1. Every complex number has form \( a + bi \), where \( a \) and \( b \) are real numbers. In addition, every complex number other than 0 has a multiplicative inverse, i.e., another complex number \( c + di \) so that \( (a + bi)(c + di) = 1 \). Suppose that \( a \) and \( b \) are real numbers, and at least one of them is not 0. Find the real numbers \( c \) and \( d \) so that \( c + di \) is the multiplicative inverse of \( a + bi \). Hint: start by thinking of \( c + di \) as the number 

\[
    c + di = \frac{1}{a + bi}.
\]

Then use a calculus trick to rewrite this fraction with no complex numbers in the denominator.*

2. Find two distinct square roots of \( i \).

3. Matrices \( A \), \( B \) and \( C \) are given by

\[
    A = \begin{pmatrix}
    1 & 1 & -3 \\
    2 & 1 & -4
    \end{pmatrix}, \quad B = \begin{pmatrix}
    4 & 1 & 0 \\
    -1 & 3 & 1 \\
    0 & -5 & 1
    \end{pmatrix}, \quad \text{and} \quad C = \begin{pmatrix}
    0 & 0 & -1 \\
    4 & 2 & 0 \\
    -1 & 3 & 1
    \end{pmatrix}.
\]

(a) Find \( A^\top \).
(b) Calculate \( AB \).
(c) Calculate \( B - C \).
(d) Find the matrix \( 3C \).

4. Matrix \( A \) has size \( 3 \times 5 \); \( B \) and \( C \) are both \( 5 \times 7 \); and \( D \) has size \( 3 \times 3 \). If the following calculations are possible, determine the size of the resulting matrix. Otherwise, write “not defined”.

(a) \( AB \)
(b) \( BA \)
(c) \( B + C \)
(d) \( A^\top D \)

5. Suppose that \( A \) is an \( m \times n \) matrix, and that \( B \) is a matrix so that \( AB \) and \( BA \) are both defined. Prove that \( B \) has size \( n \times m \).

*The multiplicative inverse of a complex number is unique, but I am not requiring you to prove this.