1. Find the coordinates of each of $f_2(x) = x^2$, $f_1(x) = x$, and $f_0(x) = 1$ with respect to each of the following bases for $\mathcal{P}_2(\mathbb{R})$:

   (a) $B_1 = (x^2,\ x,\ 1)$

   (b) $B_2 = (2x - 3,\ x^2 + 1,\ 2x^2 - x)$

2. The vectors

$$e_{11} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix},\ e_{12} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix},\ \text{and } e_{21} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$$

   form a basis B for the space $\mathfrak{sl}(2, \mathbb{R})$ of 2×2 trace 0 matrices with real entries. We may extend this basis to a basis $\hat{B}$ for all of $\mathcal{M}_2(\mathbb{R})$ by adjoining vector

$$e_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}.$$

   (a) Find the coordinates of

$$v = \begin{pmatrix} 7 & -3 \\ 1 & -7 \end{pmatrix}$$

   as a vector in $\mathfrak{sl}(2, \mathbb{R})$ with respect to

$$B = (e_{11},\ e_{12},\ e_{21}).$$

   (b) Find the coordinates of v *as a vector in* $\mathcal{M}_2(\mathbb{R})$ with respect to $\hat{B} = (e_{11},\ e_{12},\ e_{21},\ e_1)$.

3. Let V be a finite dimensional vector space over $\mathbb{F}$ with basis

$$B = (v_1,\ v_2,\ \ldots,\ v_n).$$

   Prove the following assertions:

   (a) $(u + v)_B = (u)_B + (v)_B$ for all $u, v \in V$

   (b) $(\lambda u)_B = \lambda(u)_B$ for all $u \in V$, $\lambda \in \mathbb{F}$.

4. For each of the following functions, determine if the function is a linear transformation on the given spaces (please justify your conclusions):

   (a) $S : \mathbb{R}^2 \to \mathbb{R}$,

$$S\left(\begin{pmatrix} x \\ y \end{pmatrix}\right) = \sqrt{x^2 + y^2}.$$

   (b) $D : \mathcal{M}_2(\mathbb{R}) \to \mathbb{R}$,

$$D\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = ad - bc.$$

   (c) For fixed $b \in \mathbb{R}^3$, $P_b : \mathbb{R}^3 \to \mathbb{R}^3$ is defined by projection onto b, that is

$$P_b(a) = \left(\frac{a \cdot b}{|b|^2}\right) b.$$