Statistical Characterization of the Indoor MIMO Channel Based on LOS/NLOS Measurements*

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Abstract

Measurements taken at the campus of Brigham Young University (BYU) are used to investigate the statistical properties of the indoor MIMO channel. The Ricean K parameter is estimated along a path in both LOS and NLOS. These values are related to the received power and the spatial spectrum calculated using the conventional beamformer. Furthermore, statistical tests are applied to investigate if the data belongs to a multivariate normal distribution. It is found that the univariate statistics can be approximated by a complex normal distribution but only small MIMO systems may be approximated as multivariate normally distributed.

1. Introduction

The wireless communications industry has experienced an explosive growth the last decades. However, the available spectrum for wireless communications has not grown at the same rate. Increased cost of acquiring spectrum to accommodate users has resulted in a increased interest in spectrum efficient techniques. One of the most promising spectrum efficient techniques is Multiple-Input Multiple-Output (MIMO) systems that employ multiple transmit and receive antennas. These systems exploit the spatial dimension to a larger extent than previous systems and have been shown to be capable of supporting very high data rates without increasing the bandwidth [2].

Since these MIMO systems rely more on the spatial characteristics than previous systems, obtaining knowledge of the channel is of great importance. Several indoor MIMO measurement campaigns have recently been reported in the literature [4, 7, 9, 10]. Most of these measurements were collected at different locations and the combined statistics of all these measurements were studied. However, by combining measurements for different locations, the resulting channel characteristics may differ from those of the local environment. This paper avoids those issues by using highly oversampled measurements of the MIMO channel matrix when moving along corridors. With up to a hundred samples per wavelength, it is possible to address issues such as coherence distances of MIMO measurements and the statistical properties of the local area (fast-fading).

Furthermore, the Ricean K parameter is estimated from measurement data and its value along a measurement path is studied. These K values are also related to the directional information offered by applying the conventional beamformer. Finally, formal statistical hypothesis tests are used to investigate whether the data belongs to a normal distribution. Tests for both UniVariate Normality (UVN) and MultiVariate Normality (MVN) indicate that although the individual channel coefficients are close to normal, the overall MIMO system may not be MVN distributed.

2. Measurement Setup

A narrowband custom made MIMO communications system designed and built at Brigham Young University (BYU) in Utah was used to collect measurements. The system was equipped with ten monopoles forming a uniform circular array at each end. However, since the elements were mounted over a ground plane the monopoles behave as dipoles and essentially have the same radiation patterns as dipoles. Furthermore, the elements were positioned in a circle with radius 0.86 wavelengths that approximately gives an element separation of a half wavelength. The operating frequency was 2.43GHz. For a detailed description of the measurement equipment, see [9].

Measurements were collected within the Clyde building at the BYU campus. The sampling rate was set to 2.5ms in order to get a highly oversampled channel with many samples per wavelength. With a walking speed of 1m/s this results in about 50 samples per wavelength which is enough

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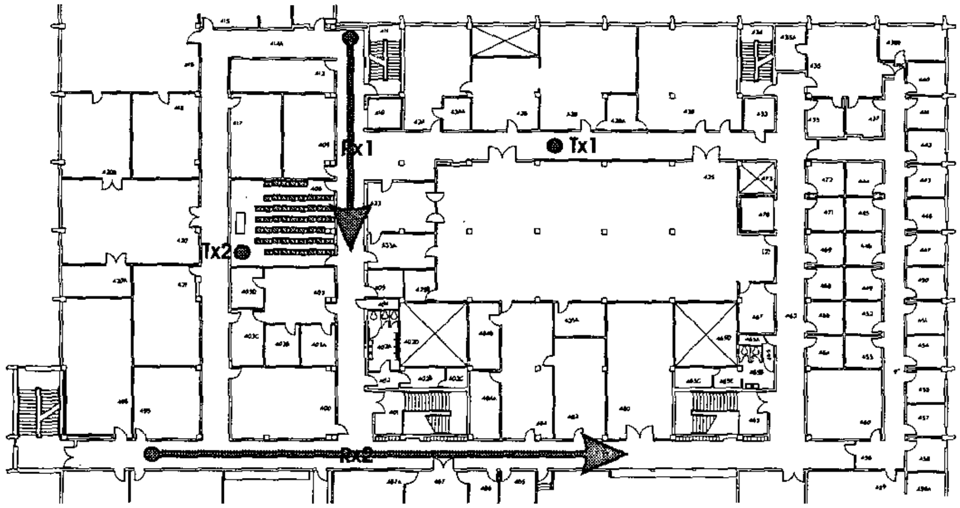

Figure 1: Layout of the fourth floor of the Clyde building with the measurement paths indicated.

to study the local area statistics.

