1. Find a pair of 2×2 invertible matrices A and B so that $(AB)^{-1} \neq A^{-1}B^{-1}$.

2. Let A, B be $n \times n$. Prove that $(AB)^\top = B^\top A^\top$ (I am only asking you to prove for square matrices, but the theorem is true as long as the product AB is defined).

3. Find a pair of 2×2 symmetric matrices A and B so that AB *is not* symmetric.

4. Let A and B be symmetric matrices. Find a condition on A and B that is equivalent to the statement "AB is symmetric", and prove it. Your result should say "If A and B are symmetric, then AB is symmetric if and only if...".

5. An $n \times n$ matrix A is called *skew hermitian* if $A^* = -A$. Find an example of a 3×3 skew hermitian matrix, *none* of whose entries is strictly real.

6. Every $n \times n$ matrix A can be written in the form
$$A = A_S + A_H,$$
where A_S is skew hermitian and A_H is hermitian. Find formulas for A_S and A_H. (Note: just as $(A^\top)^\top = A$, it is easy to see that $(A^*)^* = A$).

7. Let A be an $n \times n$ matrix with strictly real entries. If A is also skew hermitian, what can we say about the diagonal entries of A?

8. Let A be an $n \times n$ matrix, and r a positive integer. We define powers of A in a natural way:
$$A^1 = A,\ A^2 = A \cdot A,\ \ldots,\ A^r = \underbrace{A \cdot A \cdot \ldots \cdot A}_{r \text{ factors}}.$$
Then it is clear that the usual exponential rules hold, i.e.
$$A^r A^s = A^{r+s} \text{ and } (A^r)^s = A^{rs}.$$
Use induction to prove that, if A is invertible, r a positive integer, then A^r is invertible as well, and
$$(A^r)^{-1} = (A^{-1})^r.$$

9. Prove that if $A = [a_{ij}]$ is an $n \times n$ upper triangular matrix, r a positive integer, then the diagonal entries of A^r have form a_{ii}^r.