KR 2018, Tempe, AZ, USA

Knowledge, Strategies, and Know-How

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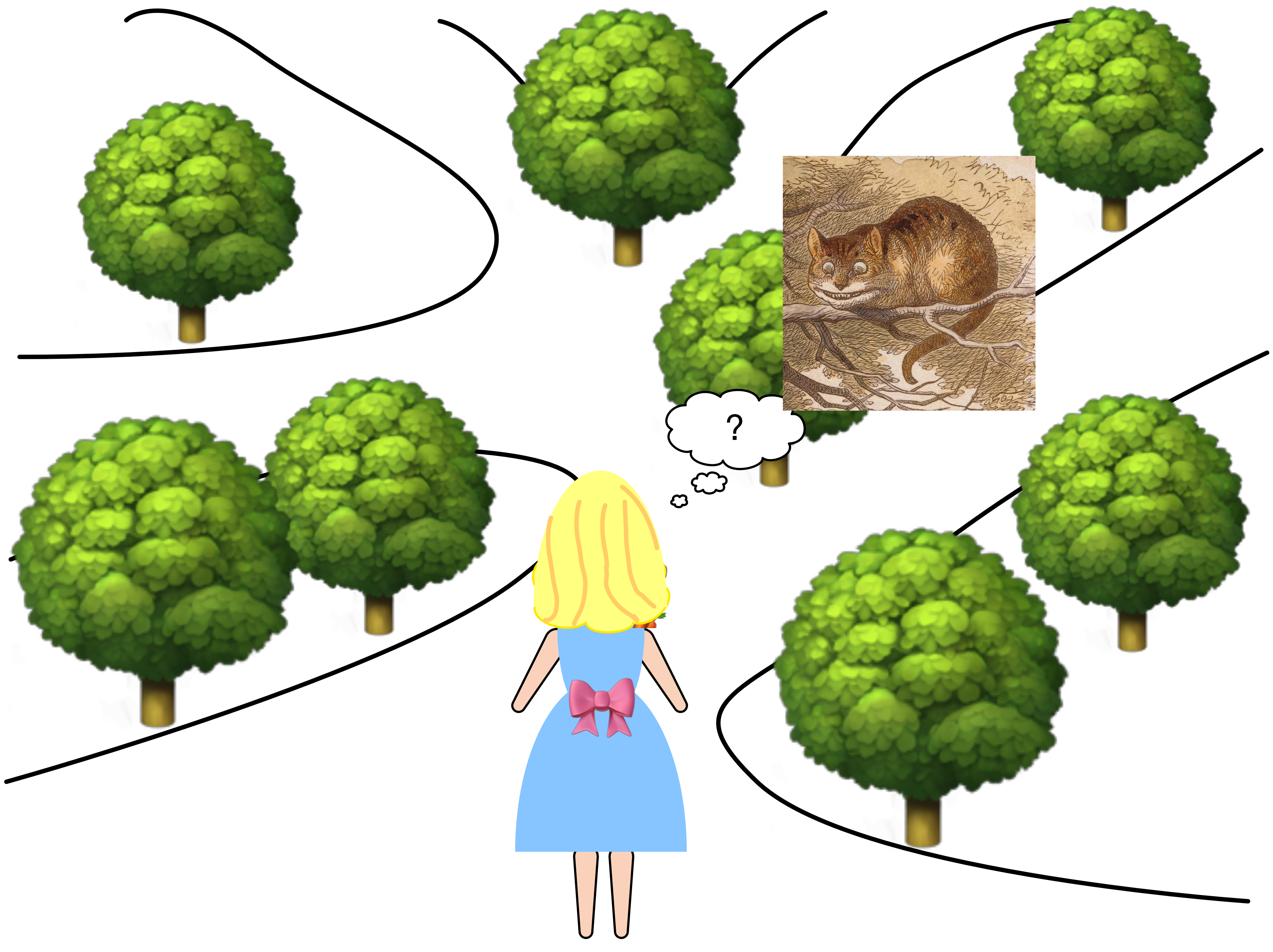
http://reasoning.eas.asu.edu/kr2018/

Knowledge, Strategies, Know-How



Epistemic Logic

Alice

K_A(Alice has a pumpkin mask)

Bob

$\neg K_B$(Alice has a pumpkin mask)

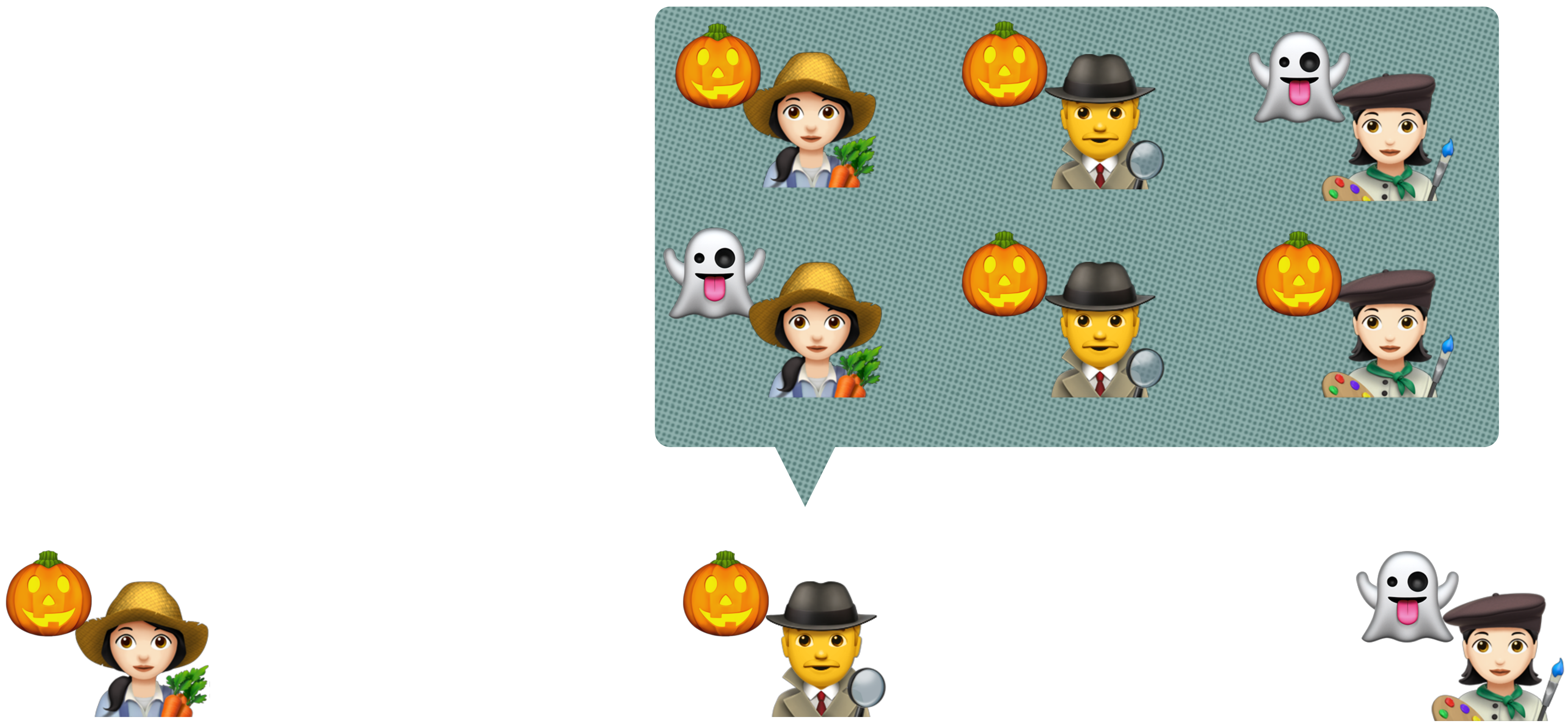
Cathy

K_C(Alice has a pumpkin mask)

$K_C \neg K_B$(Alice has a pumpkin mask)

$K_A(K_B$(Alice has a pumpkin mask) $\vee$ K_C(Alice has a pumpkin mask))

Epistemic Logic



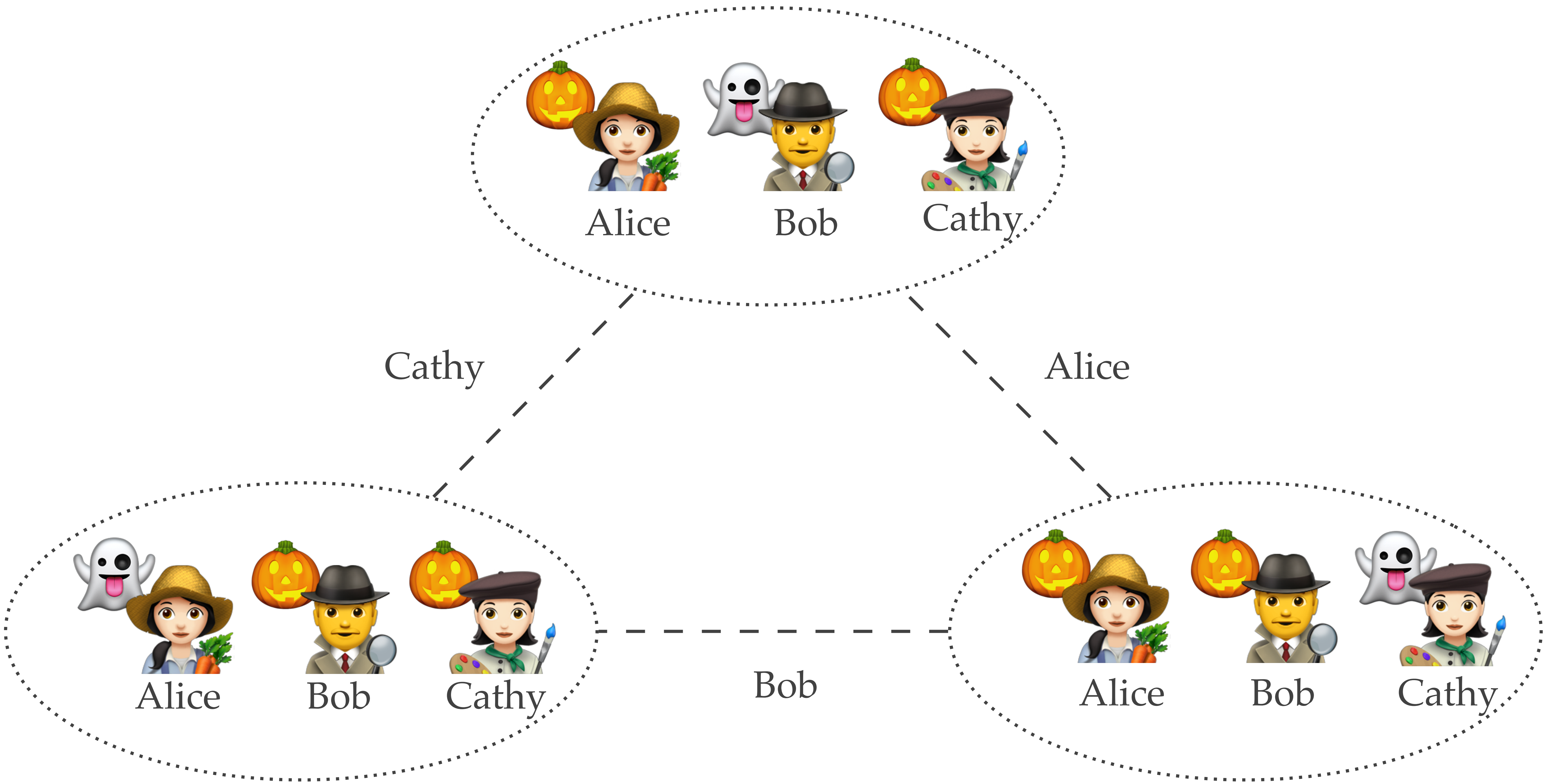
Alice Bob Cathy

$\neg K_B$(Alice has a pumpkin mask)

Epistemic Logic

$K_A(K_B(\text{Alice has a pumpkin mask}) \vee K_C(\text{Alice has a pumpkin mask}))$

Epistemic States



Epistemic Model

$(W, \{ \sim_a \}_{a \in \mathscr{A}}, \pi)$

W is a set of states

$\sim_a$ is an indistinguishability equivalence relation

π is a valuation function

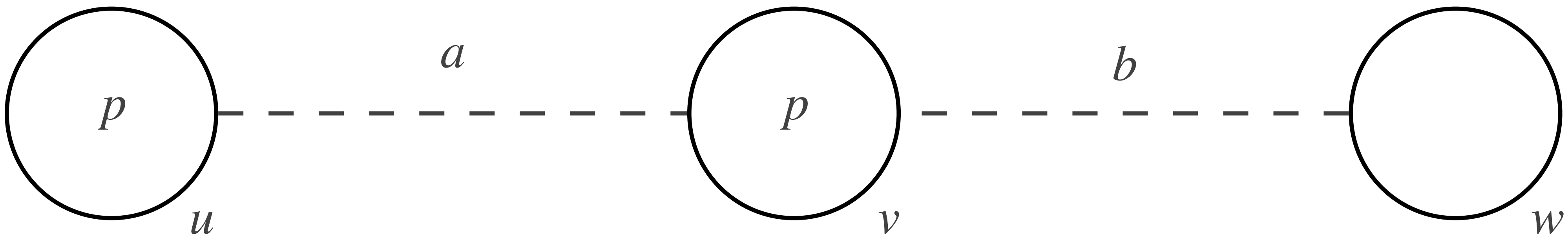
$w \Vdash p$ iff $w \in \pi(p)$

$w \Vdash \neg\varphi$ iff $w \nVdash \varphi$

$w \Vdash \varphi \rightarrow \psi$ iff $w \nVdash \varphi$ or $w \Vdash \psi$

$w \Vdash \mathsf{K}_a\varphi$ iff $u \Vdash \varphi$ for all $u \in W$ such that $w \sim_a u$

