The function e^x has Taylor series expansion given by

$$\sum_{i=0}^{\infty} \frac{x^n}{n!} = 1 + x + \frac{x^2}{2!} + \dots.$$

That is, the series above converges to the number e^x for all complex numbers x.

We can use the Taylor series expansion of e^x to define the *matrix exponential*: for any $n \times n$ matrix A, set

$$\exp(A) = \sum_{i=0}^{\infty} \frac{A^n}{n!} = I + A + \frac{A^2}{2!} + \dots,$$

where we define $A^0 = I$.

It turns out that the matrix exponential converges for every $n \times n$ matrix $A \in \mathcal{M}_n(\mathbb{C})$ (due to the convergence of the exponential for complexes, combined with some facts about norms).

1. Given

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix},$$

calculate $\exp(A)$.

2. Given

$$A(\theta) = \begin{pmatrix} 0 & \theta \\ -\theta & 0 \end{pmatrix},$$

calculate $\exp(A(\theta))$.

3. Calculate the traces of each of the matrices A and $A(\theta)$ above, and calculate the determinants of $\exp(A)$ and $\exp(A(\theta))$.

4. Complete the conjecture, based on the computations you made above: Let A be a matrix so that $\mathrm{tr}\,(A) = 0$. The exponential $a = \exp(A)$ has $\det(a) =$__________.