1. Every complex number has form $a+bi$, where a and b are *real* numbers. In addition, every complex number other than 0 has a multiplicative inverse, i.e., another complex number $c+di$ so that $(a+bi)(c+di)=1$. Suppose that a and b are real numbers, and at least one of them is not 0. Find the real numbers c and d so that $c+di$ is the multiplicative inverse of $a+bi$. Hint: start by thinking of $c+di$ as the number

$$c+di=\frac{1}{a+bi}.$$

Then use a calculus trick to rewrite this fraction with no complex numbers in the denominator.*

2. Find two distinct square roots of i.

3. Matrices A, B and C are given by

$$A=\begin{pmatrix}1 & 1 & -3\\ 2 & 1 & -4\end{pmatrix},\ B=\begin{pmatrix}4 & 1 & 0\\ -1 & 3 & 1\\ 0 & -5 & 1\end{pmatrix},\ \text{and } C=\begin{pmatrix}0 & 0 & -1\\ 4 & 2 & 0\\ -1 & 3 & 1\end{pmatrix}.$$

   (a) Find $A^{\top}$.
   (b) Calculate AB.
   (c) Calculate $B-C$.
   (d) Find the matrix $3C$.

4. Matrix A has size 3×5; B and C are both 5×7; and D has size 3×3. If the following calculations are possible, determine the size of the resulting matrix. Otherwise, write "not defined".

   (a) AB
   (b) BA
   (c) $B+C$
   (d) $A^{\top}D$

5. Suppose that A is an $m\times n$ matrix, and that B is a matrix so that AB and BA are both defined. Prove that B has size $n\times m$.

*The multiplicitive inverse of a complex number is unique, but I am not requiring you to prove this.