Black Holes as Teabags in the Cosmic Cup

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Digest of the paper "Higgs Cosmology and Dark Matter" by Ian G. Moss and Ruth Gregory [arXiv:1809.00131].

Preface

The paper which is the focus of this digest links together some of the hottest topics in particle physics and cosmology: black holes, dark matter, and the Higgs boson. It is also a very theoretically-oriented (as opposed to experimentally-oriented) paper which presumes a significant amount of prior knowledge on the part of the reader. Thus, in order to understand the main point of this paper, there are a few topics that we need to review first. First, we need to review the difference between stable, unstable, and metastable equilibria. Second, we need to review the difference between first- and second-order phase transitions. Third, we need to review certain properties of the Higgs boson.

Stable, Unstable, and Metastable Equilibria

An object is said to be in mechanical equilibrium when the net force acting on it is zero. If small perturbations to the system won't lead to a drastic departure from equilibrium, the equilibrium state is said to be stable. Examples of stable equilibrium include a mass hanging at the end of a spring and a ball at the bottom of a bowl. By contrast, if such perturbations do lead to a drastic departure from equilibrium, the equilibrium state is said to be unstable. Examples of unstable equilibrium include a pencil perfectly balanced on its point on a tabletop and a ball sitting at the top of a hill. The difference between these two types of equilibrium is illustrated in Fig. 1.

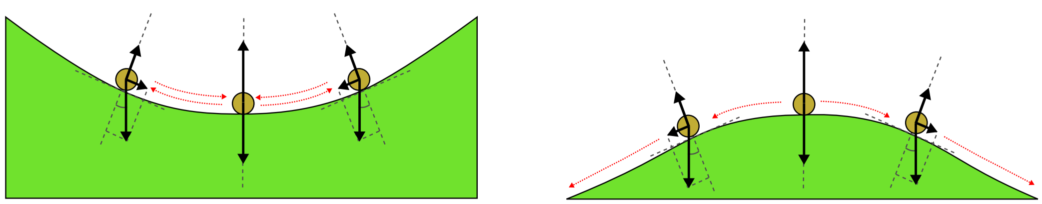

FIG. 1: An illustration of a ball in stable equilibrium (left panel) and unstable equilibrium (right panel).

It is also useful to think about equilibrium states from the viewpoint of potential energy. When the force acting on an object is a conservative force, we can define a potential-energy function $U(\vec{\mathbf{r}})$ which depends only on the position $\vec{\mathbf{r}}$ and not on the path the object took to get there. The force acting on the object at any particular position is related to the slope of the potential. In particular, for motion in one dimension we have

$$F(x) = -\frac{dU(x)}{dx} .$$

For example, if a small block of mass m is attached to the end of a spring with spring constant k, the force of the spring on the block is $F(x) = -kx$, according to Hooke's law. The potential-energy function is therefore

$$U(x) = -\int F(x)dx = \frac{1}{2}kx^2 .$$

In other words, $U(x)$ is a parabola which is concave up and has its minimum at $x = 0$. Since $dU(x)/dx = 0$ at any local minimum of $U(x)$, we know that $F(x) = -dU(x)/dx = 0$ there. The system is therefore at equilibrium when

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the box is at $x = 0$. This is a stable equilibrium point because the potential increases in both directions when the box moves away from the equilibrium point. This is equivalent to saying the if the box is displaced in either direction away from $x = 0$, the "restoring" force $F(x) = -dU(x)/dx$ from the spring pushes it back toward its equilibrium point. If no other forces act on the box, it simply oscillates back and forth around the $x = 0$ point. If dissipative forces like friction or air resistance act on it, it loses energy and settles back down into equilibrium.

Potential-energy functions are not always as simple as they are for a box on a spring, however. For example, sometimes $U(\vec{\mathbf{r}})$ for a given physical system has a number of different equilibrium points. An example of such a potential is shown in Fig. 2. This potential has a global minimum at point A, which corresponds to a stable equilibrium point. It also has a local maximum at point B, which corresponds to an unstable equilibrium point. Finally, the potential has a local minimum at point C. This is also nominally a stable equilibrium point, since there is a restoring force that would counteract small displacements of the system away from point C. However, point C is not the lowest-energy state of the system. If the system were disturbed by a not-so-small perturbation, it could be excited over the "potential barrier" at point B and drop down to the true lowest-energy state of the system. A nominally stable equilibrium point like point C, which is a local minimum but not a global minimum of $U(x)$, is called a *metastable* equilibrium point.