Model Example



$u \Vdash p$

$u \Vdash \mathsf{K}_a p$

$v \Vdash \mathsf{K}_a p$

$v \nVdash \mathsf{K}_b p$

$w \Vdash \neg p$

$w \Vdash \mathsf{K}_a \neg p$

$w \Vdash \neg \mathsf{K}_b \neg p$

$w \Vdash \mathsf{K}_b(\mathsf{K}_a \neg p \vee \mathsf{K}_a p)$

Indistinguishability Relation

- Reflexive $u \sim_a u$
- Symmetric $u \sim_a v \to v \sim_a u$
- Transitive $u \sim_a v \wedge v \sim_a w \to u \sim_a w$

Multiagent S5

all propositional tautologies

$\mathsf{K}_a\varphi \to \varphi$

~~$\mathsf{K}_a\varphi \to \mathsf{K}_a\mathsf{K}_a\varphi$~~

$\neg\mathsf{K}_a\varphi \to \mathsf{K}_a\neg\mathsf{K}_a\varphi$

$\mathsf{K}_a(\varphi \to \psi) \to (\mathsf{K}_a\varphi \to \mathsf{K}_a\psi)$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{K}_a\varphi}$$

Proof of Positive Introspection

Lemma 1

$\vdash K_a\varphi \to K_aK_a\varphi$

Proof

$\vdash K_a\neg K_a\varphi \to \neg K_a\varphi$

$\vdash K_a\varphi \to \neg K_a\neg K_a\varphi$

$\vdash \neg K_a\neg K_a\varphi \to K_a\neg K_a\neg K_a\varphi$

$\vdash K_a\varphi \to K_a\neg K_a\neg K_a\varphi$

$\vdash \neg K_a\varphi \to K_a\neg K_a\varphi$

$\vdash \neg K_a\neg K_a\varphi \to K_a\varphi$

$\vdash K_a(\neg K_a\neg K_a\varphi \to K_a\varphi)$

$\vdash K_a\neg K_a\neg K_a\varphi \to K_aK_a\varphi$

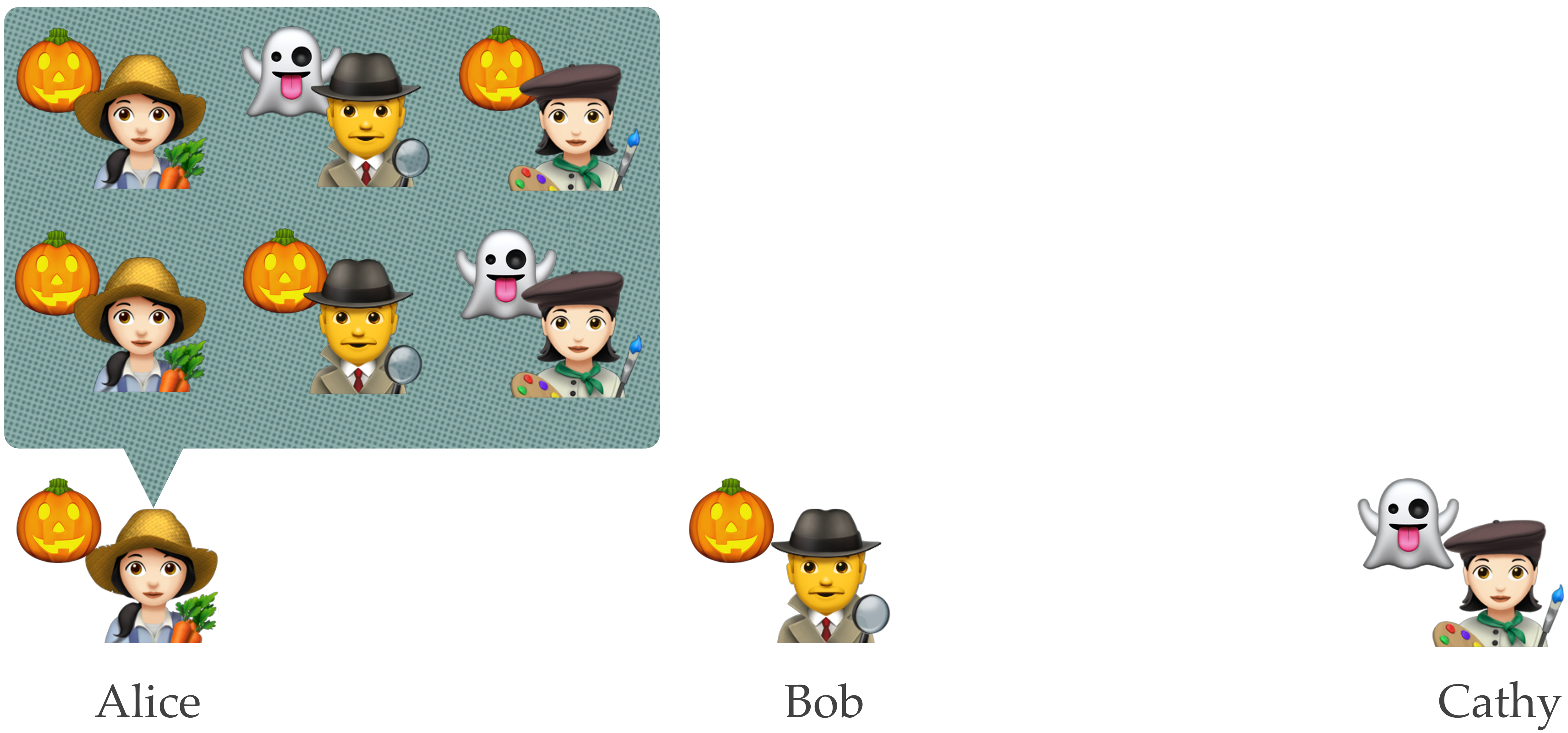
$\vdash K_a\varphi \to K_aK_a\varphi$

Soundness and Completeness Theorems

If $\vdash \varphi$, then $w \Vdash \varphi$ for each state w of each epistemic model.

If $w \Vdash \varphi$ for each state w of each epistemic model, then $\vdash \varphi$.

Alice's Knowledge



K_A("Either Bob or Cathy has a ghost mask.")

$\neg K_A$("Cathy has a ghost mask.")

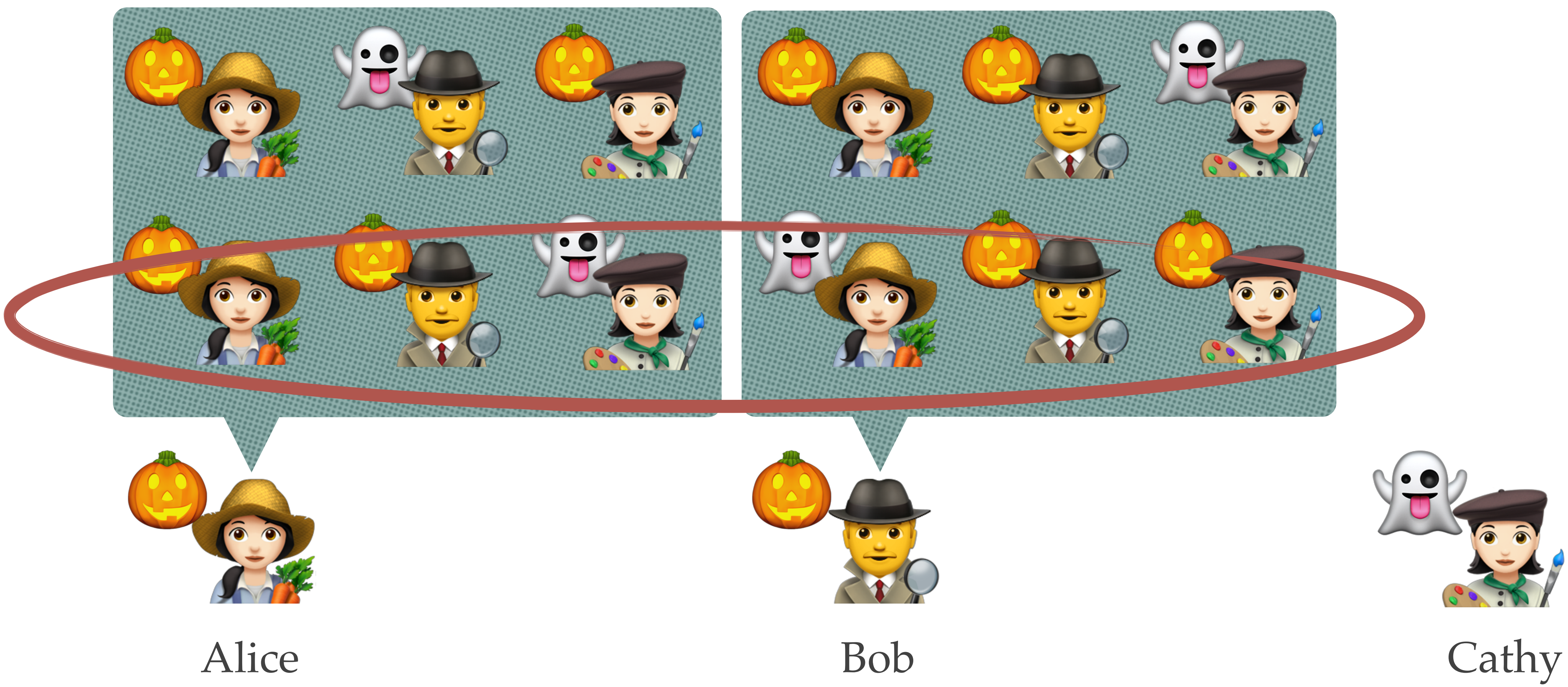
Bob's Knowledge

Alice Bob Cathy

K_B("Either Alice or Cathy has a ghost mask.")

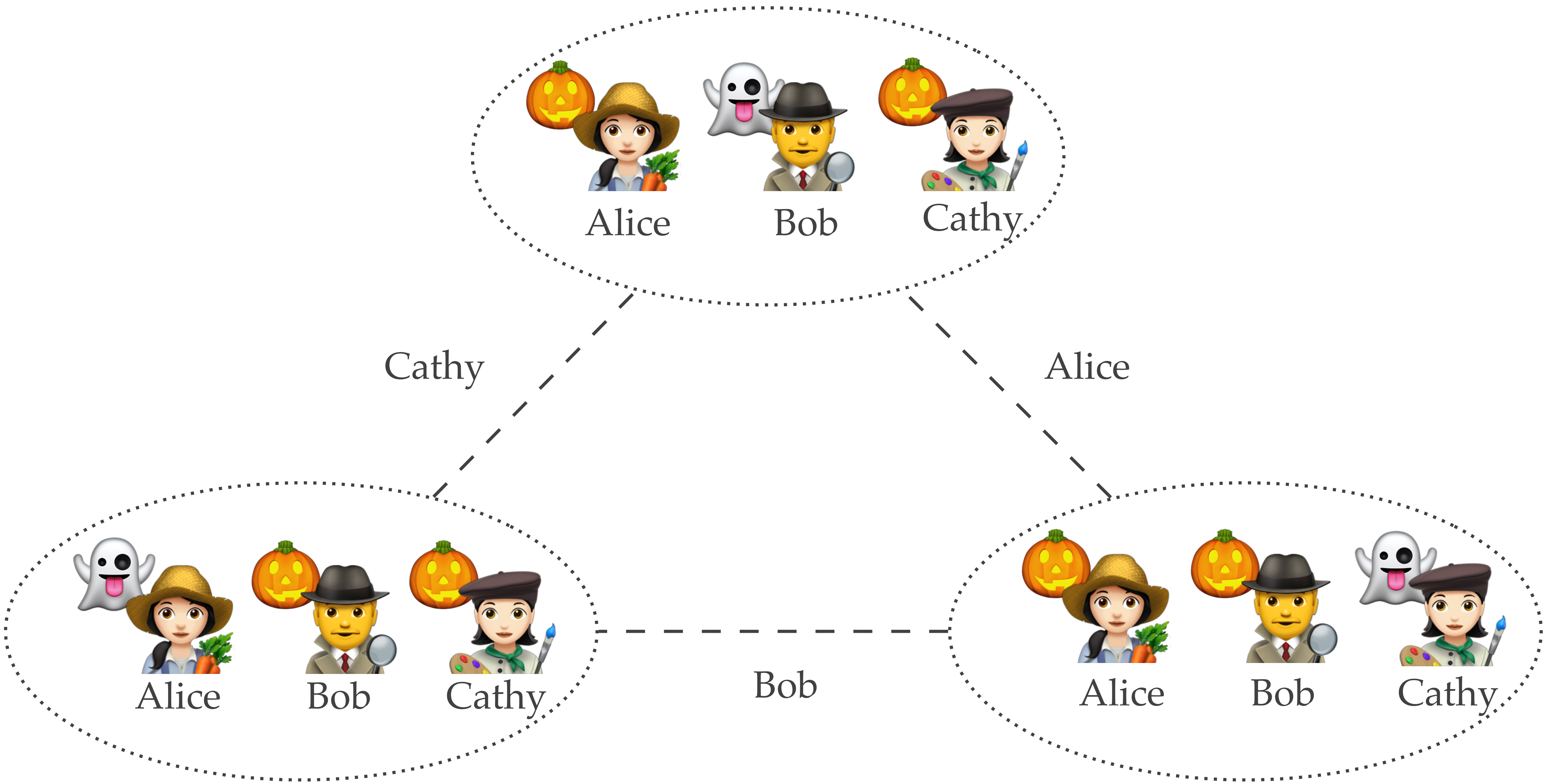
$\neg K_B$("Cathy has a ghost mask.")