Measurements were collected in both Non Line Of Sight (NLOS) and Line Of Sight (LOS) to study the difference in statistics between the two different environments. A layout of the floor plan of the fourth floor of the Clyde building is shown in Figure 1. In all the measurements, the transmitter was stationary while the receiver was moved. The first measurement scenario represents a situation where the receiver first is in NLOS and moves into LOS and then moves into NLOS again. See Figure 1, where the transmitter position is indicated by Tx1 and the receiver path is indicated by Rx1. Another measurement was performed for the application of a statistical test for MVN since this test requires data that is more stable than the data obtained by moving in and out of LOS. Hence, the second measurement scenario represents a path completely in NLOS. This scenario is also indicated in Figure 1.

3. Measurement Results

3.1. Normalization

Before any analysis of the measurements is possible, it is necessary to establish a reasonable normalization of the measurements. Several different normalization have been suggested in the literature. Previous studies of Single-Input Single-Output (SISO) channels often used a running mean or running median normalization [8]. However, it is not trivial to choose the length of the averaging window. Depending on the scenario, window lengths between 2λ and 64λ have been used [8]. In this paper, another approach is used that takes advantage of the MIMO property of the measurements. The channel matrix is simply normalized by the local average obtained by calculating the average channel coefficient magnitude for each position. Consider a MIMO system with N_t transmit antennas and N_r receive antennas. If the unnormalized $N_r \times N_t$ matrix of channel coefficients is denoted $\widetilde{\mathbf{H}}$, the normalized channel matrix $\mathbf{H}$ is obtained as

$$\mathbf{H} = \widetilde{\mathbf{H}}/\eta = \frac{\widetilde{\mathbf{H}}}{\frac{1}{N_t N_r}\sum_{k=1}^{N_r}\sum_{l=1}^{N_t}|\widetilde{\mathbf{H}}_{k,l}|}, \tag{1}$$

where η is the average magnitude of the channel coefficients $\mathbf{H}_{ij}$. This normalization is a localized version of the running mean normalization and represents the local average over the array aperture which is about $1.7\lambda \times 1.7\lambda$. It is similar to but not the same as the traditional normalization in MIMO literature $||H||_F = \sqrt{N_r N_t}$ where $||\cdot||_F$ denotes the Frobenius matrix norm. The reason for choosing this normalization is to reduce the impact of large values by avoiding the squared magnitudes that the Frobenius norm employs.

In Figure 2, the average magnitude η is shown for the first measurement path indicated in Figure 1. The total length of the first measurement path was found to be about 19m which at a carrier frequency of 2.45GHz corresponds to about 156 λ. It is clear that the average magnitude is stable and can be used for normalization purposes. A sharp rise in magnitude is observed when moving into LOS, as expected. There is also a smaller side peak at around 100λ that may be due to a pillar obstructing the LOS, see Figure 1. In the following analysis, the normalization in (1) will be used.

Figure 2 also shows the estimated local Ricean K parameter of the data which is a good indicator of LOS conditions. For each element of the normalized matrix $\mathbf{H}$, the K factor was estimated using a moment based method [1]

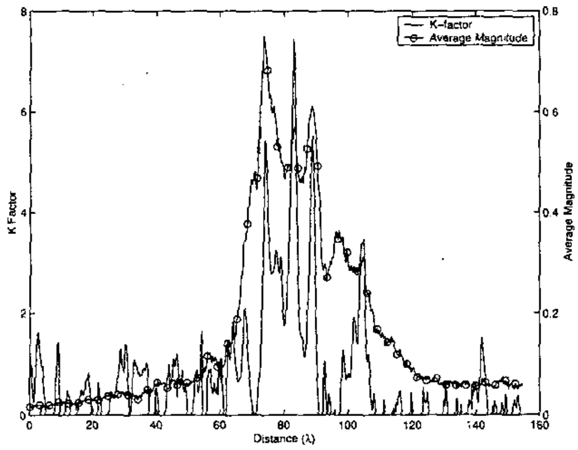

Figure 2: The average channel coefficient magnitude η for the first measurement path in Figure 1. Also shown is the estimated Ricean K parameter for the same path.