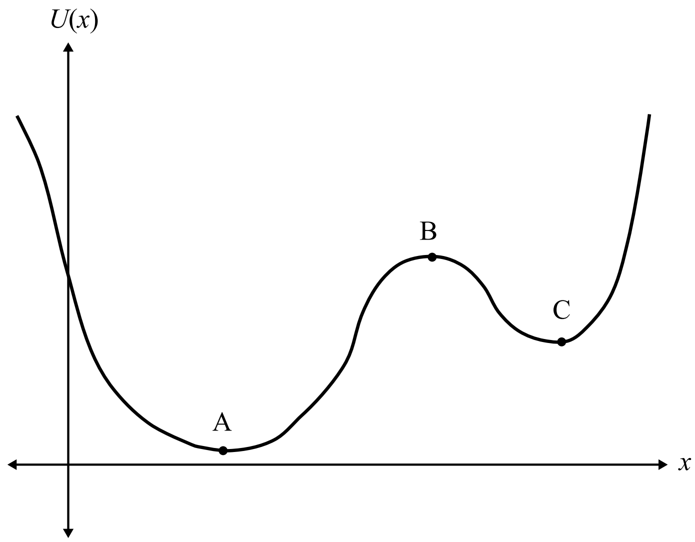

FIG. 2: A potential-energy function $U(x)$ with three equilibrium points. Point A is a stable equilibrium point which represents the global minimum of the potential. Point B is an unstable equilibrium point which represents a local maximum. Point C is a stable equilibrium point which represents a local minimum.

Since the systems in which we are interested here involve very small time, energy, and distance scales, quantum mechanics comes into play. In classical physics, in order for a system to make a transition from a metastable local minimum to a global minimum of the potential, extra energy would need to be supplied to the system in order to excite the system over the energy barrier that separates them. However, in a quantum-mechanical system, there exists a non-zero probability that the system could "tunnel" from the metastable minimum to the global minimum. Such quantum tunneling does not require any additional energy input, since the system tunnels through the barrier rather than climbing over it.

Phase Transitions

A phase transition is a transition between different states of matter. Familiar examples of phase transitions include the boiling, melting, and freezing transitions between solid ice, liquid water, and water vapor. Less familiar examples include the transition between the ferromagnetic and paramagnetic phases of magnetic materials and the emergence of superconductivity in some materials at low temperatures.

The way in which a material changes from one phase to another when it undergoes a phase transition can be classified according to what is called the *order* of the phase transition. Phase transitions are typically classified as being either first-order or second order. In a first-order phase transition, the material absorbs or releases some amount of energy per unit volume in the process as it changes phase. In such phase transitions, the entire material doesn't make the transition to the new phase all at once; rather, parts of the material make the transition to the new phase at different times. An example of such a phase transition is water boiling in a saucepan. The water doesn't immediately turn into water vapor at the instant it reaches the boiling temperature. Instead, localized bubbles of water vapor form in various places within the liquid. By contrast, in a second-order phase transition, the transition between the two phases is continuous. The entire material transitions smoothly from one phase to the other without generating localized bubbles.

When a first-order phase transition occurs in a fluid, it is often energetically favorable for bubbles, droplets, *etc.*, of the new phase to form along the surfaces of dust particles or granules embedded in the fluid, along the edges of the container in which it is kept, *etc.* These surfaces are collectively referred to as *nucleation sites*. Often, this phenomenon is crucial for getting the phase transition going in the first place. One example of this from everyday life occurs when air that is already saturated with water vapor is further cooled and becomes supersaturated. If there are dust particles in the air, water droplets tend to nucleate around these particles and form clouds. Another example of this phenomenon occurs when water is superheated in a microwave oven to a temperature slightly above its boiling point. If a teabag is dropped into the water when it is in this state, the water next to the surface of the teabag boils vigorously as a result of bubbles nucleating along its rough surface. Similar kinds of "bubbles" also form during other, less familiar kinds of first-order phase transitions, as we shall see below.

The Higgs Boson

The Higgs boson, which was discovered by the ATLAS and CMS collaborations at the LHC in 2012, is a particle with very special properties. Many of these properties stem from the potential-energy function associated with the Higgs boson. The potential-energy function that a particle experiences is often the result of its interactions with fields. One example of this is when a charged particle acquires a potential energy due to its Coulomb interactions the electric field $\vec{\mathbf{E}}$. In particle physics, all particles — including Higgs bosons — can actually be thought of as excitations of quantum fields. This means that interactions between different particles can give rise to a potential-energy function for a particle.