Distributed Knowledge



$K_{A,B}$("Cathy has a ghost mask.")

Epistemic States



Epistemic Model

$(W, \{ \sim_a \}_{a \in \mathscr{A}}, \pi)$

W is a set of states

$\sim_a$ is an indistinguishability equivalence relation

π is a valuation function

Notation: $w \sim_C u$ if $\forall a \in C (w \sim_a u)$.

$w \Vdash p$ iff $w \in \pi(p)$

$w \Vdash \neg\varphi$ iff $w \nVdash \varphi$

$w \Vdash \varphi \to \psi$ iff $w \nVdash \varphi$ or $w \Vdash \psi$

$w \Vdash \mathsf{K}_C\varphi$ iff $u \Vdash \varphi$ for all $u \in W$ such that $w \sim_C u$

Distributed Knowledge Axioms

all propositional tautologies

$$\mathsf{K}_C\varphi \to \varphi$$

$$\neg\mathsf{K}_C\varphi \to \mathsf{K}_C\neg\mathsf{K}_C\varphi$$

$$\mathsf{K}_C(\varphi \to \psi) \to (\mathsf{K}_C\varphi \to \mathsf{K}_C\psi)$$

$$\mathsf{K}_C\varphi \to \mathsf{K}_D\varphi, \text{ where } C \subseteq D$$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{K}_C\varphi}$$

Soundness and Completeness Theorems

If $\vdash \varphi$, then $w \Vdash \varphi$ for each state w of each epistemic model.

If $w \Vdash \varphi$ for each state w of each epistemic model, then $\vdash \varphi$.

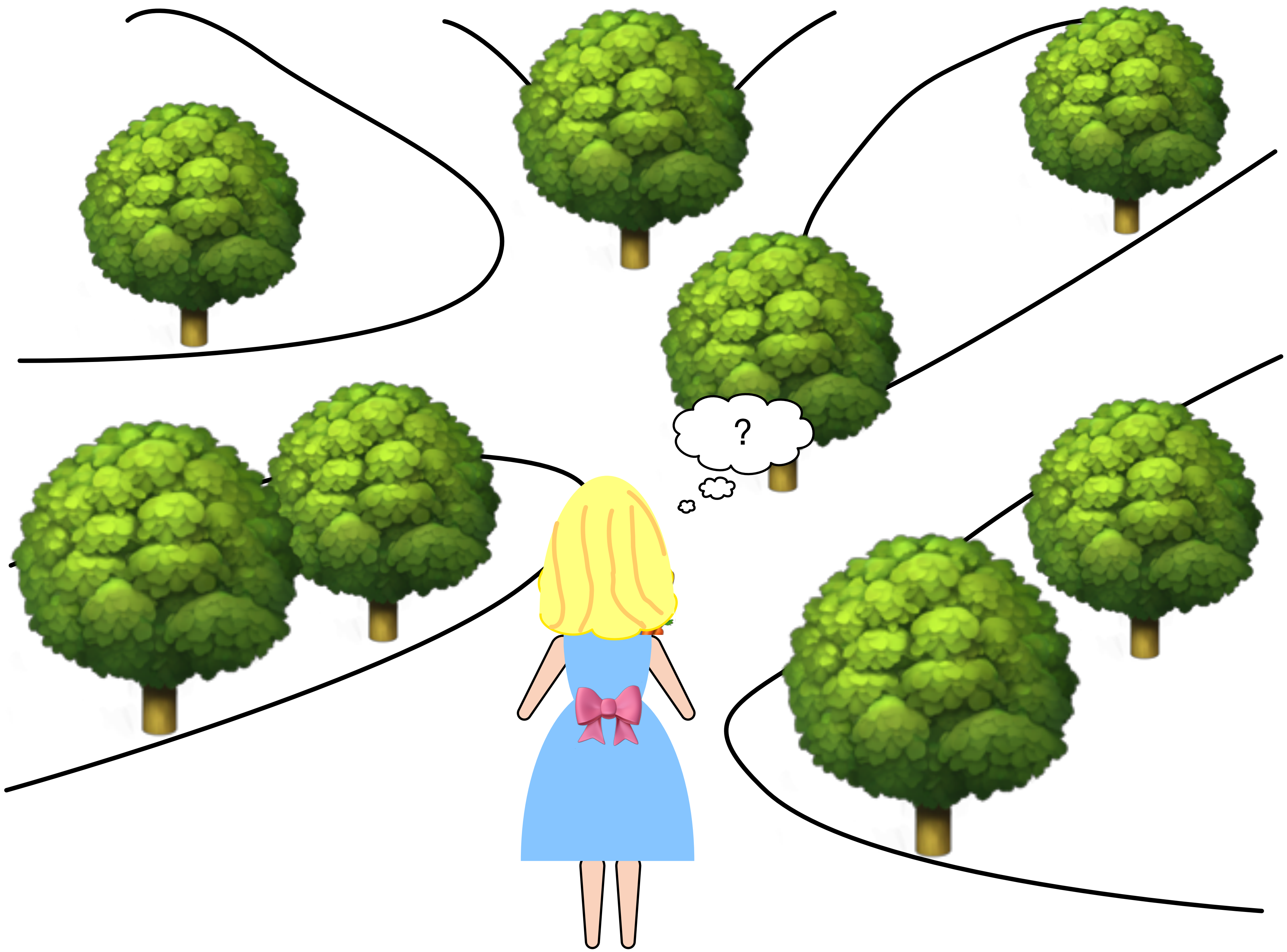
Group Knowledge

Forms of Group Knowledge

- Individual Knowledge
  - Alice and Bob both know the value of x
- Distributed Knowledge
  - Alice knows x, Bob knows y, they distributedly know the value of x+y
- Common Knowledge
  - two generals problem

Logic of Coalition Power

Strategies



Coalition Strategies

0/1

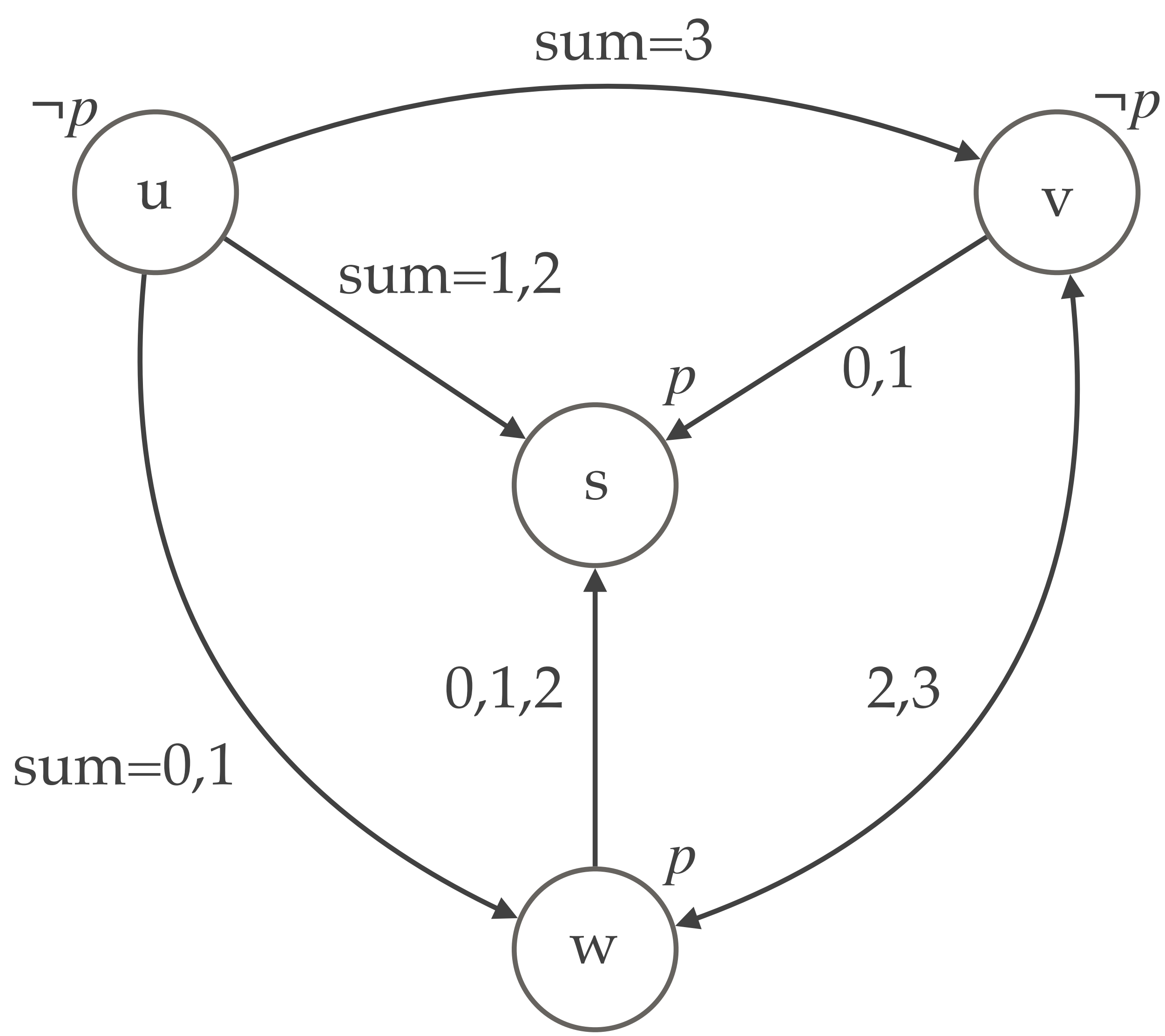
Alice

0/1

Bob

0/1

Cathy



$u \Vdash \mathsf{S}_{abc}\mathsf{S}_{\varnothing}p$

$u \Vdash \mathsf{S}_a p$

$u \Vdash \neg\mathsf{S}_a \neg p$

$u \Vdash \mathsf{S}_{abc} \neg p$

$v \Vdash \neg\mathsf{S}_{abc} \neg p$

$v \Vdash \mathsf{S}_{\varnothing} p$

$w \Vdash \mathsf{S}_{abc} \neg p$

$w \Vdash \mathsf{S}_{ab} p$

$s \Vdash \mathsf{S}_a \neg p$

$s \Vdash \mathsf{S}_a \bot$

Game Definition

(W, Δ, M, π)

W is a set of states.

Δ is a nonempty domain of actions.

$M \subseteq W \times \Delta^{\mathscr{A}} \times W$ is a mechanism.

π is a function from propositional variables into subsets of W.

Formal Semantics

$w \Vdash p$ iff $w \in \pi(p)$,

$w \Vdash \neg\varphi$ iff $w \nVdash \varphi$,

$w \Vdash \varphi \to \psi$ iff $w \nVdash \varphi$ or $w \Vdash \psi$,

$w \Vdash \mathsf{S}_C\varphi$ iff there is an action profile $s \in \Delta^C$ of coalition C

such that for each complete action profile $\delta \in \Delta^{\mathscr{A}}$ and each state $u \in W$

if $s =_C \delta$ and $(w, \delta, u) \in M$, then $u \Vdash \varphi$.

Examples of Statements

$\mathsf{S}_a p$ = Alice has an action that guarantees p in the next state.

$\mathsf{S}_\emptyset p$ = p is unavoidable in the next state.

$\mathsf{S}_\emptyset \bot$ = there is no next state.

$\mathsf{S}_a \neg \mathsf{S}_b p$ = Alice has an action after which Bob will not have an action that guarantees p

$\neg(\mathsf{S}_a p \lor \mathsf{S}_b p) \land \mathsf{S}_{ab} p$ = Although neither Alice nor Bob has an action that guarantees p they have a joint action that does .