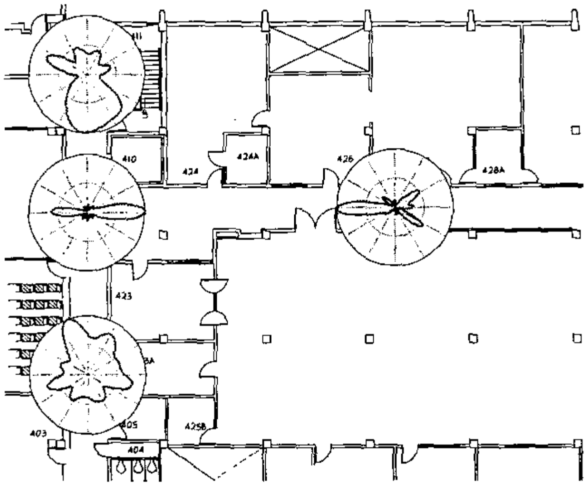

Figure 3: Spatial spectra along the first measurement path in Figure 1.

over a segment of one wavelength. By sliding this window along the measurement path, a sequence of K values was obtained. The plotted K factor in Figure 2 is the average of all the **H** coefficients which for the 10×10 array equals 100 coefficients.

For K values close to zero, the Ricean distribution coincides with the Rayleigh distribution which is known to be a good model for NLOS conditions [8]. In LOS conditions, the channel matrix **H** is dominated by the LOS part that changes much slower than the diffuse NLOS part. Hence, a high K value has been found in those scenarios [8]. This is confirmed in Figure 2 where larger values are obtained in the LOS portion which also has larger magnitude. In the NLOS portions, lower K values as well as magnitudes were obtained. However, the K values are not zero which indicates that there is some component in **H** that is more stationary. Possible explanations for this may be strong reflections, refractions, or wave-guiding effects.

3.2. Beamforming

It is interesting to relate the properties of the channel to the physical environment. To study this, the received energy versus the angle, i.e. the spatial spectrum, is plotted using the conventional beamformer in Figure 3. Spectra for three positions of the receive array and one position for the transmitter (LOS position) are shown. Note that the antenna arrays were mounted on top of carts that were pulled along the hallway which lead to some inaccuracies in position and alignment along the path. It is clear that in LOS there is a focused beam along the LOS direction. But there is also a backlobe at the receiver that may be due to a back wall reflection. Hence, there are at least two rays in the LOS position that gives K values larger than zero but less than in outdoor scenarios with just one dominant ray [8].

At the NLOS positions of the receiver, the energy is arriving from a much wider angular band. Hence, no single ray dominates and low K values result as found in Figure 1. In the NLOS portion of the path, the spatial spectrum also varies rapidly where strong reflectors such as metal bars at times dominate the spatial spectrum. At other times, the main part of the energy arrives in the direction of the hallway. This may be a waveguide effect, especially at larger distances.

3.3. Correlation

Another important characteristic of the channel is the coherence distance, i.e. the distance required for the correlation to drop below a certain value. It is expected that the NLOS section should exhibit a short coherence distance since there are many diffuse contributions that change rapidly with distance. For the LOS positions, a longer coherence distance is expected since there is a stable LOS component. This is confirmed in Figure 4, where the correlation coefficient versus separation distance is shown for a LOS and NLOS section. Here, the NLOS section consists of the last 40 wavelengths of the first path, see Figure 1 and 2. The last NLOS segment was chosen since the K factor was in Figure 2 found to be lower than that of the first NLOS part. As LOS section, a 20 λ section centered around a total distance of 80 λ that has high K values was chosen, see Figure 2. The correlation curves represent the average over all the 100 coefficients.

