We have several operations on the space $\mathcal{M}_n(\mathbb{C})$ of $n \times n$ matrices with complex entries:

1. Scalar multiplication of a matrix by a number
2. Addition of a matrices
3. Multiplication of matrices.

We are going to define a *new operation* on $\mathcal{M}_n(\mathbb{C})$, generally referred to as the *bracket*, and defined by

$$[X, Y] = XY - YX,$$

for matrices X, $Y \in \mathcal{M}_n(\mathbb{C})$; the notation "$XY$" indicates the matrix product of X with Y.

1. Given matrices
$$X = \begin{pmatrix} 1 & 4 \\ 2 & -1 \end{pmatrix} \text{ and } Y = \begin{pmatrix} 2 & 0 \\ -1 & -2 \end{pmatrix},$$
calculate $[X, Y]$, as well as the traces of all matrices involved.

2. Is $[\cdot, \cdot]$ commutative, i.e. does
$$[X, Y] = [Y, X] \ \forall \ X, \ Y \in \mathcal{M}_n?$$
If so, prove it. If not, find a counterexample.

3. Is $[\cdot, \cdot]$ associative, in the sense that
$$[X, [Y, Z]] = [[X, Y], Z] \ \forall \ X, \ Y, \ Z \in \mathcal{M}_n?$$
If so, prove it. If not, find a counterexample.