$= \underbrace{\overline{a}_T^H}_{\overline{X}^H} \underbrace{(\overline{\overline{I}} - \overline{\overline{S}}_{TT}^H \overline{\overline{S}}_{TT})}_{\overline{\overline{A}}} \underbrace{\overline{a}_T}_{\overline{X}}, \tag{9}$$

where  $\overline{A}$  is defined as the *coherence matrix* and  $\overline{X}$  denotes a transmit signal vector. For zero mean signals, the average radiated power is given by

$$P_T = \mathrm{E}\left\{P_T^{\mathrm{inst}}\right\} = \mathrm{Tr}(\overline{K}_X \overline{A}),\tag{10}$$

which corresponds to the radiated power constraint considered in Section 2. It is noteworthy that while the waterfilling solution must be modified to incorporate this power constraint, the uninformed transmit solution remains unchanged for uncorrelated transmit streams.

## 4 Network Channel Models

Capacity analysis of MIMO systems requires that the communication channel be formulated within the network description adopted in this work. The following subsections describe the channel modeling framework and present two basic noise models for analysis.

#### 4.1 Channel Representation

Consider transmit and receive arrays consisting of  $N_T$ and  $N_R$  antenna elements, respectively, embedded in a linear scattering medium. The inward-traveling and outwardtraveling waves at the transmitter are defined as  $\overline{a}_T$  and  $\overline{b}_T$ , respectively, while those at the receiver are defined as  $\overline{a}_R$ and  $\overline{b}_R$ . The  $(N_T + N_R) \times (N_T + N_R)$  S-parameter matrix for the transmit and receive ports completely characterizes the propagation channel, and may may be partitioned into the signal representation

$$\begin{bmatrix} \overline{b}_T \\ \overline{b}_R \end{bmatrix} = \underbrace{\begin{bmatrix} \overline{\overline{S}}_{TT} & \overline{\overline{S}}_{TR} \\ \overline{\overline{S}}_{RT} & \overline{\overline{S}}_{RR} \end{bmatrix}}_{\overline{\overline{S}}_{R}} \begin{bmatrix} \overline{a}_T \\ \overline{a}_R \end{bmatrix}.$$
(11)

For this analysis, we assume that  $\overline{\overline{S}}_{TR} = \overline{\overline{0}}$ , which means that power reflected from the receive antennas does not couple significantly back into the transmit antennas.



Figure 2. Network model for the entire MIMO communication system.

#### 4.2 Communication System Model

Figure 2 depicts a realistic communication system incorporating elements discussed thus far. The  $N_T$  transmit antennas (the  $N_T$  input ports to  $\overline{\overline{S}}_H$ ) are excited by generators with arbitrary phases and magnitudes. A unit gain element that is matched to the reference impedance  $Z_0$  is included to allow the addition of noise in the receiver. Each port in the chain is then terminated by a matched load  $Z_0$ , and the voltage across this load is sampled to obtain  $\overline{v}_R$ . Because the output ports of the matching network  $(\overline{\overline{S}}_M)$  are terminated in  $Z_0$ , only the outward-traveling wave  $\overline{b}'_R$  will exist at this point.

In the noiseless case, the sampled voltages are related to the transmit signal according to

$$\underbrace{\overline{v}_R}_{\overline{Y}_R} = \underbrace{Z_0^{1/2} \,\overline{\overline{S}}_{21} (\overline{\overline{I}} - \overline{\overline{S}}_{RR} \overline{\overline{S}}_{11})^{-1} \overline{\overline{S}}_{RT}}_{\overline{\overline{H}} (\overline{\overline{S}}_M)} \underbrace{\overline{\overline{A}}_R}_{\overline{\overline{X}}_R}, \quad (12)$$

where the underbraces indicate the relationship to the simple MIMO model in Section 2. This relationship indicates that the effective channel is a function of the matching network employed at the receiver. Thus, a true definition of capacity will in general require a maximization of the mutual information of  $\overline{X}$  and  $\overline{Y}$  not only over all possible transmit excitations, but also over all allowed matching networks. This maximization is dependent on the type of noise model assumed. Therefore, we consider two realistic noise models for existing microwave systems.

