Integration Techniques Cheat Sheet

When given an integral to evaluate with no indication as to which technique would be appropriate, it may be quite difficult to choose the proper technique. I've summarized the integration methods below; the list may help you choose an appropriate method:

- 1. Basic rules: use if the function to be integrated is a basic function, or if it can be rewritten as a basic function.
- 2. U-substitution: the function to be integrated becomes simpler when we replace some portion of the function with u.
- 3. Integration by parts: the function is a product of two or more functions, at least one of which can be integrated.
- 4. Partial fractions: the function to be integrated contains a fraction that could be rewritten in a simpler form.
- 5. Trigonometric integrals: the function to be integrated contains only trig functions. Might be helpful to rewrite using $\sin x$ and $\cos x$, and there is a good chance that you will need the identities

$$\sin^2(\alpha x) = \frac{1 - \cos 2\alpha x}{2}$$
 and $\cos^2(\alpha x) = \frac{1 + \cos 2\alpha x}{2}$.

6. Trig substitution: some factor of the function to be integrated has form $\sqrt{\pm x^2 \pm a^2}$.

There are several other things to keep in mind when evaluating integrals. First of all, rewriting or simplifying the original function is often extremely helpful, so keep in mind that this may be necessary.

Second, you may need to use multiple techniques to evaluate an integral; we have seen examples where we used both u-substitution and integration by parts. With this point in mind, note that if the technique you chose did not yield an answer, but did at least seem to simplify the problem to some extent, there is a good chance that you are going in the right direction. On the other hand, if the problem seems more complicated than it was originally, you probably need to go back to the beginning and try a different technique.