

1. Let λ be a variable which takes on real or complex values, and consider the matrix $A - \lambda I$, where A and I are both $n \times n^*$. Calculate the matrix $A - \lambda I$ and its determinant $\det(A - \lambda I)$ for each of the following choices of A :

(a) $A = (4)$

(b) $A = \begin{pmatrix} 2 & 1 \\ 4 & 1 \end{pmatrix}$

(c) $A = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 0 \end{pmatrix}$

2. Given an $n \times n$ matrix A , set $p(\lambda) = \det(A - \lambda I)$. Calculate $p(0)$ for each of the matrices A from part (1), and determine what information $p(0)$ provides about A .
3. Consider the set \mathbb{R}_+ of all positive real numbers, with the following definitions for addition and scalar multiplication (to avoid confusion, we use the symbol \square to refer to the addition operation, and \wedge to refer to the operation of scalar multiplication):

$$a \square b = ab \text{ and } \lambda \wedge a = a^\lambda.$$

In other words, our “addition” in \mathbb{R}_+ is just real number multiplication, and our “scalar multiplication” raises a given vector in \mathbb{R}_+ to the designated scalar power.

\mathbb{R}_+ together with these operations is a vector space over \mathbb{R} .

- (a) Calculate $3 \square 5$.
- (b) Calculate $-2 \wedge 3$.
- (c) Show that \mathbb{R}_+ is closed under scalar multiplication by elements of \mathbb{R} .
- (d) What number serves as the additive identity in \mathbb{R}_+ ?
- (e) Given an element a of \mathbb{R}_+ , determine the form of the additive inverse of a .
- (f) \mathbb{R}_+ is a subset of elements of \mathbb{R} . Is \mathbb{R}_+ with the operations defined above a subspace of \mathbb{R} (under the normal operations on \mathbb{R})? Explain why/why not.

*For any $n \times n$ matrix A , and λ a real or complex variable, the function $\det(A - \lambda I)$ is a degree n polynomial.