Marc Pauly's Logic of Coalitional Power

all propositional tautologies

$$\mathsf{S}_C(\varphi \to \psi) \to (\mathsf{S}_D\varphi \to \mathsf{S}_{C\cup D}\psi), \text{ where } C \cap D = \emptyset$$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad\qquad \frac{\varphi}{\mathsf{S}_C\varphi}$$

M. Pauly, A modal logic for coalitional power in games, Journal of Logic and Computation (2002)

Example of Derivation

Lemma 1. $\vdash \mathsf{S}_C\varphi \to \mathsf{S}_D\varphi$, where $C \subseteq D$.

Proof.

$\vdash \varphi \to \varphi$

$\vdash S_{D\setminus C}(\varphi \to \varphi)$

Since $C \subseteq D$, by the Cooperation axiom,

$\vdash S_{D\setminus C}(\varphi \to \varphi) \to (S_C\varphi \to S_D\varphi)$

$\vdash S_C\varphi \to S_D\varphi$

Soundness and Completeness Theorems

If $\vdash \varphi$, then $w \Vdash \varphi$ for each state w of each game.

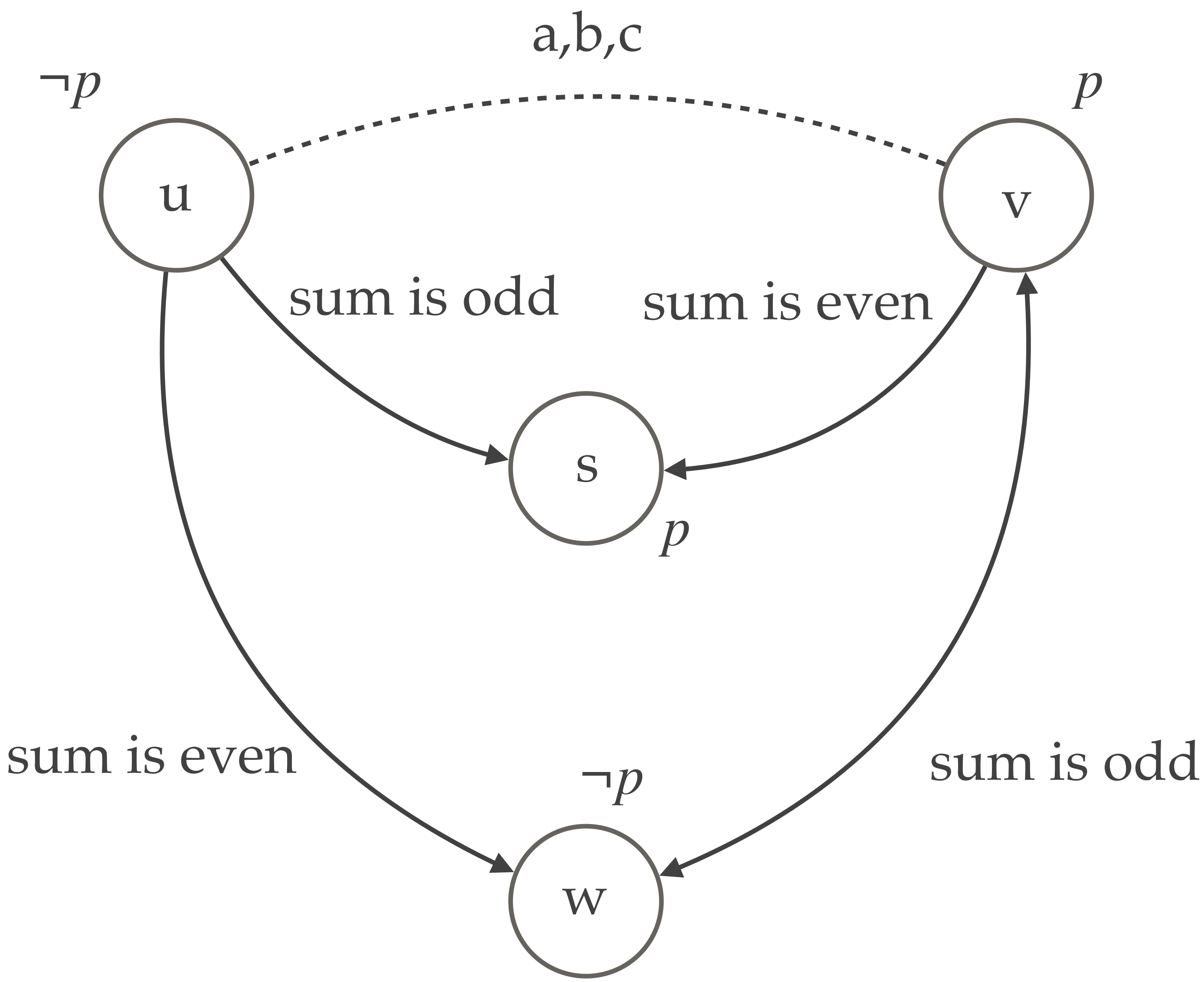
If $w \Vdash \varphi$ for each state w of each game, then $\vdash \varphi$.

Coalition Strategies

Alice 0/1

Bob 0/1

Cathy 0/1



$u \Vdash \mathsf{S}_{abc} p$?

Knowledge and Strategies

Game with Imperfect Information

0/1

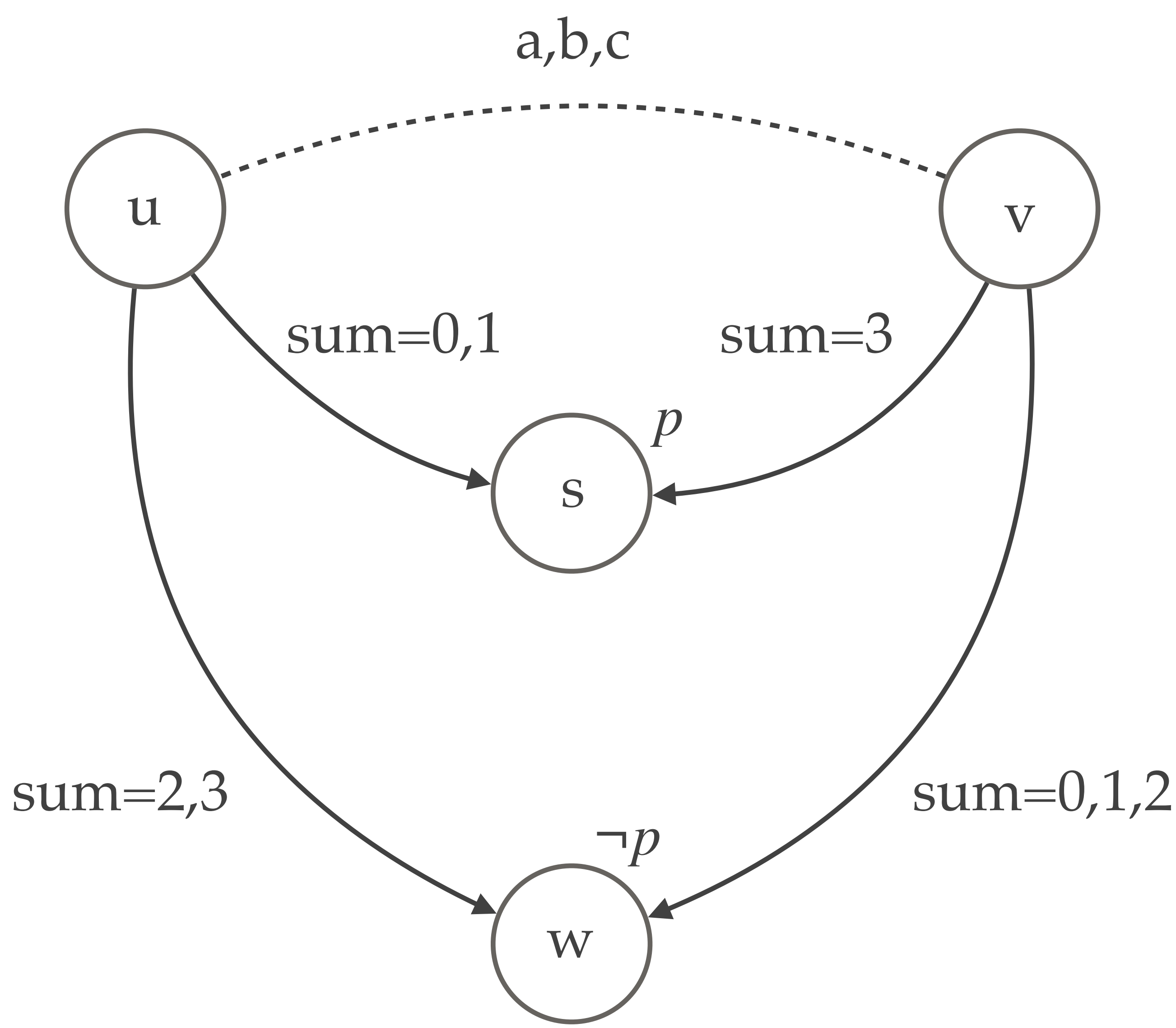
Alice

0/1

Bob

0/1

Cathy



$u \Vdash \neg \mathsf{S}_a p$

$u \Vdash \mathsf{S}_{a,b} p$

$v \Vdash \neg \mathsf{S}_{a,b} p$

$u \Vdash \neg \mathsf{K}_{a,b} \mathsf{S}_{a,b} p$

$u \Vdash \mathsf{S}_{a,b,c} p$

$v \Vdash \mathsf{S}_{a,b,c} p$

$u \Vdash \mathsf{K}_{a,b,c} \mathsf{S}_{a,b,c} p$

Game with Imperfect Information

$(W, \{ \sim_a \}_{a \in \mathscr{A}}, \Delta, M, \pi)$

W is a set of states.

$\sim_a$ is an indistinguishability equivalence relation.

Δ is a nonempty domain of actions.

$M \subseteq W \times \Delta^{\mathscr{A}} \times W$ is a mechanism.

π is a function from propositional variables into subsets of W.

Thomas Ågotnes and Natasha Alechina (2012). "Epistemic Coalition Logic: Completeness and Complexity." International Conference on Autonomous Agents and Multiagent Systems

Formal Semantics

$w \Vdash p$ iff $w \in \pi(p)$,

$w \Vdash \neg\varphi$ iff $w \nVdash \varphi$,

$w \Vdash \varphi \to \psi$ iff $w \nVdash \varphi$ or $w \Vdash \psi$,

$w \Vdash \mathsf{S}_C\varphi$ iff there is an action profile $s \in \Delta^C$ of coalition C

such that for each complete action profile $\delta \in \Delta^{\mathscr{A}}$ and each state $u \in W$

if $s =_C \delta$ and $(w, \delta, u) \in M$, then $u \Vdash \varphi$,

$w \Vdash \mathsf{K}_C\varphi$ iff $u \Vdash \varphi$ for all $u \in W$ such that $w \sim_C u$.

Distributed Knowledge and Strategies

all propositional tautologies

$\mathsf{K}_C\varphi \to \varphi$

$\neg\mathsf{K}_C\varphi \to \mathsf{K}_C\neg\mathsf{K}_C\varphi$

$\mathsf{K}_C(\varphi \to \psi) \to (\mathsf{K}_C\varphi \to \mathsf{K}_C\psi)$

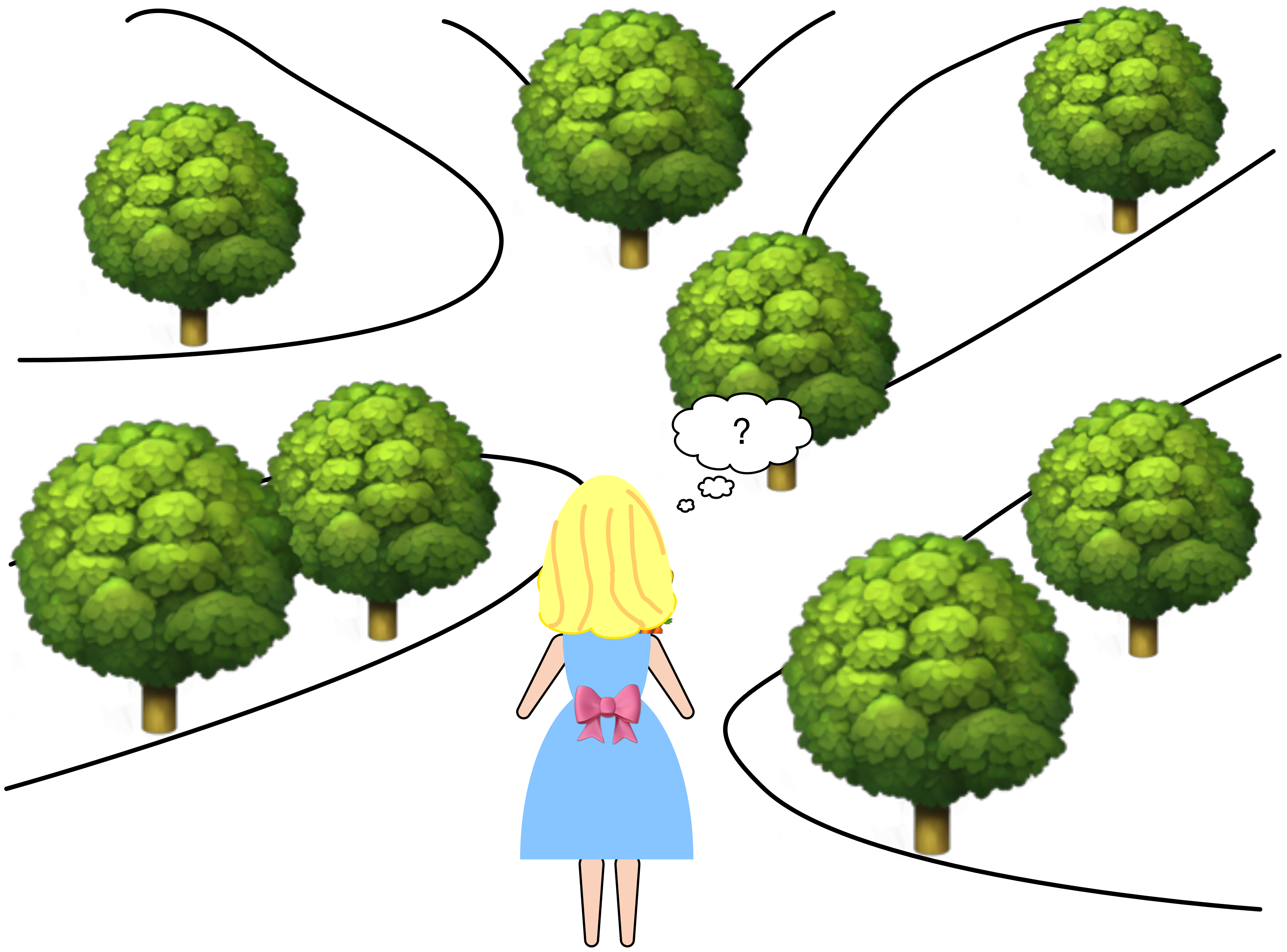
$\mathsf{K}_C\varphi \to \mathsf{K}_D\varphi$, where $C \subseteq D$