## 4.3 Channel Noise Model

If the dominant source of noise in the system is from the channel (co-channel interference, channel instability, cosmic radiation, etc.), we may neglect noise additions in the receiver. When no signal is present and the receive antenna ports are terminated in  $Z_0$ , we define the resulting forward traveling noise wave on the *i*th receive port as  $b_{RN,i} = Z_0^{-1/2} N_i$ , where  $N_i$  is an effective noise voltage. With the matching network inserted, the forward traveling wave becomes

$$\overline{b}_{RN} = Z_0^{-1/2} (\overline{\overline{I}} - \overline{\overline{S}}_{RR} \, \overline{\overline{S}}_{11})^{-1} \overline{N}.$$
(13)

Superimposing the signal and noise vectors yields the result

$$\overline{b}_R = (\overline{\overline{I}} - \overline{\overline{S}}_{RR} \,\overline{\overline{S}}_{11})^{-1} (\overline{\overline{S}}_{RT} \,\overline{a}_T + Z_0^{-1/2} \overline{N}), \quad (14)$$

leading to the channel equation

$$\underbrace{\overline{v}_R}_{\overline{Y}} = \underbrace{\overline{\overline{S}}_{21}(\overline{\overline{I}} - \overline{\overline{S}}_{RR}\overline{\overline{S}}_{11})^{-1}}_{\overline{\overline{P}}} (\underbrace{Z_0^{1/2}\overline{\overline{S}}_{RT}}_{\overline{\overline{H}}}, \underbrace{\overline{a}_T}_{\overline{X}} + \overline{N}).$$
(15)

We may now compute capacity by assuming optimal Gaussian signaling at the transmitter and maximizing the resulting mutual-information expression. However, since both the noise and signal are scaled by the same factor  $\overline{\overline{P}}$ , matching does not change the mutual-information as long as  $\overline{\overline{P}}$  is full rank. For capacity computations, we may therefore simply remove the matching network ( $\overline{\overline{S}}_{11} = \overline{\overline{0}}$  and  $\overline{\overline{S}}_{21} = \overline{\overline{I}}$ ). The resulting mutual-information expression is equivalent to (2) with the channel matrix  $\overline{\overline{H}}$  replaced with  $\overline{\overline{S}}_{RT}$ . Capacity may then be computed using standard techniques.

## 4.4 Receiver Noise Model

In single-user point-to-point transmission systems, the receiver front end is often the major source of noise. In this case, the amplifiers in Figure 2 contribute the noise vector  $\overline{N}$  at the output, leading to the relation

$$\underbrace{\overline{v}_R}_{\overline{Y}} = \underbrace{Z_0^{1/2}\overline{\overline{S}}_{21}(\overline{\overline{I}} - \overline{\overline{S}}_{RR}\overline{\overline{S}}_{11})^{-1}\overline{\overline{S}}_{RT}}_{\overline{\overline{H}}(\overline{\overline{S}}_M)} \underbrace{\overline{\overline{a}}_T}_{\overline{\overline{X}}} + \overline{N}. \quad (16)$$

In this case, the mutual information expression is

$$I(\overline{Y};\overline{X}) = \log_2 \left| \frac{\overline{\overline{H}}(\overline{\overline{S}}_M)\overline{\overline{K}}_X\overline{\overline{H}}(\overline{\overline{S}}_M)^H}{\sigma^2} + \overline{\overline{I}} \right|$$
$$= \log_2 \left| \frac{\overline{\overline{W}}(\overline{\overline{S}}_{11})\overline{\overline{M}}}{\sigma^2} + \overline{\overline{I}} \right|, \qquad (17)$$

where  $\overline{\overline{W}}(\overline{\overline{S}}_{11})$  is given in (8), the noise vector is i.i.d. complex Gaussian with single element variance  $\sigma^2$ , and

$$\overline{\overline{M}} = \overline{\overline{S}}_{RT} \overline{\overline{K}}_X \overline{\overline{S}}_{RT}^H, \tag{18}$$