It turns out that the potential-energy function for the Higgs field ϕ depends primarily on the value of the Higgs field itself. Hence, the potential-energy function can be written as $U(\phi)$. If the minimum of this potential-energy function were located at $\phi = 0$, this would mean that the strength of the Higgs field is zero at equilibrium. By contrast, if the minimum of $U(\phi)$ were to be found at some other value of ϕ, the strength of the Higgs field would be non-zero at equilibrium. It turns out that in order for the Higgs boson to play the role that we know it plays in giving mass to the other particles of the Standard model, it must be the case that $\phi \neq 0$ at equilibrium. Physically, the fact that $\phi \neq 0$ when the Higgs is in its equilibrium state implies that there is a constant "background" of Higgs field everywhere in the universe even when there are no actual Higgs *particles* — *i.e.*, excitations of the Higgs field — around. As an analogy, you can think of this background value for the Higgs field as the height of the water in a calm sea above the seafloor and the particles/excitations in the Higgs field as waves or ripples on the surface of the water. Particles that interact with the Higgs field are affected by this background (commonly called a "vacuum expectation value," or "VEV" for short) in such a way that each effectively acquires a mass. This mass is proportional to both the background value of ϕ and to the strength of the coupling between that particular particle and the Higgs. This is how the Higgs boson gives mass to all of the matter particles in the Standard Model.

However, when the Higgs boson interacts with other particles, there are also feedback effects in which these particles contribute to the potential energy of the Higgs boson itself. The top quark is the heaviest matter particle in the Standard Model because it has the largest coupling to the Higgs boson. This also means that the top quark has the largest back-reaction on the Higgs potential. The value of the top mass m_t is therefore very import in determining whether the background value of ϕ we observe in our universe is associated with a global minimum of $U(\phi)$ or merely a local minimum. The mass m_h of the Higgs boson itself also plays an important role insetting the shape of the potential-energy function. If this VEV is associated with a global minimum, then it is absolutely stable. On the other hand, if the VEV is associated with a local rather than a global minimum, this VEV — and by extension the masses of all the matter particles and therefore the entire structure of the universe as we know it — is metastable. Eventually, our universe will undergo a phase transition to a different vacuum state with a different VEV for ϕ. It is a bit of an understatement to say that if and when this phase transition happens, none of us will survive it.

Right now, however, we don't know whether the potential minimum that the Higgs boson currently resides in is a global minimum or merely a local minimum. If we knew exactly what the values of all of the relevant physical parameters that determined that potential as a function of ϕ — and in particular, m_h and m_t — we would be able to say for certain. However, there is still a bit of experimental uncertainty about what m_h and m_t actually are. The contour plot in Fig. 3 shows shows the regions of (m_h, m_t)-space in which the Higgs vacuum state is stable (green region), metastable (orange region), or completely unstable (red region). The dashed blue ellipse shows the region of (m_h, m_t)-space which were consistent with data from the Tevatron at 95% confidence level. The dashed green ellipse shows the region of parameter space consistent with current LHC data at 95% confidence level. The fact that this ellipse straddles the orange and green regions indicates that we currently do not know with any real certainty whether the Higgs is currently situated in a global potential minimum or merely a local one. The small red ellipse shows a projection for how a proposed future collider called the ILC, which is currently under consideration for construction in Japan, could further narrow down the uncertainties in m_h and m_t, thereby potentially teaching us whether the Higgs boson currently resides in a stable minimum of its potential or merely in a metastable one. It's important to note that the actual location of this small ellipse within the currently allowed region of (m_h, m_t)-space is unknown, and that its location on the contour plot in Fig. 3 simply represents one possible scenario.

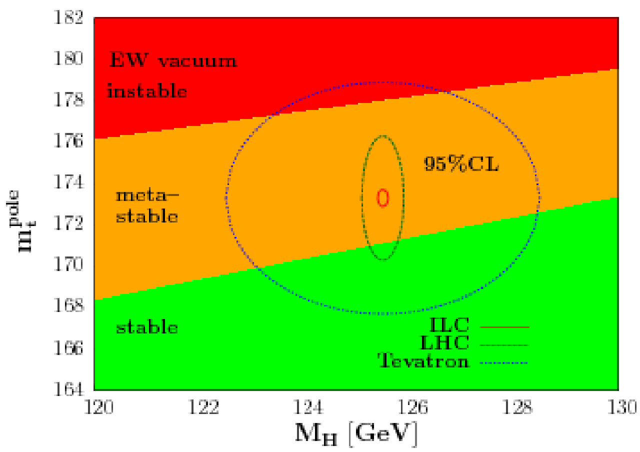

FIG. 3: Contour plot taken from Ref. [2] showing the regions of (m_h, m_t)-space in which the Higgs vacuum state is stable (green region), metastable (orange region), or completely unstable (red region). The ellipses show the 95% confidence-level uncertainty contours associated with different particle colliders past, present, and future in measuring the values of m_t and m_h.

