In the last challenge problem, we defined a new operation on the space $\mathcal{M}_n(\mathbb{C})$ of $n \times n$ matrices with complex entries; for matrices X, $Y \in \mathcal{M}_n(\mathbb{C})$, the bracket $[X, Y]$ of X and Y is given by

$$[X, Y] = XY - YX,$$

where the notation "XY" indicates the matrix product of X with Y.

We have already proved that the bracket operation is neither commutative nor associative; indeed, we saw that

1. $[X, Y] = -[Y, X]$, and that
2. $[X, [Y, Z]] \neq [[X, Y], Z]$.

In this challenge problem, we will discuss several properties that the bracket operation *does* satisfy.

1. Show that $[X, X] = \mathbf{0}$ for all $n \times n$ matrices X.
2. Let
$$X = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \ Y = \begin{pmatrix} 2 & 1 \\ 0 & 1 \end{pmatrix} \text{ and } Z = \begin{pmatrix} -1 & 0 \\ 1 & 1 \end{pmatrix}.$$
   Calculate:
   (a) $[[X, Y], Z]$
   (b) $[[Y, Z], X]$
   (c) $[[Z, X], Y]$
   (d) $[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y]$
3. Show that the bracket operation on $\mathcal{M}_n(\mathbb{C})$ satisfies the Jacobi identity:
$$[[X, Y], Z] + [[Y, Z], X] + [[Z, X], Y] = \mathbf{0}.$$
   (We can think of this identity as a type of associative law–just not the one we are use to).