$\mathsf{S}_C(\varphi \to \psi) \to (\mathsf{S}_D\varphi \to \mathsf{S}_{C\cup D}\psi)$, where $C \cap D = \emptyset$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{K}_C\varphi} \qquad \frac{\varphi}{\mathsf{S}_C\varphi}$$

Completeness: Thomas Ågotnes and Natasha Alechina (2012). "Epistemic Coalition Logic: Completeness and Complexity." International Conference on Autonomous Agents and Multiagent Systems

Know-How



Know-How

0/1

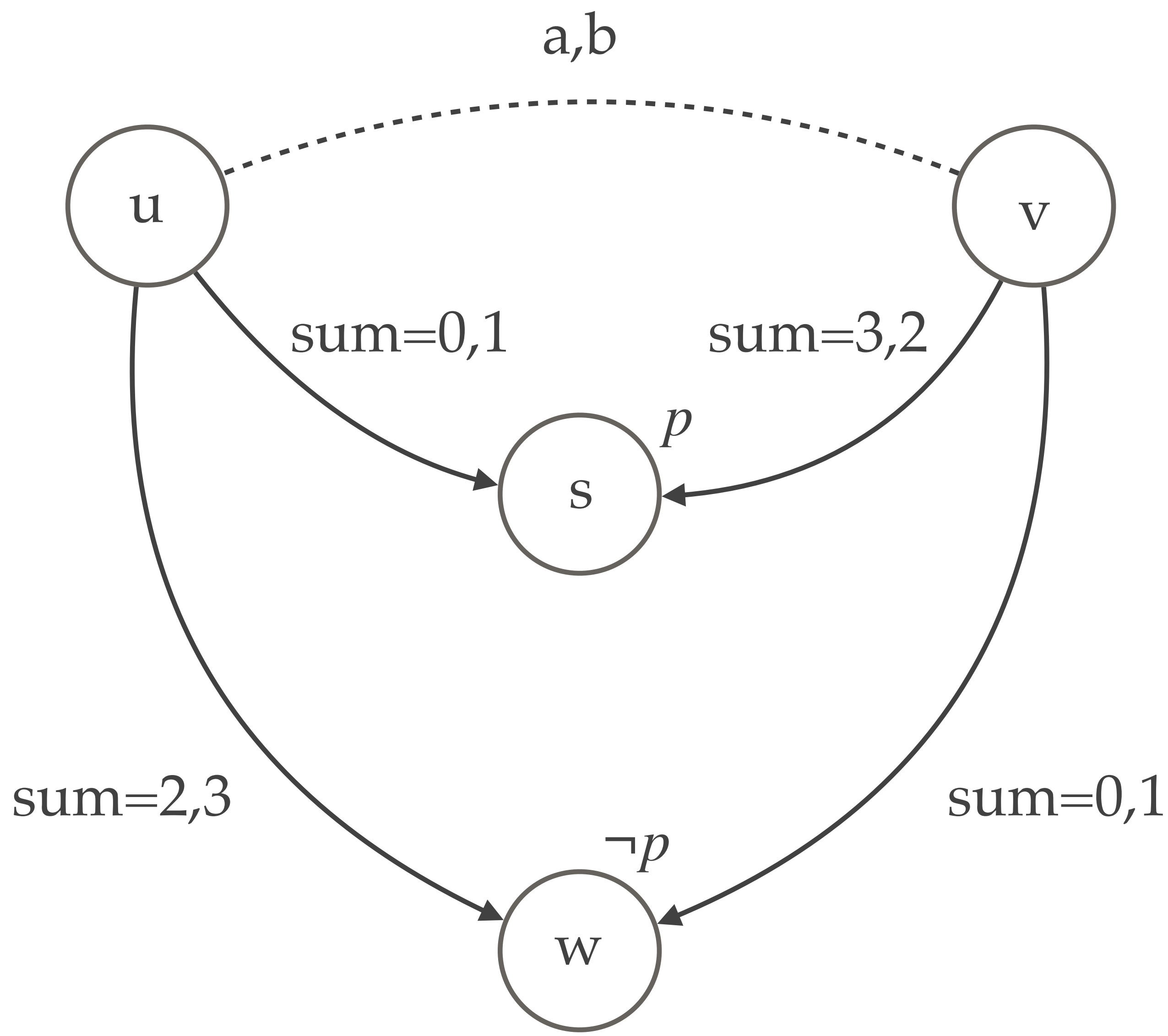
Alice

0/1

Bob

0/1

Cathy



$u \Vdash \mathsf{S}_{a,b}p$

$v \Vdash \mathsf{S}_{a,b}p$

$u \Vdash \mathsf{K}_{a,b}\mathsf{S}_{a,b}p$

$u \Vdash \neg\mathsf{H}_{a,b}p$

$u \Vdash \mathsf{H}_{a,b,c}p$

Another Example

0/1

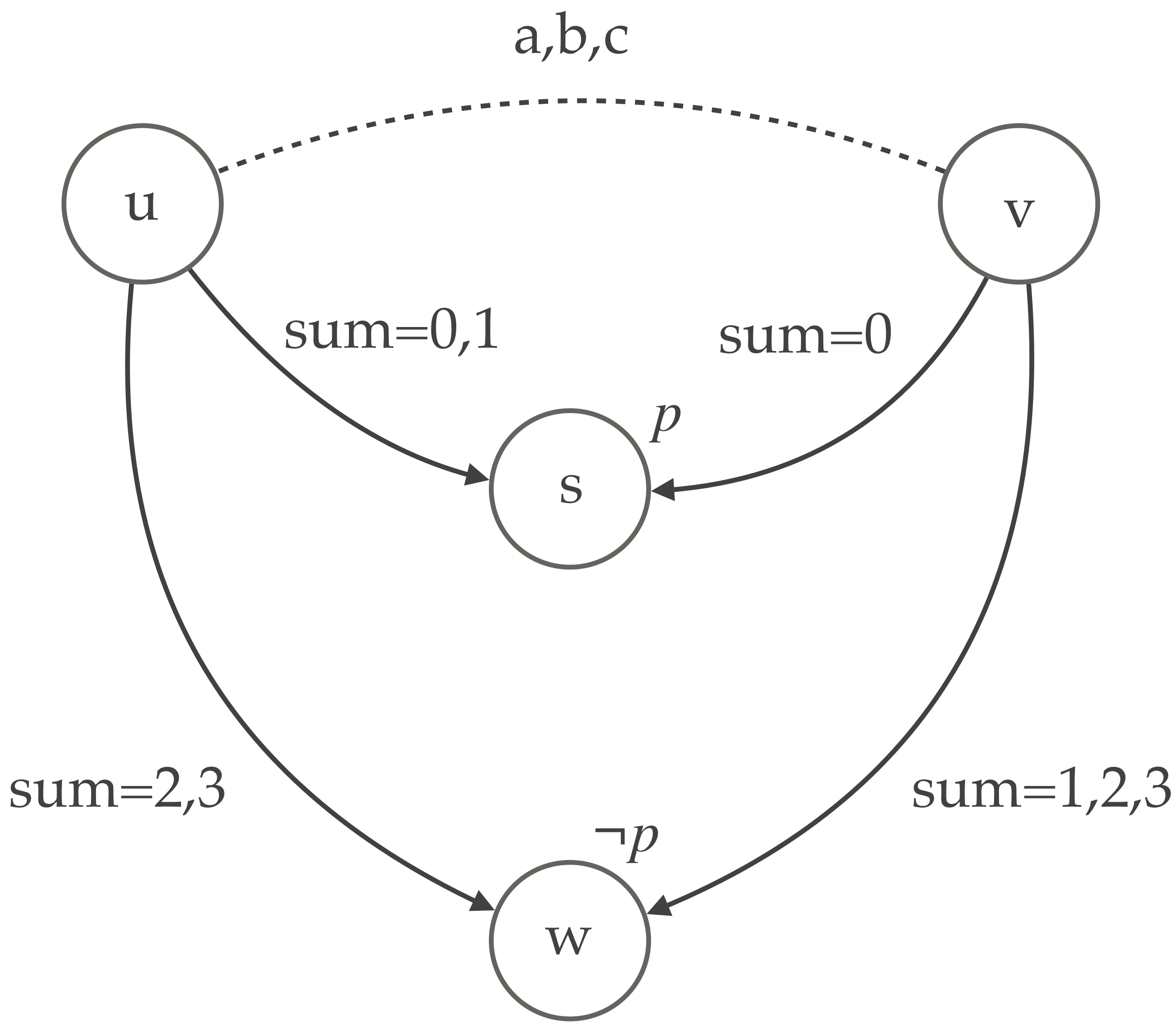
Alice

0/1

Bob

0/1

Cathy



$u \Vdash \mathsf{S}_{a,b}p$

$v \Vdash \neg\mathsf{S}_{a,b}p$

$u \Vdash \neg\mathsf{H}_{a,b}p$

$u \Vdash \mathsf{H}_{a,b,c}p$

Formal Semantics

$w \Vdash \mathsf{S}_C\varphi$ iff there is an action profile $s \in \Delta^C$ of coalition C such that

for each complete action profile $\delta \in \Delta^{\mathscr{A}}$

and each state $u \in W$,

if $s =_C \delta$ and $(w, \delta, u) \in M$, then $u \Vdash \varphi$

$w \Vdash \mathsf{H}_C\varphi$ iff there is an action profile $s \in \Delta^C$ of coalition C such that

for each state v and each complete action profile $\delta \in \Delta^{\mathscr{A}}$

and each state $u \in W$,

if $w \sim_C v$, $s =_C \delta$, and $(v, \delta, u) \in M$, then $u \Vdash \varphi$

Term “Know-How” Strategies

- “uniform” strategies - van Benthem (2001)
- “difference between an agent knowing that he has a suitable strategy and knowing the strategy itself” - Jamroga and van der Hoek (2004)
- “knowledge to identify and execute a strategy” - Jamroga and Ågotnes (2007)
- “knowingly doing” Broersen (2008)
- “knowing how” - Wang (2015)
- “knows how” or “knowledge *de re*” - Ågotnes and Alechina (2016)
- “executable”or “know-how” strategy - Naumov and Tao (2017)

Knowledge and Know-How

all propositional tautologies

$\mathsf{K}_C\varphi \to \varphi$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{K}_C\varphi} \qquad \frac{\varphi}{\mathsf{H}_C\varphi}$$

$\neg\mathsf{K}_C\varphi \to \mathsf{K}_C\neg\mathsf{K}_C\varphi$

$\mathsf{K}_C(\varphi \to \psi) \to (\mathsf{K}_C\varphi \to \mathsf{K}_C\psi)$

$\mathsf{K}_C\varphi \to \mathsf{K}_D\varphi,$ where $C \subseteq D$

$\mathsf{H}_C(\varphi \to \psi) \to (\mathsf{H}_D\varphi \to \mathsf{H}_{C\cup D}\psi),$ where $C \cap D = \varnothing$

$\mathsf{H}_C\varphi \to \mathsf{K}_C\mathsf{H}_C\varphi$

$\mathsf{K}_\varnothing\varphi \to \mathsf{H}_\varnothing\varphi$

Strategic Negative Introspection

Lemma 1. (Alechina, private communication)

$$\vdash \neg H_C\varphi \to K_C\neg H_C\varphi$$

Proof.