The separation required to drop to a correlation of 0.5 is 0.3 λ in NLOS and 1 λ in LOS. The NLOS separation distance is close to the distance obtained using the classic Jakes model with a uniform distribution surrounding the receiver

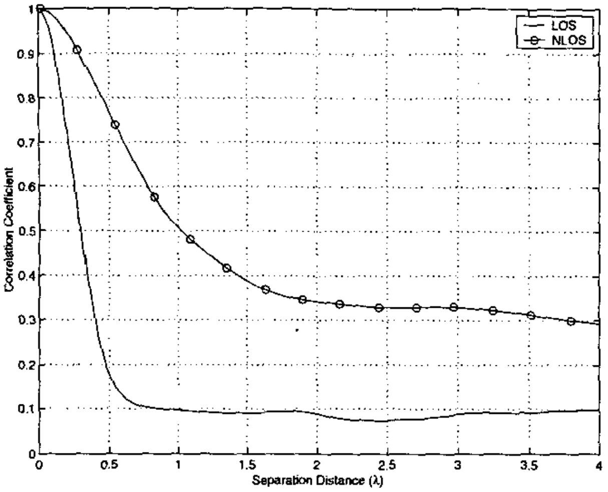

Figure 4: Correlation versus separation distance for NLOS and LOS positions along the first path in Figure 1.

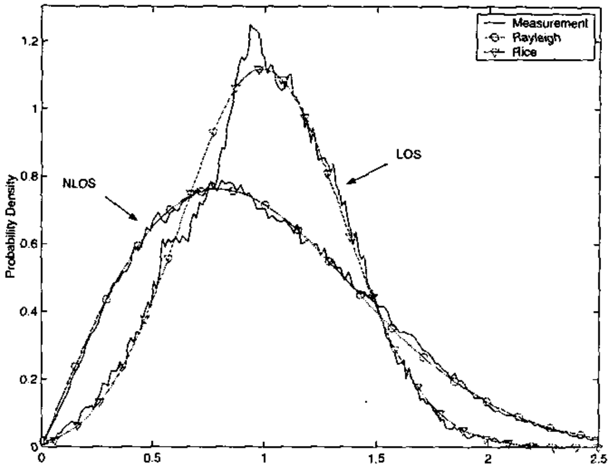

Figure 5: Empirical and estimated probability density functions for magnitude of channel coefficients of the NLOS and LOS part of the first measurement path in Figure 1.

$d = J_0^{-1}(0.5)/(2\pi) \approx 0.25\lambda$. Here, $J_0(\cdot)$ denotes the zeroth order Bessel function of the first kind. This indicates that the signal arrives at the receiver from many different angles, as found in the beamforming analysis in Figure 3.

3.4. Multivariate Statistics

Most analysis of MIMO systems assumes that the statistical distribution of channel coefficients is multivariate normal. Few have, however, investigated the validity of this assumption although some studies of the marginal distributions, i.e. the univariate distribution of each coefficient have appeared [9, 10]. Therein, the magnitude (envelope) is fitted to a Rayleigh distribution and the phase is found to be reasonably well modeled by a uniform distribution. This suggests that a complex normal distribution would fit the measurements well.

But the fact that each coefficient individually is well described by a complex normal distribution does not mean that the joint multivariate distribution must be complex normal. In fact, it will be shown that for the MIMO data under study, the data appears to marginally normal but not MVN. However, for any multivariate normal distribution, all the univariate distributions must be normal. Hence, tests for univariate normality will be applied together with a test for multivariate normality in this section.

Figure 5 shows the distribution of the channel coefficient magnitude for the the NLOS and LOS segments of the first measurement path that was analyzed in Section 3.3. The results are obtained by averaging over all channel coefficients. A Rayleigh distribution is fitted to the NLOS data while a Ricean distribution with K=3.1 is fitted to the LOS data. The Rice and Rayleigh distributions appears to fit the data reasonably well. It should be noted that the empirical Probability Density Function (PDF) for the LOS case is estimated using less data than the NLOS PDF. It was also found that the phase is reasonably well approximated with a uniform distribution, although the LOS case again showed some deviation due to less data.

