Here is a detailed proof of Proposition 7.2 from *Parametrized Tutte polynomials of graphs and matroids*.

To verify that the identities of the generalized Zaslavsky-Bollobás-Riordan theorem for matroids hold in $\mathcal{M}''$ we must verify that if $\{e_0, e_1\}$ is a digon in $\mathcal{M}''$ then $T(\emptyset)\cdot(X_{e_0''}y_{e_1}+Y_{e_0''}x_{e_1}) = T(\emptyset)\cdot(X_{e_1}y_{e_0''}+Y_{e_1}x_{e_0''})$, if $\{e_0,e_1,e_2\}$ is a triangle or triad in $\mathcal{M}''$ then $T(\emptyset)\cdot Z_{e_2}x_{e_1}(Y_{e_0''}-y_{e_0''}) = T(\emptyset)\cdot Z_{e_2}x_{e_0''}(Y_{e_1}-y_{e_1})$, where Z_{e_2} is X_{e_2} or Y_{e_2}, and if $\{e_0,e_1,e_2\}$ is a triangle or triad in $\mathcal{M}''$ then $T(\emptyset)\cdot Z_{e_0''}x_{e_1}(Y_{e_2}-y_{e_2}) = T(\emptyset)\cdot Z_{e_0''}x_{e_2}(Y_{e_1}-y_{e_1})$, where $Z_{e_0''}$ is $X_{e_0''}$ or $Y_{e_0''}$.

By hypothesis (i) the third of these identities follows immediately from the fact that $Z_{e_0''}$ is a sum of a multiple of X_{e_0} and a multiple of Y_{e_0}.

The second identity is

$$\begin{aligned}
& T(\emptyset)\cdot Z_{e_2}x_{e_1}(Y_{e_0''}-y_{e_0''}) \\
= \ & T(\emptyset)\cdot Z_{e_2}x_{e_1}(Y_{e_0}X_{e_0}y_{e_0}+x_{e_0}Y_{e_0}^2-X_{e_0}Y_{e_0}^2-y_{e_0}^2x_{e_0})T(M_2/e_0)-T(\emptyset)\cdot Z_{e_2}x_{e_1}(y_{e_0}-Y_{e_0})y_{e_0}^2T(M_2-e_0) \\
= \ & T(\emptyset)\cdot Z_{e_2}x_{e_1}(Y_{e_0}X_{e_0}(y_{e_0}-Y_{e_0})+x_{e_0}(Y_{e_0}^2-y_{e_0}^2))T(M_2/e_0)-T(\emptyset)\cdot Z_{e_2}x_{e_0}(y_{e_1}-Y_{e_1})y_{e_0}^2T(M_2-e_0) \\
= \ & T(\emptyset)\cdot Z_{e_2}x_{e_1}(Y_{e_0}-y_{e_0})(-Y_{e_0}X_{e_0}+x_{e_0}(Y_{e_0}+y_{e_0}))T(M_2/e_0)+T(\emptyset)\cdot Z_{e_2}x_{e_0}(Y_{e_1}-y_{e_1})y_{e_0}^2T(M_2-e_0) \\
= \ & T(\emptyset)\cdot Z_{e_2}x_{e_0}(Y_{e_1}-y_{e_1})(-Y_{e_0}X_{e_0}+x_{e_0}(Y_{e_0}+y_{e_0}))T(M_2/e_0)+T(\emptyset)\cdot Z_{e_2}x_{e_0}(Y_{e_1}-y_{e_1})y_{e_0}^2T(M_2-e_0) \\
= \ & T(\emptyset)\cdot Z_{e_2}x_{e_0''}(Y_{e_1}-y_{e_1}).
\end{aligned}$$

Here $T(\emptyset)\cdot Z_{e_2}x_{e_1}(y_{e_0}-Y_{e_0}) = T(\emptyset)\cdot Z_{e_2}x_{e_0}(y_{e_1}-Y_{e_1})$ and $T(\emptyset)\cdot Z_{e_2}x_{e_1}(Y_{e_0}-y_{e_0}) = T(\emptyset)\cdot Z_{e_2}x_{e_0}(Y_{e_1}-y_{e_1})$ are simply the corresponding triangle or triad identity in $\mathcal{M}$.

The digon identity is much harder to verify. Observe that

$$\begin{aligned}
& T(\emptyset)\cdot(X_{e_0''}y_{e_1}+Y_{e_0''}x_{e_1}) \\
&= T(\emptyset)\cdot(Y_{e_0}x_{e_1}+x_{e_0}y_{e_1})(X_{e_0}y_{e_0}+x_{e_0}Y_{e_0}-X_{e_0}Y_{e_0})T(M_2/e_0) \\
&+ T(\emptyset)\cdot y_{e_0}y_{e_1}(X_{e_0}y_{e_0}+x_{e_0}Y_{e_0}-X_{e_0}Y_{e_0})T(M_2-e_0) \\
&= T(\emptyset)\cdot X_{e_0}Y_{e_0}x_{e_1}(y_{e_0}-Y_{e_0})T(M_2/e_0)+T(\emptyset)\cdot Y_{e_0}x_{e_1}x_{e_0}Y_{e_0}T(M_2/e_0) \\
&\quad + T(\emptyset)\cdot x_{e_0}y_{e_1}(X_{e_0}y_{e_0}+x_{e_0}Y_{e_0}-X_{e_0}Y_{e_0})T(M_2/e_0) \\
&+ T(\emptyset)\cdot y_{e_0}(X_{e_0}y_{e_0}y_{e_1}+Y_{e_0}y_{e_1}(x_{e_0}-X_{e_0}))T(M_2-e_0).
\end{aligned}$$