In general,  $\overline{\overline{M}}$  is a Hermitian positive semi-definite matrix, so that we can use the eigenvalue decomposition (EVD) of  $\overline{\overline{M}}$  to write

$$\overline{\overline{M}} = \overline{\overline{\xi}}_M \overline{\overline{\Lambda}}_M \overline{\overline{\xi}}_M^H = \overline{\overline{M}}^{1/2} \overline{\overline{M}}^{(1/2)H}$$
(19)

with

$$M^{1/2} = \overline{\overline{\xi}}_M \overline{\overline{\Lambda}}_M^{1/2}.$$
 (20)

Thus, maximization of the mutual-information for a fixed (but arbitrary)  $\overline{\overline{K}}_X$  requires maximization of

$$I(\overline{Y};\overline{X}) = \log_2 \left| \frac{\overline{\overline{M}}^{(1/2)H}\overline{\overline{W}}(\overline{\overline{S}}_{11})\overline{\overline{M}}^{1/2}}{\sigma^2} + \overline{\overline{I}} \right|$$
(21)

over all possible values of  $\overline{\overline{S}}_{11}$  and  $\overline{\overline{K}}_X$ .

The maximization is accomplished by recognizing that the Hermitian match condition will always maximize (21) for fixed but arbitrary  $\overline{\overline{K}}_X$ . To show this, we use the result from Section 3.1 that

$$\overline{x}^{H}\overline{\overline{W}}(\overline{\overline{S}}_{RR}^{H})\overline{x} \ge \overline{x}^{H}\overline{\overline{W}}(\overline{\overline{S}}_{11})\overline{x}$$
(22)

for all possible values of  $\overline{\overline{S}}_{11}$  and  $\overline{x}$ . Letting  $\overline{x} = \overline{\overline{M}}^{1/2}\overline{y}$  and  $\overline{\overline{W}}'(\overline{\overline{S}}) = \overline{\overline{M}}^{(1/2)H} \overline{\overline{W}}(\overline{\overline{S}}) \overline{\overline{M}}^{1/2}$ , we obtain

$$\overline{y}^H \overline{\overline{W}}' (\overline{\overline{S}}_{RR}^H) \overline{y} \ge \overline{y}^H \overline{\overline{W}}' (\overline{\overline{S}}_{11}) \overline{y}$$

and therefore

$$\overline{y}^{H}\left[\frac{\overline{\overline{W}}'(\overline{\overline{S}}_{RR}^{H})}{\sigma^{2}} + \overline{\overline{I}}\right]\overline{y} \ge \overline{y}^{H}\left[\frac{\overline{\overline{W}}'(\overline{\overline{S}}_{11})}{\sigma^{2}} + \overline{\overline{I}}\right]\overline{y}.$$
 (23)

Since the bracketed expressions in (23) are positive definite, we also have [4]

$$\left|\frac{\overline{\overline{W}'}(\overline{\overline{S}}_{RR}^{H})}{\sigma^{2}} + \overline{\overline{I}}\right| \ge \left|\frac{\overline{\overline{W}'}(\overline{\overline{S}}_{11})}{\sigma^{2}} + \overline{\overline{I}}\right|.$$
 (24)

These results prove that for arbitrary  $\overline{K}_X$ ,  $\overline{S}_{11} = \overline{S}_{RR}^H$  will maximize (21). Therefore, we compute capacity by first finding  $\overline{\overline{H}}(\overline{\overline{S}}_M)$  with  $\overline{\overline{S}}_{11} = \overline{\overline{S}}_{RR}^H$  to obtain a fixed channel transfer matrix. Then, other capacity solutions (waterfilling, modified water-filling, uniformed transmit) may be used to find the optimal  $\overline{\overline{K}}_X$  and compute the capacity.

