1. A skew-symmetric matrix Y has the property that $Y^\top = -Y$; so any 2×2 skew-symmetric matrix may be written in the form

$$Y = \begin{pmatrix} 0 & \theta \\ -\theta & 0 \end{pmatrix} = \theta \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

   Setting

$$X = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},$$

   we have $Y = \theta X$ as desired; again, *any* 2×2 skew-symmetric matrix may be written in this form.

2. In challenge problem 1, you actually calculated $\exp(\theta X)$; indeed,

$$\exp(\theta X) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}.$$

3. The bonus problem on Homework 3 was to show that *any* real 2×2 determinant 1 orthogonal matrix can be written in the form

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$$

   for some $\theta \in \mathbb{R}$. Thus the image of $\mathfrak{o}(2,\mathbb{R})$ under exp is precisely $\mathrm{SO}(2,\mathbb{R})$. The map

$$\exp : \mathfrak{o}(2,\mathbb{R}) \mapsto \mathrm{SO}(2,\mathbb{R})$$

   is onto, but definitely *not* one-to-one.