Unit 2, Section 1: Introduction to Vector Spaces

Definitions 1.18-1.19. A vector space $V$ over a field $F$ is a set $V$ of objects call vectors, along with two operations defined on $V$:

1. **Closure of $V$ under addition**: the operation “+” assigns a vector $u + v$ in $V$ to every pair $u, v$ of vectors in $V$.

2. **Closure of $V$ under scalar multiplication**: the operation of scalar multiplication assigns a vector $\lambda u$ to every pair $\lambda, u$, where $\lambda$ is an element of $F$ and $u$ is a vector in $V$.

The operations of vector addition and scalar multiplication must satisfy the following rules:

3. **Commutativity**: $u + v = v + u \ \forall \ u, v \in V$

4. **Associativity**: $(u + v) + w = u + (v + w) \ \forall \ u, v, w \in V$

5. **Additive Identity**: There is an element $0$ in $V$, called the zero vector, so that

$$u + 0 = 0 + u = u$$

for every $u \in V$.

6. **Additive Inverse**: For each element $u$ in $V$, there is another element $w$ in $V$, called an additive inverse of $u$, so that $u + w = 0$.

7. **Multiplicative Identity**: The number $1 \in F$ has the property that $1u = u$ for all $u \in V$.

8. **Distribution of Scalar Multiplication over Vector Addition**: $\alpha(u + v) = \alpha u + \alpha v$ for all $u, v \in V$, all $\alpha \in F$.

9. **Distribution of Scalar Multiplication over Scalar Addition**: $(\alpha + \gamma)u = \alpha u + \gamma u$ for all $u \in V$, all $\alpha, \gamma \in F$.