1. Recall that the vector space $\mathfrak{sl}(2,\mathbb{R})$ of 2×2 trace 0 matrices is a subspace of $\mathcal{M}_2(\mathbb{R})$. The list
$$\left(\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}, \begin{pmatrix}0 & 1\\ 0 & 0\end{pmatrix}, \begin{pmatrix}0 & 0\\ 1 & 0\end{pmatrix}\right).$$
Find *two different* subspaces W_1 and W_2 of $\mathcal{M}_2(\mathbb{R})$ so that
$$\mathcal{M}_2(\mathbb{R}) = \mathfrak{sl}(2,\mathbb{R})\oplus W_1 \text{ and } \mathfrak{sl}(2,\mathbb{R})\oplus W_2.$$

2. Let V be the subspace of $\mathbb{R}^4$ spanned by the vectors
$$\left(\begin{pmatrix}11\\4\\1\\10\end{pmatrix}, \begin{pmatrix}2\\1\\1\\1\end{pmatrix}, \begin{pmatrix}8\\1\\-5\\13\end{pmatrix}, \begin{pmatrix}-1\\0\\1\\-2\end{pmatrix}\right).$$
   (a) Determine the dimension of V.
   (b) Find a basis for V.

3. Let
$$A = \begin{pmatrix}1 & -1 & 4\\ 2 & -1 & 5\\ 1 & 0 & 1\end{pmatrix}.$$
We can think of the columns of A as vectors in $\mathbb{R}^3$. The subspace of $\mathbb{R}^3$ spanned by the columns of A is called the *column space* of A, denoted by column (A).
   (a) Find two nonzero vectors in column (A).
   (b) Show that column $(A) \neq \mathbb{R}^3$ using an argument on the determinant of A.
   (c) Find a basis for column (A) and determine the dimension of column (A).

4. Let V be the subspace of $\mathcal{P}_4(\mathbb{R})$ of all vectors $p\in\mathcal{P}_4(\mathbb{R})$ so that
$$p''(1/2) = 0.$$
   (a) Find a basis for V.
   (b) Extend the basis to a basis for $\mathcal{P}_4(\mathbb{R})$.

5. Prove that every subspace of $\mathbb{R}^3$ is either $\{\mathbf{0}\}$, a line through the origin, or a plane through the origin.

6. Prove that if U and W are both 5 dimensional subspaces of $\mathbb{R}^9$, then $U\cap W\neq\{\mathbf{0}\}$.

7. The notation
$$\dim_{\mathbb{F}}(V)$$
denotes the dimension of V as a vector space over field $\mathbb{F}$.

   Let V be any vector space over $\mathbb{C}$. Since $\mathbb{R}\subset\mathbb{C}$, V is also a vector space over $\mathbb{R}$. Prove that
$$\dim_{\mathbb{R}}(V) = 2\dim_{\mathbb{C}}(V).$$