$\vdash H_C\varphi \to K_C H_C\varphi$

$\vdash \neg K_C H_C\varphi \to \neg H_C\varphi$

$\vdash K_C(\neg K_C H_C\varphi \to \neg H_C\varphi)$

$\vdash K_C\neg K_C H_C\varphi \to K_C\neg H_C\varphi$

$\vdash \neg K_C H_C\varphi \to K_C\neg K_C H_C\varphi$

$\vdash \neg K_C H_C\varphi \to K_C\neg H_C\varphi$

$\vdash K_C H_C\varphi \to H_C\varphi$

$\vdash \neg H_C\varphi \to \neg K_C H_C\varphi$

$\vdash \neg H_C\varphi \to K_C\neg H_C\varphi$

Strategic Monotonicity

Lemma 2.

$\vdash \mathsf{H}_C\varphi \to \mathsf{H}_D\varphi$, where $C \subseteq D$.

Proof.

$\vdash \varphi \to \varphi$

$\vdash \mathsf{H}_{D\setminus C}(\varphi \to \varphi)$

$\vdash \mathsf{H}_{D\setminus C}(\varphi \to \varphi) \to (\mathsf{H}_C\varphi \to \mathsf{H}_{(D\setminus C)\cup C}\varphi)$

$\vdash \mathsf{H}_C\varphi \to \mathsf{H}_{(D\setminus C)\cup C}\varphi$

$\vdash \mathsf{H}_C\varphi \to \mathsf{H}_D\varphi$

Knowledge and Know-How

all propositional tautologies

$$\mathsf{K}_C\varphi \to \varphi$$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{K}_C\varphi} \qquad \frac{\varphi}{\mathsf{H}_C\varphi}$$

$$\neg\mathsf{K}_C\varphi \to \mathsf{K}_C\neg\mathsf{K}_C\varphi$$

$$\mathsf{K}_C(\varphi \to \psi) \to (\mathsf{K}_C\varphi \to \mathsf{K}_C\psi)$$

$$\mathsf{K}_C\varphi \to \mathsf{K}_D\varphi, \text{ where } C \subseteq D$$

$$\mathsf{H}_C(\varphi \to \psi) \to (\mathsf{H}_D\varphi \to \mathsf{H}_{C\cup D}\psi), \text{ where } C \cap D = \varnothing$$

$$\mathsf{H}_C\varphi \to \mathsf{K}_C\mathsf{H}_C\varphi \qquad \mathsf{K}_\varnothing\varphi \to \mathsf{H}_\varnothing\varphi$$

Soundness and Completeness Theorems

If $\vdash \varphi$, then $w \Vdash \varphi$ for each state w of each game with imperfect information .

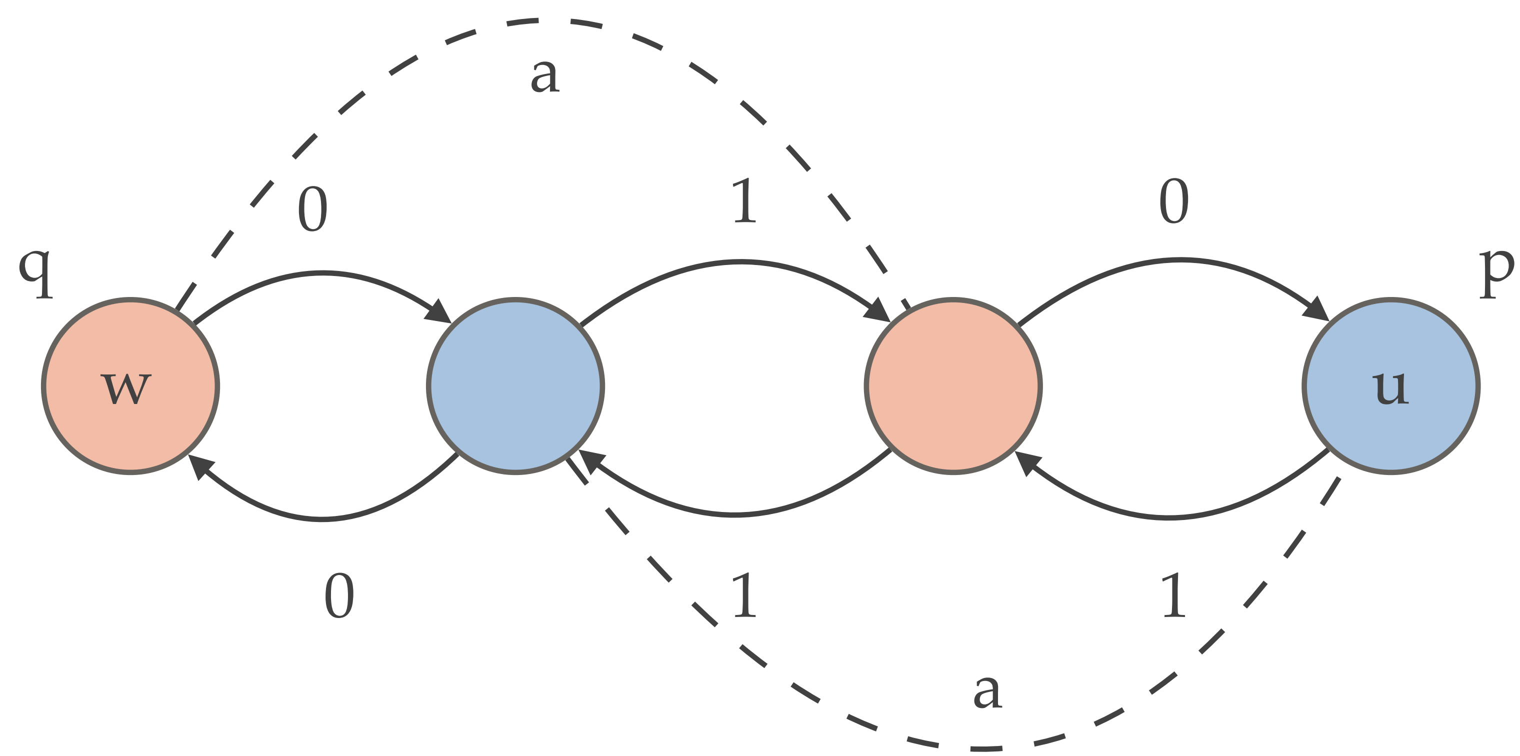
If $w \Vdash \varphi$ for each state w of each game with imperfect information, then $\vdash \varphi$.

Break

Multi-Step Strategies

0/1

Alice



$w \Vdash \mathsf{S}_a p$ $\quad$ $u \Vdash \mathsf{S}_a q$

$w \Vdash \mathsf{H}_a p$ $\quad$ $u \Vdash \neg \mathsf{H}_a q$

Single-Player Multi-Step Strategies to Achieve a Goal

all propositional tautologies

$\mathsf{K}\varphi \to \varphi$

$\neg\mathsf{K}\varphi \to \mathsf{K}\neg\mathsf{K}\varphi$

$\mathsf{K}(\varphi \to \psi) \to (\mathsf{K}\varphi \to \mathsf{K}\psi)$

$\mathsf{H}\varphi \to \mathsf{KH}\varphi$

$\mathsf{HH}\varphi \to \mathsf{H}\varphi$

$\mathsf{H}\varphi \to \mathsf{HK}\varphi$

$\mathsf{K}\varphi \to \mathsf{H}\varphi$

$\neg\mathsf{H}\bot$

(due to verifiability)

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{K}\varphi} \qquad \frac{\varphi \to \psi}{\mathsf{H}\varphi \to \mathsf{H}\psi} \qquad \frac{\varphi(p)}{\varphi[\psi/p]}$$

Completeness: Raul Fervari, Andreas Herzig, Yanjun Li, Yanjing Wang (2017), Strategically Knowing How, International Joint Conference on Artificial Intelligence (IJCAI)

Coalitional Multi-Step Strategies to Maintain a Goal

all propositional tautologies

$\mathsf{K}_C\varphi \to \varphi$

$\neg\mathsf{K}_C\varphi \to \mathsf{K}_C\neg\mathsf{K}_C\varphi$

$\mathsf{K}_C(\varphi \to \psi) \to (\mathsf{K}_C\varphi \to \mathsf{K}_C\psi)$

$\mathsf{K}_C\varphi \to \mathsf{K}_D\varphi$, where $C \subseteq D$

$\mathsf{H}_C(\varphi \to \psi) \to (\mathsf{H}_D\varphi \to \mathsf{H}_{C\cup D}\psi)$, where $C \cap D = \varnothing$

$\mathsf{H}_C\varphi \to \mathsf{K}_C\varphi$

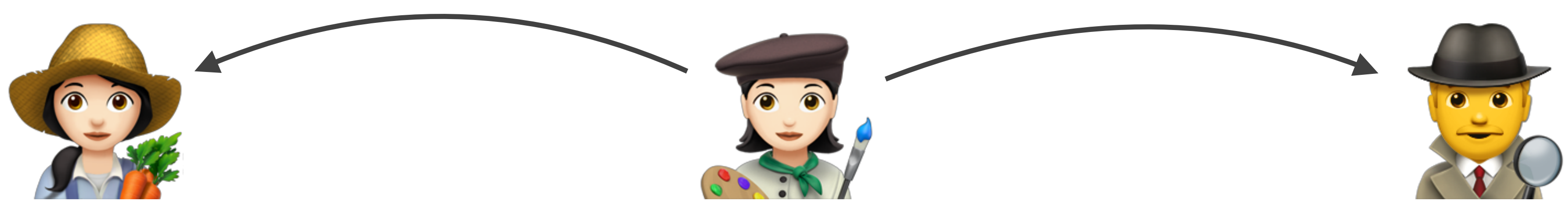
$\mathsf{H}_C\varphi \to \mathsf{H}_C\mathsf{H}_C\varphi$

$\mathsf{K}_\varnothing\varphi \to \mathsf{H}_\varnothing\varphi$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad\qquad \frac{\varphi}{\mathsf{H}_C\varphi}$$

Completeness: Pavel Naumov and Jia Tao (2017), Coalition Power in Epistemic Transition Systems, International Conference on Autonomous Agents and Multiagent Systems (AAMAS)

Blind Date Failure



K_a("date is at 6pm")

K_aK_b("date is at 6pm")

$K_aK_bK_a$("date is at 6pm")

$K_aK_bK_aK_b$("date is at 6pm")

K_b("date is at 6pm")

K_bK_a("date is at 6pm")

$K_bK_aK_b$("date is at 6pm")

$K_bK_aK_bK_a$("date is at 6pm")

Common Knowledge

$C_{a,b,c}$("date is at 6pm")

Common Knowledge and Strategies

all propositional tautologies

$\mathsf{K}_a\varphi \to \varphi$

$\neg\mathsf{K}_a\varphi \to \mathsf{K}_a\neg\mathsf{K}_a\varphi$

$\mathsf{K}_a(\varphi \to \psi) \to (\mathsf{K}_a\varphi \to \mathsf{K}_a\psi)$

$$\mathsf{C}_A\varphi \to \bigwedge_{a\in A}\mathsf{K}_a(\varphi \wedge \mathsf{C}_A\varphi)$$

$\mathsf{S}_A(\varphi \to \psi) \to (\mathsf{S}_B\varphi \to \mathsf{S}_{A\cup B}\psi)$, where $A \cap B = \varnothing$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{C}_A\varphi} \qquad \frac{\varphi}{\mathsf{S}_A\varphi}$$

$$\frac{\psi \to \bigwedge_{a\in A}\mathsf{K}_a(\varphi \wedge \psi)}{\psi \to \mathsf{C}_A\varphi}$$

Completeness: Thomas Ågotnes and Natasha Alechina (2012). "Epistemic Coalition Logic: Completeness and Complexity." International Conference on Autonomous Agents and Multiagent Systems

Common-Know-How?

$$\mathsf{CH}_A(\varphi \to \psi) \to (\mathsf{CH}_B(\varphi \to \mathsf{CH}_{A \cup B}\psi), \text{ where } A \cap B = \emptyset$$

Distributed Knowledge, Strategies, and Know-How

all propositional tautologies

$\mathsf{K}_C\varphi \to \varphi$

$\neg\mathsf{K}_C\varphi \to \mathsf{K}_C\neg\mathsf{K}_C\varphi$

$\mathsf{K}_C(\varphi \to \psi) \to (\mathsf{K}_C\varphi \to \mathsf{K}_C\psi)$

$\mathsf{K}_C\varphi \to \mathsf{K}_D\varphi$, where $C \subseteq D$

$\mathsf{S}_C(\varphi \to \psi) \to (\mathsf{S}_D\varphi \to \mathsf{S}_{C\cup D}\psi)$, where $C \cap D = \varnothing$

$\mathsf{H}_C(\varphi \to \psi) \to (\mathsf{H}_D\varphi \to \mathsf{H}_{C\cup D}\psi)$, where $C \cap D = \varnothing$

$\mathsf{H}_C\varphi \to \mathsf{K}_C\mathsf{H}_C\varphi$ $\quad$ $\mathsf{K}_\varnothing\varphi \to \mathsf{H}_\varnothing\varphi$

