1. The bracket of
$$X = \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix} \text{ and } Y = \begin{pmatrix} 2 & 0 \\ -1 & -2 \end{pmatrix}$$
is given by
$$\begin{aligned} [X, Y] &= XY - YX \\ &= \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -1 & -2 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ -1 & -2 \end{pmatrix} \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix} \\ &= \begin{pmatrix} -2 & -8 \\ 5 & 2 \end{pmatrix} - \begin{pmatrix} 2 & 8 \\ -5 & -2 \end{pmatrix} \\ &= \begin{pmatrix} -4 & -16 \\ 10 & 4 \end{pmatrix}. \end{aligned}$$
Notice that X, Y, and $[X, Y]$ are all 2×2 traceless matrices.

2. $[\cdot, \cdot]$ is most definitely *not* commutative: indeed, we can use X and Y from the previous example to see that this is the case:
$$[X, Y] = \begin{pmatrix} -4 & -16 \\ 10 & 4 \end{pmatrix}, \text{ but } [Y, X] = \begin{pmatrix} 4 & 16 \\ -10 & -4 \end{pmatrix}.$$
It is interesting to note in this case that $[X, Y] = -[Y, X]$.

3. Indeed, it is easy to see that this property is true for *any* pair of $n \times n$ matrices X and Y:
$$\begin{aligned} [X, Y] &= XY - YX \\ &= -(-XY + YX) \\ &= -(YX - XY) \\ &= -[Y, X]. \end{aligned}$$

4. The bracket operation fails to be associative; keeping X and Y from above, and setting
$$Z = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$
let's calculate
$$[X, [Y, Z]] \text{ and } [[X, Y], Z] :$$

$$
\begin{aligned}
[X,[Y,Z]] &= X[Y,Z]-[Y,Z]X \\
&= X(YZ-ZY)-(YZ-ZY)X \\
&= \begin{pmatrix}1 & 4\\ 2 & -1\end{pmatrix}\left(\begin{pmatrix}2 & 0\\ -1 & -2\end{pmatrix}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}-\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}2 & 0\\ -1 & -2\end{pmatrix}\right) \\
&- \left(\begin{pmatrix}2 & 0\\ -1 & -2\end{pmatrix}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}-\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}2 & 0\\ -1 & -2\end{pmatrix}\right)\begin{pmatrix}1 & 4\\ 2 & -1\end{pmatrix} \\
&= \begin{pmatrix}1 & 4\\ 2 & -1\end{pmatrix}\begin{pmatrix}1 & 4\\ 4 & -1\end{pmatrix}-\begin{pmatrix}1 & 4\\ 4 & -1\end{pmatrix}\begin{pmatrix}1 & 4\\ 2 & -1\end{pmatrix} \\
&= \begin{pmatrix}8 & 0\\ -4 & -8\end{pmatrix}.
\end{aligned}
$$

On the other hand,

$$
\begin{aligned}
[[X,Y],Z] &= [X,Y]Z-Z[X,Y] \\
&= (XY-YX)Z-Z(XY-YX) \\
&= \left(\begin{pmatrix}1 & 4\\ 2 & -1\end{pmatrix}\begin{pmatrix}2 & 0\\ -1 & -2\end{pmatrix}-\begin{pmatrix}2 & 0\\ -1 & -2\end{pmatrix}\begin{pmatrix}1 & 4\\ 2 & -1\end{pmatrix}\right)\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix} \\
&- \begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\left(\begin{pmatrix}1 & 4\\ 2 & -1\end{pmatrix}\begin{pmatrix}2 & 0\\ -1 & -2\end{pmatrix}-\begin{pmatrix}2 & 0\\ -1 & -2\end{pmatrix}\begin{pmatrix}1 & 4\\ 2 & -1\end{pmatrix}\right) \\
&= \begin{pmatrix}-4 & -16\\ 10 & 4\end{pmatrix}\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}-\begin{pmatrix}0 & 1\\ -1 & 0\end{pmatrix}\begin{pmatrix}-4 & -16\\ 10 & 4\end{pmatrix} \\
&= \begin{pmatrix}6 & -8\\ -8 & -6\end{pmatrix}.
\end{aligned}
$$

Thus it is clear that the operation is not associative.