By hypothesis (ii) and Proposition 2.4 (Proposition 2.1 in the published article), $T(M_2-e_0)$ and $T(M_2/e_0)$ are sums of multiples of $T(\emptyset)\cdot X_{e_2}$ and $T(\emptyset)\cdot Y_{e_2}$ for some $e_2\in S_2-\{e_0\}$ which is triangular and triadic with e_0 and e_1 in $\mathcal{M}$. This implies that $x_{e_1}(y_{e_0}-Y_{e_0})T(M_2/e_0) = x_{e_0}(y_{e_1}-Y_{e_1})T(M_2/e_0)$ and $y_{e_1}(x_{e_0}-X_{e_0})T(M_2-e_0) = y_{e_0}(x_{e_1}-X_{e_1})T(M_2-e_0)$, so

$$\begin{aligned}
& T(\emptyset)\cdot(X_{e_0''}y_{e_1}+Y_{e_0''}x_{e_1}) \\
= \ & T(\emptyset)\cdot X_{e_0}Y_{e_0}x_{e_0}(y_{e_1}-Y_{e_1})T(M_2/e_0)+T(\emptyset)\cdot Y_{e_0}x_{e_1}x_{e_0}Y_{e_0}T(M_2/e_0) \\
& +T(\emptyset)\cdot x_{e_0}y_{e_1}(X_{e_0}y_{e_0}+x_{e_0}Y_{e_0}-X_{e_0}Y_{e_0})T(M_2/e_0) \\
& +T(\emptyset)\cdot y_{e_0}(X_{e_0}y_{e_0}y_{e_1}+Y_{e_0}y_{e_0}(x_{e_1}-X_{e_1}))T(M_2-e_0) \\
= \ & T(\emptyset)\cdot(-X_{e_0}Y_{e_0}x_{e_0}Y_{e_1}+Y_{e_0}x_{e_1}x_{e_0}Y_{e_0}+x_{e_0}y_{e_1}(X_{e_0}y_{e_0}+x_{e_0}Y_{e_0}))T(M_2/e_0) \\
& +T(\emptyset)\cdot y_{e_0}(y_{e_0}(X_{e_0}y_{e_1}+Y_{e_0}x_{e_1})-Y_{e_0}y_{e_0}X_{e_1})T(M_2-e_0).
\end{aligned}$$

The triad, triangle and digon identities in $\mathcal{M}$ imply $(x_{e_0}y_{e_1}+x_{e_1}Y_{e_0})T(M_2/e_0) = (x_{e_1}y_{e_0}+Y_{e_1}x_{e_0})T(M_2/e_0)$, $(X_{e_0}y_{e_1}+Y_{e_0}x_{e_1})T(M_2/e_0) = (X_{e_1}y_{e_0}+Y_{e_1}x_{e_0})T(M_2/e_0)$ and $(X_{e_0}y_{e_1} + Y_{e_0}x_{e_1})T(M_2 - e_0) = (X_{e_1}y_{e_0} + Y_{e_1}x_{e_0})T(M_2 - e_0)$. Hence

$$\begin{aligned}
& T(\emptyset)\cdot(X_{e_0''}y_{e_1} + Y_{e_0''}x_{e_1}) \\
= \; & T(\emptyset)\cdot(-X_{e_0}Y_{e_0}x_{e_0}Y_{e_1} + x_{e_0}y_{e_1}X_{e_0}y_{e_0} + x_{e_0}Y_{e_0}(x_{e_1}y_{e_0} + Y_{e_1}x_{e_0}))T(M_2/e_0) \\
& +T(\emptyset)\cdot y_{e_0}(y_{e_0}(X_{e_1}y_{e_0} + Y_{e_1}x_{e_0}) - Y_{e_0}y_{e_0}X_{e_1})T(M_2 - e_0) \\
= \; & T(\emptyset)\cdot(-X_{e_0}Y_{e_0}x_{e_0}Y_{e_1} + x_{e_0}y_{e_0}(y_{e_1}X_{e_0} + Y_{e_0}x_{e_1}) + x_{e_0}Y_{e_0}Y_{e_1}x_{e_0})T(M_2/e_0) \\
& +T(\emptyset)\cdot y_{e_0}(y_{e_0}Y_{e_1}x_{e_0} + y_{e_0}X_{e_1}(y_{e_0} - Y_{e_0}))T(M_2 - e_0) \\
= \; & T(\emptyset)\cdot(-X_{e_0}Y_{e_0}x_{e_0}Y_{e_1} + x_{e_0}y_{e_0}(X_{e_1}y_{e_0} + Y_{e_1}x_{e_0}) + x_{e_0}Y_{e_0}Y_{e_1}x_{e_0})T(M_2/e_0) \\
& +T(\emptyset)\cdot y_{e_0}(y_{e_0}Y_{e_1}x_{e_0} + y_{e_0}X_{e_1}(y_{e_0} - Y_{e_0}))T(M_2 - e_0) \\
= \; & T(\emptyset)\cdot(x_{e_0''}Y_{e_1} + y_{e_0''}X_{e_1}).
\end{aligned}$$

This verifies the identities of the generalized Z-B-R theorem in $\mathcal{M}''$.