$\dfrac{\varphi, \quad \varphi \to \psi}{\psi}$ $\quad$ $\dfrac{\varphi}{\mathsf{K}_C\varphi}$ $\quad$ ~~$\dfrac{\varphi}{\mathsf{S}_C\varphi}$~~ $\quad$ $\dfrac{\varphi}{\mathsf{H}_C\varphi}$

$\mathsf{H}_C\varphi \to \mathsf{S}_C\varphi$ $\quad$ $\neg\mathsf{S}_C\bot$

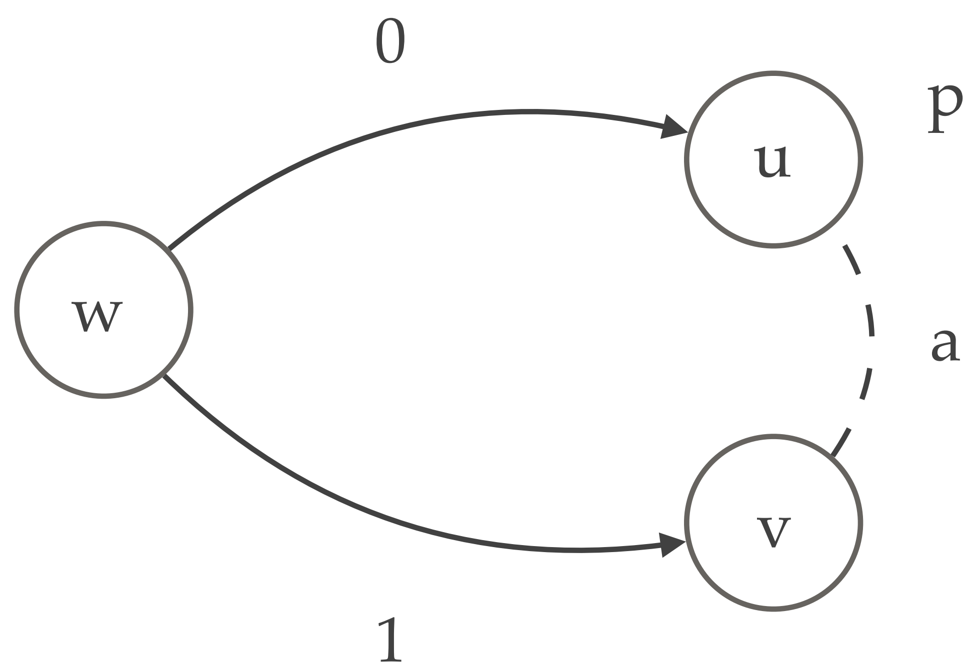
$\mathsf{H}_C(\varphi \to \psi) \to (\mathsf{K}_C\mathsf{S}_\varnothing\varphi \to \mathsf{H}_C\psi)$

Completeness: Pavel Naumov and Jia Tao (2018), Together We Know How to Achieve: An Epistemic Logic of Know-How, Journal of Artificial Intelligence

No Perfect Recall

0/1

Alice



$w \Vdash \mathsf{S}_a p$

$u \Vdash \neg \mathsf{K}_a p$

$w \Vdash \neg \mathsf{S}_a \mathsf{K}_a p$

$w \Vdash \mathsf{H}_a p$

$w \Vdash \neg \mathsf{H}_a \mathsf{K}_a p$

Perfect Recall Semantics

$(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \Vdash p$ iff $w \in \pi(p)$,

$(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \Vdash \neg\varphi$ iff $(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \nVdash \varphi$,

$(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \Vdash \varphi \to \psi$ iff $(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \nVdash \varphi$ or $(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \Vdash \psi$,

$(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \Vdash \mathsf{K}_C\varphi$ iff $(w'_0, \delta'_1, w'_1, \ldots, \delta'_n, w'_n) \Vdash \varphi$ for all $(w'_0, \delta'_1, w'_1, \ldots, \delta'_n, w'_n)$
such that $(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \sim_C (w'_0, \delta'_1, w'_1, \ldots, \delta'_n, w'_n)$

$(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \Vdash \mathsf{H}_C\varphi$ iff there is an action profile $s \in \Delta^C$ of coalition C
such that for each history $(w'_0, \delta'_1, w'_1, \ldots, \delta'_n, w'_n)$,
each complete action profile $\delta \in \Delta^{\mathscr{A}}$, and each state $u \in W$
if $(w_0, \delta_1, w_1, \ldots, \delta_n, w_n) \sim_C (w'_0, \delta'_1, w'_1, \ldots, \delta'_n, w'_n)$, $s =_C \delta$, and $(w'_n, \delta, u) \in M$,
then $(w'_0, \delta'_1, w'_1, \ldots, \delta'_n, w'_n, \delta, u) \Vdash \varphi$

Knowledge and Know-How with Perfect Recall

all propositional tautologies

$\mathsf{K}_C\varphi \to \varphi$

$\neg\mathsf{K}_C\varphi \to \mathsf{K}_C\neg\mathsf{K}_C\varphi$

$\mathsf{K}_C(\varphi \to \psi) \to (\mathsf{K}_C\varphi \to \mathsf{K}_C\psi)$

$\mathsf{K}_C\varphi \to \mathsf{K}_D\varphi$, where $C \subseteq D$

$\mathsf{H}_C(\varphi \to \psi) \to (\mathsf{H}_D\varphi \to \mathsf{H}_{C\cup D}\psi)$, where $C \cap D = \varnothing$

$\mathsf{H}_C\varphi \to \mathsf{K}_C\mathsf{H}_C\varphi$

$\mathsf{K}_\varnothing\varphi \to \mathsf{H}_\varnothing\varphi$

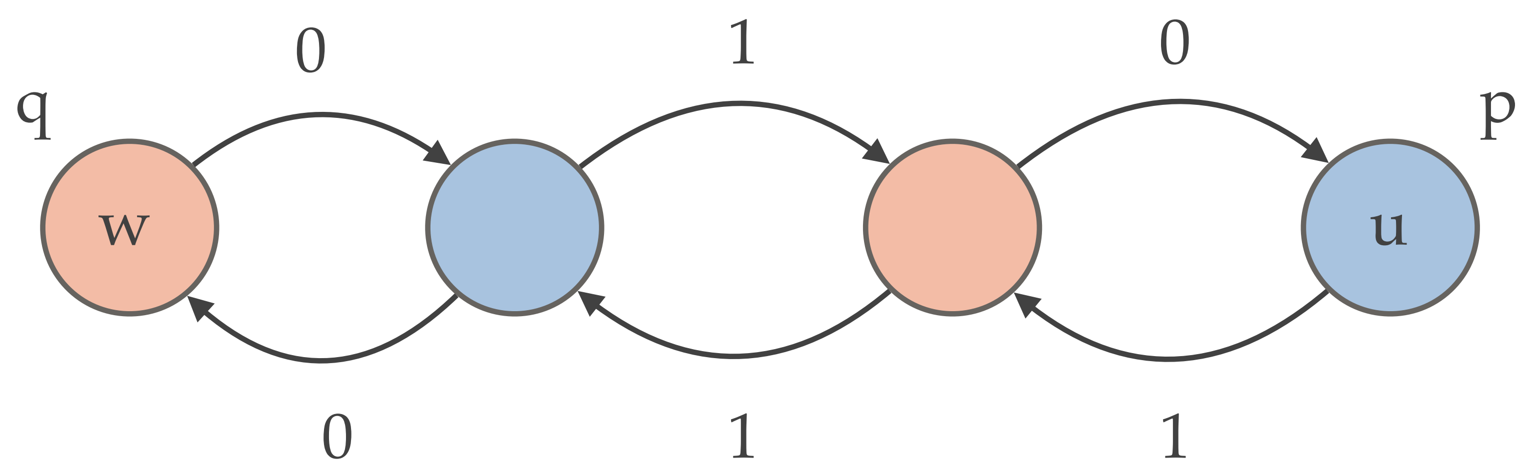
$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{K}_C\varphi} \qquad \frac{\varphi}{\mathsf{H}_C\varphi}$$

$\mathsf{H}_D\varphi \to \mathsf{H}_D\mathsf{K}_C\varphi$, where $D \subseteq C \neq \varnothing$

$\neg\mathsf{H}_C\bot$

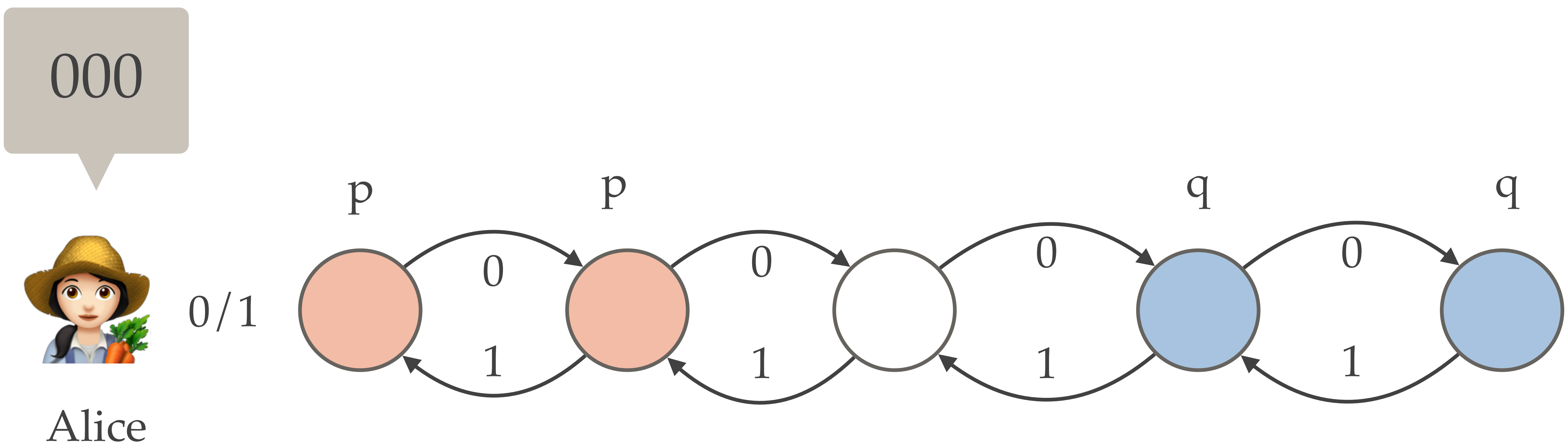
Completeness: Pavel Naumov and Jia Tao (2018), Strategic Coalitions with Perfect Recall, AAAI Conference on Artificial Intelligence (AAAI 18)

Linear Plans



H(q, p) H(p, q)

Linear Plans



H(p, q)

Axioms for Single-Agent Linear Plans

all propositional tautologies

$\mathsf{N}\varphi \to \varphi$

$\mathsf{N}(\varphi \to \psi) \to (\mathsf{N}\varphi \to \mathsf{N}\psi)$

$\mathsf{H}(\varphi, \psi) \to (\mathsf{H}(\psi, \chi) \to \mathsf{H}(\varphi, \chi))$

$\mathsf{N}(\varphi \to \psi) \to \mathsf{H}(\varphi, \psi)$

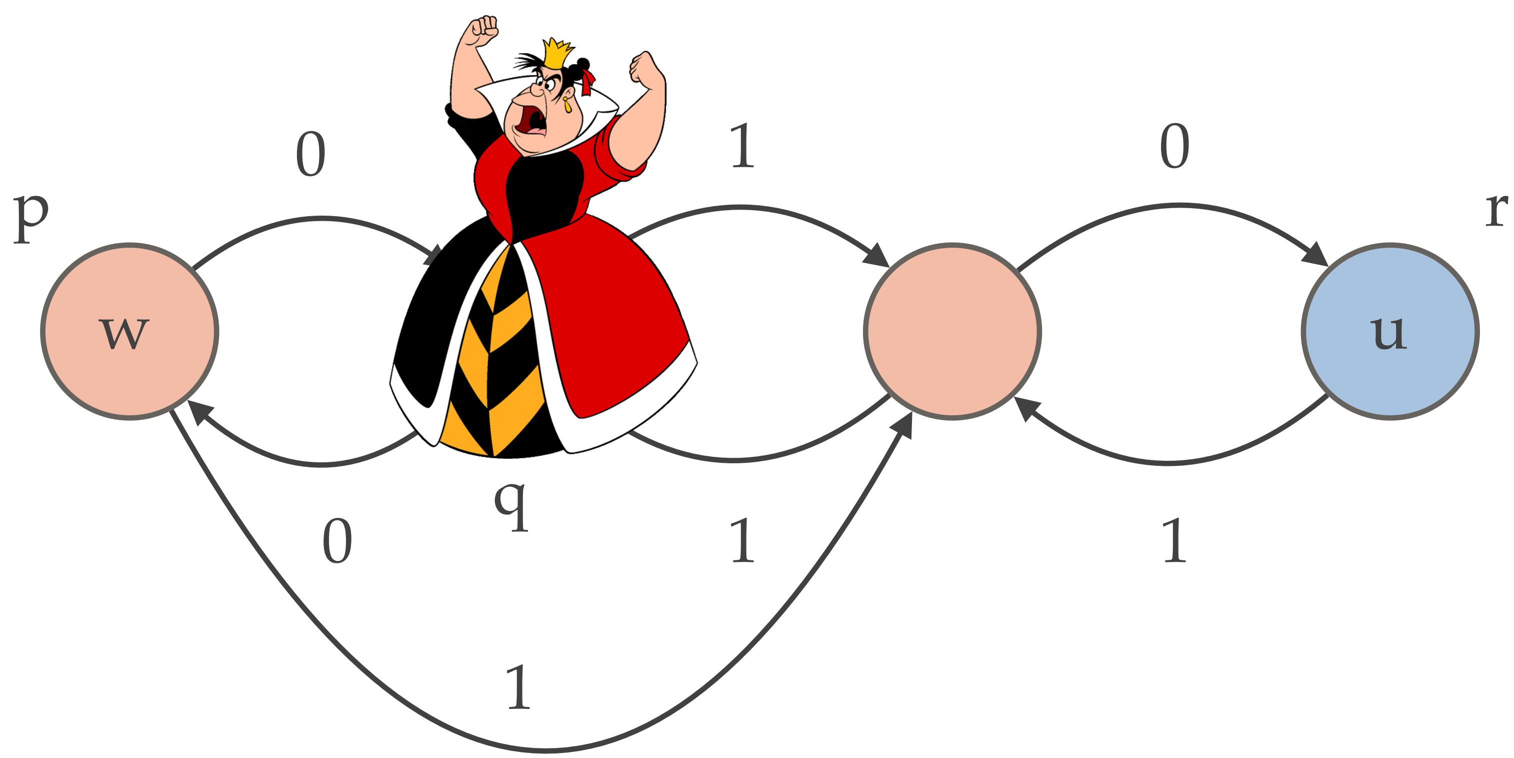
$\mathsf{H}(\varphi, \psi) \to \mathsf{NH}(\varphi, \psi)$