Black Holes as Nucleation Sites

Okay . . . so let's

If our universe is in a metastable state, with the Higgs boson sitting at a local minimum of the potential which is not the global minimum, it eventually could undergo a phase transition to the vacuum state associated with that global minimum. There is energy released in the process because the global minimum has a lower value of $U(\phi)$ than the metastable local minimum, so this phase transition is first order. This means that during such a phase transition, "bubbles" or regions of space in which the Higgs boson is in the lower-energy vacuum state need to nucleate — just like they do in superheated water when a teabag is lowered into it.

So what plays the role of the teabag here? Might there be any bodies out there in the early universe which can serve as nucleation sites at which "bubbles" of lower-energy Higgs phase can form more readily? Indeed there are: black holes. Black holes that are formed during the earliest stages of the development of the universe — typically called "primordial" black holes — can serve as nucleation sites for the lower-energy Higgs phase. The severe warping of spacetime that occurs in the vicinity of a black hole turns out to lower the energy barrier for the formation of bubbles of the lower-energy Higgs phase in much the same way as dust particles serve to lower the energy barrier for the formation of water droplets in supersaturated air. This nucleation process is illustrated in Fig.

As a result, the nucleation rate for bubbles of the ground-state Higgs phase increases in the presence of black holes.

Primordial black holes don't last forever, however. They constantly lose mass and energy by radiating away familiar particles through a process known as Hawking radiation. As a result of this emission, small black holes can simply evaporate away. The question, then, is whether a sufficient number of primordial black holes would stick around long enough to nucleate bubbles of the Higgs vacuum, thereby precipitating the catastrophic phase transition from the metastable vacuum state to the stable vacuum state.

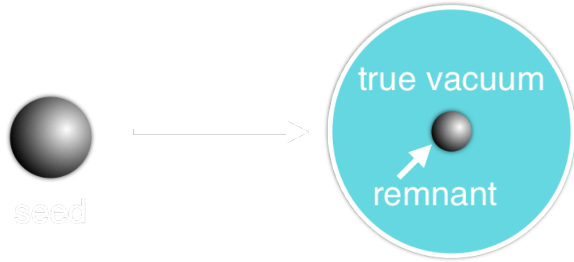

FIG. 4: Cartoon from Ref. [1] showing how a primordial black hole can serve as a nucleation site for a phase transition in which the Higgs field transitions from a metastable state to a stable, lower-energy state.

The authors of Ref. [1] find that for small black holes with masses in the range 10^{-4} kg $\lesssim M_{\rm BH} \lesssim 1$ kg, the nucleation rate for bubbles of the lower-energy Higgs phase always exceeds the rate at which these black holes evaporate. In other words, such small black holes are dangerous: they last long enough to serve as nucleation sites for "bubbles" of the stable Higgs vacuum. Moreover, even heavier black holes which start out with masses above this dangerous range will decrease in mass due to the emission Hawking radiation and eventually enter that range. The authors find that only black holes with masses $M_{\rm BH} \gtrsim 10^{12}$ kg are sufficiently heavy as not to pose a risk to the metastable Higgs vacuum in this way.

This has an important consequence for dark-matter physics. Primordial black holes might make up some fraction of the dark matter in our universe. If the Higgs vacuum really is metastable, the catastrophic effects of small primordial black holes on that vacuum would imply that such black holes were not generated in any significant number in the early universe. Thus, the authors argue, only black holes with masses in the range $M_{\rm BH} \gtrsim 10^{12}$ kg can possibly contribute a non-negligible fraction of the dark matter today. The authors also go on to discuss some additional gravitational effects that could occur in certain models of the very early universe that can also destabilize the metastable minimum in the potential-energy function for the Higgs field.

[1] I. G. Moss and R. Gregory, arXiv:1809.00131 [hep-ph].
[2] S. Alekhin, A. Djouadi and S. Moch, Phys. Lett. B **716**, 214 (2012) doi:10.1016/j.physletb.2012.08.024 [arXiv:1207.0980 [hep-ph]].