## **5** Capacity Simulations

To demonstrate application of the analysis framework developed in this paper and to illustrate the impact of mutual coupling on the capacity of MIMO systems, we will explore transmit and receive arrays consisting of two coupled dipoles. Specifically, we focus on the receiver noise capacity expression from Section 4.4. Antenna network Sparameter descriptions and radiation patterns obtained from full-wave FDTD simulations are combined with a simple path-based channel model to construct the effective channel matrix.

#### 5.1 FDTD Antenna Simulations

FDTD simulations were run for half-wave (total-length) dipoles with wire radius  $0.01\lambda$  and separated by a variable distance d. These simulations were necessary to obtain realistic values of  $\overline{S}_{RR}$  and  $\overline{S}_{TT}$  to compute the capacity with mutual-coupling. In the simulations, single-frequency antenna excitation was used. The FDTD grid used 80 cells per wavelength in the  $\hat{z}$  direction and 200 cells per wavelength in the  $\hat{x}$  and  $\hat{y}$  directions. This finer resolution was required to adequately model the current variations on the finite-radius wire for close antenna spacings. A quarter-wavelength buffer region was placed between the antennas and the terminating 8-cell perfectly matched layer (PML) absorbing boundary condition.

## 5.2 Path-based Channel Model

Realistic values of  $\overline{\overline{S}}_{RT}$  were computed by assuming a simple path-based channel model. Here we assume that the incident electric field at the receiver may be written as a sum of plane waves, whose amplitude and phase depend on  $\overline{\overline{S}}_{RR}$ ,  $\overline{\overline{S}}_{TT}$ , the far-field patterns of the antennas, and the direction-of-departure (DOD) and direction-of-arrival (DOA) of the multipath components.

In our simulations, we assumed 4 multipath components with Rayleigh i.i.d. amplitude, uniform i.i.d. distributed phase on  $[0, 2\pi]$ , and uniform i.i.d. DOA and DOD on  $[0, 2\pi]$ . For each antenna spacing considered, 7000 random channel realizations were generated, the capacity was computed for each realization, and the mean was taken. Total transmit power was set to an arbitrary constant, and noise power was fixed at the level required to obtain an average of 20 dB SNR for a single antenna system.

Figure 3 demonstrates the combined effect of mutual coupling at transmit and receive. Here, the transmit and receive antenna spacings were equal and capacity was computed for ideal antennas with no mutual-coupling (nmc), mutual coupling at transmit and receive with an optimal match (mc), and mutual coupling with a sub-optimal self-impedance match (mcsi). For spacings between  $0.1\lambda$  and  $0.3\lambda$ , mutual coupling provides an obvious capacity benefit. For spacings below  $0.1\lambda$ , mutual coupling can actually degrade capacity. Finally, the sub-optimal matching network yields a modest capacity degradation. Thus, the rigorous network theory framework confirms the observations made by previous studies (e.g., [7]) that mutual coupling can increase channel capacity.

## 6 Conclusion

We have presented a rigorous network-theory framework for the analysis of mutual coupling in MIMO wireless com-



Figure 3. Capacity vs. transmit/receive antenna spacing.

munications. A detailed network model was used to develop a new mutual information expression and radiated power constraint accounting for this antenna coupling. Unlike previous analyses, this new method includes the effect of mutual coupling, and the resulting capacity expression provides a true upper bound on system performance. We analyzed a simple yet realistic  $2 \times 2$  MIMO system by combining full-wave FDTD simulations with a path-based channel model. Before more general conclusions can be drawn concerning the effect of mutual coupling, more extensive simulations using increased array sizes and various array configurations must be performed.

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