1. Let $\sum_{n=1}^{\infty} a_n$ be a series with sequence of partial sums $\{s_1, s_2, s_3, \ldots\}$. Answer the following questions about the series:

   a. If $\lim_{n \to \infty} s_n = 5$, what can we say about $\sum_{n=1}^{\infty} a_n$? **Series converges to 5**

   b. If $\lim_{n \to \infty} a_n = 5$, what can we say about $\sum_{n=1}^{\infty} a_n$? **Series diverges by term test**

   c. If $\lim_{n \to \infty} s_n = 0$, what can we say about $\sum_{n=1}^{\infty} a_n$? **Series converges to 0**

   d. If $\lim_{n \to \infty} a_n = 0$, what can we say about $\sum_{n=1}^{\infty} a_n$? **Nothing**

   e. If $\sum_{n=1}^{\infty} a_n$ converges to 1, what can we say about $\lim_{n \to \infty} s_n$? **$\lim_{n \to \infty} s_n = 1$**

   f. If $\sum_{n=1}^{\infty} a_n$ converges to 1, what can we say about $\lim_{n \to \infty} a_n$? **$\lim_{n \to \infty} a_n = 0$**

2. Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be series with positive terms so that $\sum_{n=1}^{\infty} b_n$ converges to 2.

   a. If $a_n > b_n$ for all $n$, what does the direct comparison test say about $\sum_{n=1}^{\infty} a_n$? **Nothing**

   b. If $\lim_{n \to \infty} \frac{a_n}{b_n} = 3$, what does the limit comparison test say about $\sum_{n=1}^{\infty} a_n$? **Converges**

   c. Could we use the limit comparison test with the series $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ to show that $\sum_{n=1}^{\infty} a_n$ diverges? **No**

3. Which of the following series would be appropriate to test for convergence using the integral test? If the integral test is not appropriate, explain why, and suggest an appropriate convergence test.

   a. $\sum_{n=1}^{\infty} \frac{\ln n}{n}$ **Integral test**

   b. $\sum_{n=1}^{\infty} \frac{1}{n^n}$ **No, can’t integrate, Try direct comp.**

4. True or False

   a. The integral test may tell us that a series converges or diverges. **True**
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b. The $n$th term test may tell us that a series converges or diverges. \(\mathcal{F}\)

c. The alternating series test is an appropriate test to apply to \(\sum_{n=1}^{\infty} \frac{\sin n}{\cos n}\). \(\mathcal{F}\)

5. Let \(\sum_{n=1}^{\infty} a_n\) be a series with positive and negative terms.

a. Can we test \(\sum_{n=1}^{\infty} a_n\) for convergence using the limit comparison test? \(\text{No}\)

b. If \(\sum_{n=1}^{\infty} |a_n|\) converges by the limit comparison test, what can we say about \(\sum_{n=1}^{\infty} a_n\)? \(\text{Converges}\)

c. If \(\sum_{n=1}^{\infty} |a_n|\) diverges by the limit comparison test, what can we say about \(\sum_{n=1}^{\infty} a_n\)? \(\text{Nothing}\)

6. Each of the following series may be tested for convergence using one of the comparison tests.

For each series, suggest an appropriate comparison series and comparison test (limit or direct).

- \(\sum_{n=1}^{\infty} \left(\frac{2}{3}\right)^n\) \(\text{limit comp.}\) \(\sum_{n=1}^{\infty} \frac{1}{n^2}\)

- \(\sum_{n=1}^{\infty} \frac{n^2 + 2^n}{3^n}\) \(\text{limit comp.}\) \(\sum_{n=1}^{\infty} \frac{2n^2}{n^5}\)

- \(\sum_{n=1}^{\infty} \frac{n^2 + 2^n}{3^n}\) \(\text{limit comp.}\) \(\sum_{n=1}^{\infty} \frac{2^n}{n^2}\)

- \(\sum_{n=1}^{\infty} \frac{\sin n}{2^n}\) \(\text{direct comp.}\) \(\sum_{n=1}^{\infty} \frac{1}{n^2}\)

7. The series \(\sum_{n=1}^{\infty} \frac{(-1)^n}{(n^2 + 1)(n^3 + 1)}\) converges conditionally. What does this tell us about the series \(\sum_{n=1}^{\infty} \frac{1}{(n^2 + 1)(n^3 + 1)}\)? \(\text{It diverges}\)