**Definition.** A power series centered at \( x = a \) is an infinite polynomial of the form

\[
\sum_{n=0}^{\infty} c_n (x - a)^n = c_0 + c_1 (x - a) + c_2 (x - a)^2 + c_3 (x - a)^3 + \ldots
\]

where \( a \) and each \( c_i \) are constants.

**Theorem.** For a given power series

\[
\sum_{n=1}^{\infty} c_n (x - a)^n,
\]

there are only three possibilities:

1. The series converges only when \( x = a \).

2. The series converges (absolutely) for every \( x \).

3. There is a number \( R > 0 \) so that the series converges absolutely for all \( x \) so that \( |x - a| < R \) and diverges for all \( x \) so that \( |x - a| > R \). The series may or may not converge at the points \( x = a - R \) and \( x = a + R \).

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We put all of this information together to get a method for determining where the power series

\[
\sum_{n=1}^{\infty} c_n (x - a)^n
\]

converges:

1. Ignore the original series and consider the absolute value of the series \( \sum_{n=1}^{\infty} |c_n (x - a)^n| \).

2. Test the series of absolute values for convergence, usually by using the ratio test or the root test. If we find that the series converges whenever \( |x - a| < R \), then the series converges for all \( x \) so that \( a - R < x < a + R \).

3. Test separately for convergence of the series at the endpoints \( a - R \) and \( a + R \) by evaluating the series at these endpoints and using tests from 11.2-11.6.

4. The series converges when \( a - R < x < a + R \), diverges when \( x < a - R \) or \( x > a + R \), and may converge or diverge at the endpoints \( a - R \) and \( a + R \) depending on the solutions found in the previous step.