	NLOS		LOS	
	Real	Imag	Real	Imag
Lilliefors	8%	11%	5%	9%
Shapiro-Wilks	11%	12%	7%	12%
Bera-Jarque	3%	7%	3%	3%

Table 1: Univariate test results for the NLOS and LOS segments along the first path in Figure 1.

To verify that the channel coefficients obey a complex normal distribution, tests for univariate normality were used. Tests were applied to the real part and the imaginary part of the channel coefficients which were found to be approximately uncorrelated. Three test were considered, the Lilliefors modification of the Kolmogorov-Smirnov test, the Bera-Jarque test, and the Shapiro-Wilks test [5]. The Shapiro-Wilks test was constructed by considering the regression of ordered sample values on corresponding expected normal order statistics. It is is known to have good power properties and is considered an omnibus test. The Bera-Jarque test is based on skewness and kurtosis but is an asymptotic test and is expected to perform slightly worse for small samples. Lilliefors test is based on measuring the departure of the Cumulative Distribution Function (CDF) from the hypothesis CDF. Unfortunately, the powers of Bera-Jarque and Lilliefors is not as well known as that of Shapiro-Wilks.

The results of the hypothesis test for the NLOS and LOS

	2×2	3×3	4×4	5×5	6×6
Real	10%	14%	32%	100%	100%
Imag	2%	5%	25%	100%	100%

Table 2: Results from the Henze-Zirkler MVN test applied to data from the second path in Figure 1.

cases are shown in Table 1. Here, the NLOS coefficients were sampled at every 0.3λ and the LOS every λ. These sampling distances corresponds to the 0.5 correlation level found in Figure 4. Thus, the NLOS set consists of 136 samples of each coefficient while the shorter LOS set consists of 41 samples. A significance level of 5% was used to determine the critical values and the results presented in Table 1 represents the average over one hundred coefficients.

Only mild deviations from non-normality are present since the rejection rates are above the 5% significance level but not significantly higher. Note that the rejection rate when performing 100 tests fluctuates quite a bit even under the true hypothesis. Thus, the null hypothesis of normally distributed data can not be rejected as was suspected from the empirical PDF plots in Figure 5.

Many tests have been designed to assess multivariate normality. A recent survey of tests for MVN [6] found that the Henze-Zirkler test [3] has overall good power against alternative distributions. This test is based on measuring the deviation between the characteristic function of the empirical and hypothesis distributions. This test will be used to examine the validity of the assumption of MVN of the measured MIMO data. To test MIMO systems of increasing orders which requires more data, the second measurement path in Figure 1 was used for these tests. This path is in NLOS and is significantly longer but also more stable since no LOS sections are present.

A window of 40λ was used to test the different hypothesis. By sliding this window along the measurement path, statistics of the performance were collected. The data was downsampled to one sample per wavelength to ensure low correlation between samples. Before investigating MVN, the univariate tests that were applied to the first path were applied also to this path. Those test yielded similar results with no strong indications of non-normality. The MVN results, averaged along the measurement path, are presented in Table 2. For 2×2 and 3×3 systems, the rejection rates are moderate and these systems may be approximated as MVN distributed. However, for larger systems there is strong evidence of non-normality and MVN must be rejected for these systems. Hence, it seems that MVN is an reasonable assumption only for small systems.

Performing the same test for the first measurement path, yields essentially the same results except for the segments with LOS. This is due to the fact that in LOS there is a deterministic component that varies, albeit slowly, over the LOS segment. Hence, the resulting PDF is the average distribution along the segment which differs from a MVN distribution. Measurements with longer stationary LOS segments are needed to run the hypothesis test in that case.

4. Conclusions

Measurements taken at the campus of Brigham Young University (BYU) were used to investigate the statistical properties of the indoor MIMO channel. The Ricean K parameter was estimated along a path in both LOS and NLOS. It was found that the maximum K value in LOS conditions is about seven and less than unity in NLOS. These results were related to spatial spectra calculated using the conventional beamformer which confirmed a larger multipath content in NLOS. Furthermore, statistical tests were applied to investigate if the data belongs to a multivariate normal distribution. It was found that the univariate statistics can be approximated by a complex normal distribution but only small MIMO systems can be considered MVN distributed.

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