It remains to prove $T''(M_1'') = (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})T(\emptyset)T(M)$.

Suppose $|S_1| = 2$, and $S_1 = \{e_0, e_1\}$. Then e_0 and e_1 are parallel (and in series) in M_1 and M_1'', and hence

$$\begin{aligned}
T''(M_1'') \; &= \; (x_{e_1}Y_{e_0''} + y_{e_1}X_{e_0''})T(\emptyset) \\
&= \; (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})(x_{e_1}Y_{e_0}T(M_2/e_0) + y_{e_1}T(M_2))T(\emptyset) \\
&= \; (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})((x_{e_1}Y_{e_0} + y_{e_1}x_{e_0})T(M_2/e_0) + y_{e_1}y_{e_0}T(M_2 - e_0))T(\emptyset) \\
&= \; (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})((x_{e_1}Y_{e_0} + y_{e_1}x_{e_0})T((M - e_1)/e_0) + y_{e_1}y_{e_0}T(M - e_1 - e_0))T(\emptyset) \\
&= \; (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})(y_{e_1}x_{e_0}T((M - e_1)/e_0) + y_{e_1}y_{e_0}T(M - e_1 - e_0))T(\emptyset) \\
& \quad +(X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})(x_{e_1}Y_{e_0}T((M - e_1)/e_0))T(\emptyset) \\
&= \; (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})y_{e_1}T(M - e_1)T(\emptyset) \\
& \quad +(X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})(x_{e_1}Y_{e_0}T((M/e_1) - e_0))T(\emptyset) \\
&= \; (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})(y_{e_1}T(M - e_1) + x_{e_1}T(M/e_1))T(\emptyset) \\
&= \; (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})T(M)T(\emptyset).
\end{aligned}$$

Suppose now that $|S_1| > 2$. If there is an $e_1 \in S_1$ which is neither parallel to nor in series with e_0 then Proposition 7.2 may be assumed to hold for both M/e_1 and $M - e_1$, and its validity for M follows using the deletion-contraction formula.

If there is an $e_1 \in S_1$ which is parallel to e_0 then we may assume inductively that Proposition 7.2 holds for $M - e_1$, i.e., $T''(M_1'' - e_1) = (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})T(M - e_1)T(\emptyset)$. On the other hand,

$$\begin{aligned}
& T''(M_1''/e_1) \\
= \; & Y_{e_0''}T''((M_1''/e_1) - e_0) \\
= \; & Y_{e_0}(X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})T(M_2/e_0)T((M_1/e_1) - e_0) \\
= \; & Y_{e_0}(X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})T(M_2/e_0)T((M_1/e_1) - e_0) \\
= \; & (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})T(M_2/e_0)Y_{e_0}T((M_1/e_1) - e_0).
\end{aligned}$$

As M is the parallel connection of M_1 and M_2, M/e_0 is the direct sum of M_2/e_0 and M_1/e_0. As e_1 is parallel to e_0, M_1/e_0 is the direct sum of a loop e_1 and $(M_1/e_0) - e_1$. Moreover, M/e_1 is isomorphic to M/e_0, so M/e_1 is the direct sum of M_2/e_0, a loop e_0, and $(M_1/e_0) - e_1 = (M_1/e_1) - e_0$. It follows that $T(M/e_1)T(\emptyset) = T(M_2/e_0)Y_{e_0}T((M_1/e_1) - e_0)$ and hence $T''(M_1''/e_1) = (X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})T(M/e_1)T(\emptyset)$. The validity of Proposition 7.2 for M follows, using the deletion-contraction formula.

Finally, suppose e_0 is in series with every $e_1 \in S_1$, i.e., M_1 is a circuit. Choose a particular $e_1 \neq e_0 \in S_1$. We may assume inductively that Proposition 7.2 holds for M/e_1. $M - e_1$ is the direct sum of M_2 and $M_1 - e_1 - e_0$, so $T(M - e_1)T(\emptyset) = T(M_1 - e_1 - e_0)T(M_2)$. $M_1'' - e_1$ is the direct sum of $\{e_0\}$ and $M_1'' - e_1 - e_0 = M_1 - e_1 - e_0$, so $T''(M_1'' - e_1) = T''(M_1 - e_1 - e_0)X_{e_0''} = T(M_1 - e_1 - e_0)(X_{e_0}y_{e_0} + x_{e_0}Y_{e_0} - X_{e_0}Y_{e_0})T(M_2)$, and we see that Proposition 7.2 holds for $M - e_1$. The validity of Proposition 7.2 for M follows from the deletion-contraction formula.