Definitions 1.18-1.19. A *vector space* V *over a field* $\mathbb{F}$ is a set V of objects call *vectors*, along with two operations defined on V:

1. **Closure of V under addition**: the operation "+" assigns a vector $u+v$ in V to every pair u, v of vectors in V.

2. **Closure of V under scalar multiplication**: the operation of scalar multiplication assigns a vector λu to every pair λ, u, where λ is an element of $\mathbb{F}$ and u is a vector in V.

The operations of vector addition and scalar multiplication must satisfy the following rules:

3. **Commutativity:** $u + v = v + u \ \forall \ u, v \in V$

4. **Associativity:** $(u + v) + w = u + (v + w) \ \forall \ u, v, w \in V$

5. **Additive Identity:** There is an element $\mathbf{0}$ in V, called the *zero vector*, so that

$$u + \mathbf{0} = \mathbf{0} + u = u$$

   for every $u \in V$.

6. **Additive Inverse:** For each elemnt u in V, there is another element w in V, called an *additive inverse* of u, so that $u + w = \mathbf{0}$.

7. **Multiplicative Identity:** The number $1 \in \mathbb{F}$ has the property that $1u = u$ for all $u \in V$.

8. **Distribution of Scalar Multiplication over Vector Addition:** $\alpha(u + v) = \alpha u + \alpha v$ for all $u, v \in V$, all $\alpha \in \mathbb{F}$.

9. **Distribution of Scalar Multiplication over Scalar Addition:** $(\alpha + \gamma)u = \alpha u + \gamma u$ for all $u \in V$, all $\alpha, \gamma \in \mathbb{F}$.