$\neg\mathsf{H}(\varphi, \psi) \to \mathsf{N}\neg\mathsf{H}(\varphi, \psi)$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{N}\varphi}$$

Completeness: Yanjing Wang (2016), A logic of goal-directed knowing how, Synthese

Linear Plans with Intermediate Constraints



$\mathsf{H}(p, \neg q, r)$

Axioms for Plans with Intermediate Constraints

all propositional tautologies

$$\mathsf{N}\varphi \to \varphi$$

$$\mathsf{N}(\varphi \to \psi) \to (\mathsf{N}\varphi \to \mathsf{N}\psi)$$

$$\mathsf{H}(\varphi, \tau, \psi) \to (\mathsf{H}(\psi, \tau, \chi) \to (\mathsf{N}(\psi \to \tau) \to \mathsf{H}(\varphi, \tau, \chi)))$$

$$\mathsf{N}(\varphi \to \psi) \to \mathsf{H}(\varphi, \bot, \psi)$$

$$\mathsf{H}(\varphi, \tau, \psi) \to (\neg\mathsf{H}(\varphi, \bot, \psi) \to \mathsf{H}(\varphi, \bot, \tau))$$

$$\mathsf{H}(\varphi, \tau, \psi) \to \mathsf{NH}(\varphi, \tau, \psi)$$

$$\neg\mathsf{H}(\varphi, \tau, \psi) \to \mathsf{N}\neg\mathsf{H}(\varphi, \tau, \psi)$$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{N}\varphi}$$

$$\mathsf{N}(\varphi' \to \varphi) \to (\mathsf{H}(\varphi, \tau, \psi) \to \mathsf{H}(\varphi', \tau, \psi))$$

$$\mathsf{N}(\psi \to \psi') \to (\mathsf{H}(\varphi, \tau, \psi) \to \mathsf{H}(\varphi, \tau, \psi'))$$

$$\mathsf{N}(\tau \to \tau') \to (\mathsf{H}(\varphi, \tau, \psi) \to \mathsf{H}(\varphi, \tau', \psi))$$

Completeness: Yanjun Li and Yanjing Wang (2017), Achieving while maintaining: A logic of knowing how with intermediate constraints, Indian Conference on Logic and Its Applications

Second-Order Know-How

0/1

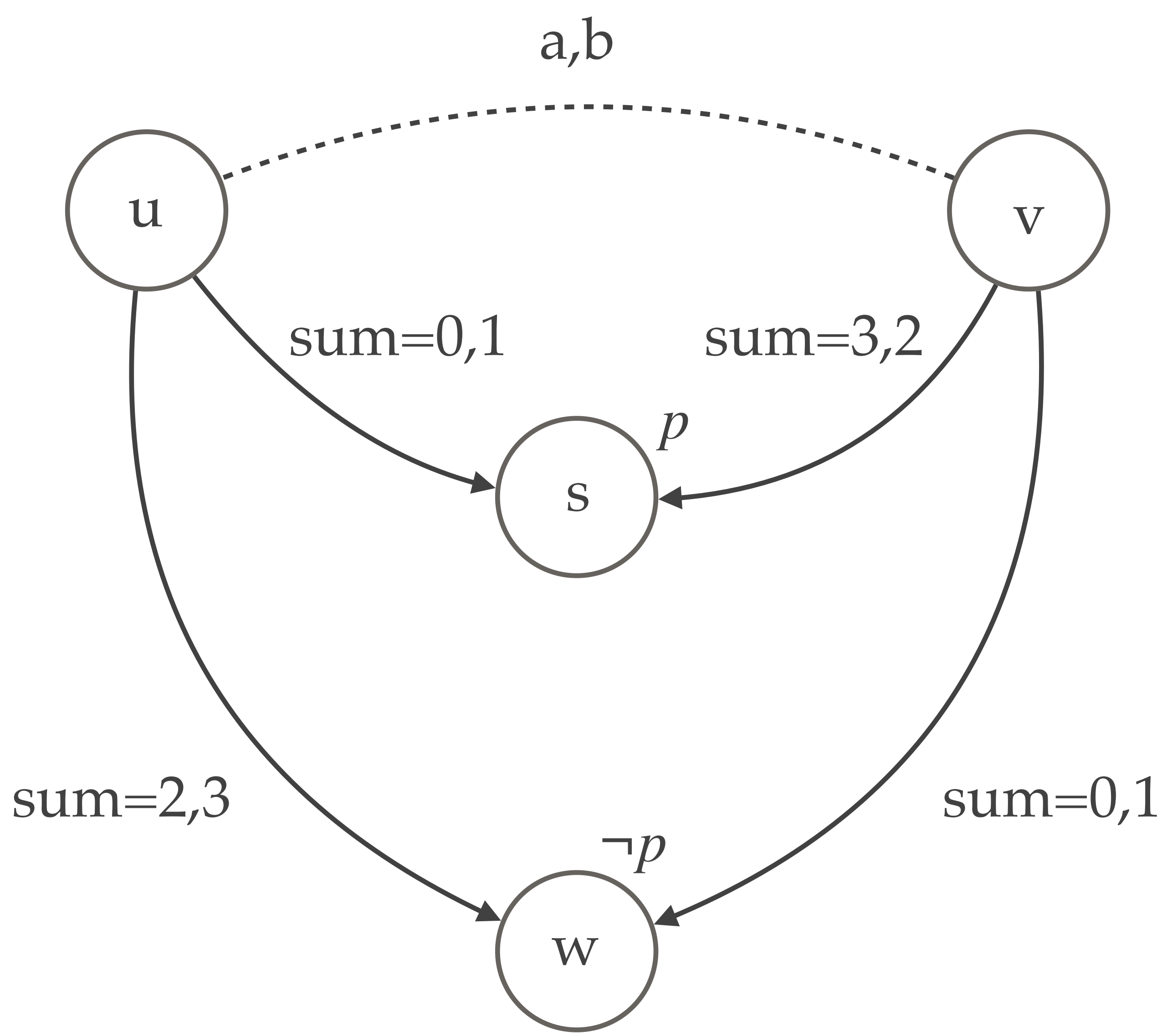
Alice

0/1

Bob

0/1

Cathy



$u \Vdash \mathsf{S}_{a,b}p$

$v \Vdash \mathsf{S}_{a,b}p$

$u \Vdash \mathsf{K}_{a,b}\mathsf{S}_{a,b}p$

$u \Vdash \neg\mathsf{H}_{a,b}p$

$u \Vdash \mathsf{H}_{a,b,c}p$

$u \Vdash \mathsf{H}_{c}^{a,b}p$

Second-Order Know-How

$w \Vdash \mathsf{H}_C\varphi$ iff there is an action profile $s \in \Delta^C$ of coalition C

such that for each state v and each complete action profile $\delta \in \Delta^{\mathscr{A}}$ and each state $u \in W$

if $w \sim_C v, s =_C \delta,$ and $(v, \delta, u) \in M,$ then $u \Vdash \varphi$

$w \Vdash \mathsf{H}_C^D\varphi$ iff there is an action profile $s \in \Delta^D$ of coalition D

such that for each state v and each complete action profile $\delta \in \Delta^{\mathscr{A}}$ and each state $u \in W$

if $w \sim_C v, s =_D \delta,$ and $(v, \delta, u) \in M,$ then $u \Vdash \varphi$

Axioms for Second-Order Know-How

all propositional tautologies

$\mathsf{K}_C\varphi \to \varphi$

$\neg\mathsf{K}_C\varphi \to \mathsf{K}_C\neg\mathsf{K}_C\varphi$

$\mathsf{K}_C(\varphi \to \psi) \to (\mathsf{K}_C\varphi \to \mathsf{K}_C\psi)$

$\mathsf{K}_C\varphi \to \mathsf{K}_D\varphi$, where $C \subseteq D$

$\mathsf{H}^{D_1}_{C_1}(\varphi \to \psi) \to (\mathsf{H}^{D_2}_{C_2}\varphi \to \mathsf{H}^{D_1\cup D_2}_{C_1\cup C_2}\psi)$, where $D_1 \cap D_2 = \varnothing$

$\mathsf{H}^D_C\varphi \to \mathsf{K}_C\mathsf{H}^D_C\varphi$

$\mathsf{K}_\varnothing\varphi \to \mathsf{H}^\varnothing_\varnothing\varphi$

$\mathsf{K}_C\mathsf{H}^\varnothing_D\varphi \to \mathsf{H}^\varnothing_C\varphi$

$$\frac{\varphi, \quad \varphi \to \psi}{\psi} \qquad \frac{\varphi}{\mathsf{K}_C\varphi} \qquad \frac{\varphi}{\mathsf{H}^D_C\varphi}$$

Completeness: Pavel Naumov and Jia Tao (2018), International Conference on Autonomous Agents and Multiagent Systems (AAMAS 18)

Strategies and Responsibility

$\mathsf{R}_a\varphi \equiv$ agent a used strategy that made φ unavoidable.

$\mathsf{N}\varphi \equiv$ statement φ is always true

$\overline{\mathsf{N}}_a\varphi \equiv \neg\mathsf{N}_a\neg\varphi$

$\mathsf{N}\varphi \to \varphi$

$\neg\mathsf{N}\varphi \to \mathsf{N}\neg\mathsf{N}\varphi$

$\mathsf{N}(\varphi \to \psi) \to (\mathsf{N}\varphi \to \mathsf{N}\psi)$

$\mathsf{R}_a\varphi \to \varphi$

$\neg\mathsf{R}_a\varphi \to \mathsf{R}_a\neg\mathsf{R}_a\varphi$

$\mathsf{R}_a(\varphi \to \psi) \to (\mathsf{R}_a\varphi \to \mathsf{R}_a\psi)$

$\mathsf{N}\varphi \to \mathsf{R}_a\varphi$

$\overline{\mathsf{N}}\mathsf{R}_{a_1}\varphi_1 \wedge \overline{\mathsf{N}}\mathsf{R}_{a_2}\varphi_2 \wedge \ldots \wedge \overline{\mathsf{N}}\mathsf{R}_{a_n}\varphi_n \to \overline{\mathsf{N}}(\mathsf{R}_{a_1}\varphi_1 \wedge \mathsf{R}_{a_2}\varphi_2 \wedge \ldots \wedge \mathsf{R}_{a_n}\varphi_n)$

Belnap N., Perloff M., Xu M., Facing the Future: Agents and Choices in our Indeterminist World, Oxford, 2001

Group Responsibility

$\mathsf{R}_C\varphi \equiv$ coalition C used strategy that made φ unavoidable.

$\mathsf{R}_\emptyset\varphi \equiv \mathsf{N}\varphi$

$\overline{\mathsf{N}}_a\varphi \equiv \neg\mathsf{N}_a\neg\varphi$

$\mathsf{R}_C\varphi \to \varphi$

$\neg\mathsf{R}_C\varphi \to \mathsf{R}_C\neg\mathsf{R}_C\varphi$

$\mathsf{R}_C(\varphi \to \psi) \to (\mathsf{R}_C\varphi \to \mathsf{R}_C\psi)$

$\mathsf{R}_C\varphi \to \mathsf{R}_D\varphi$, where $C \subseteq D$

$\mathsf{R}_C\mathsf{R}_{\overline{C}}\varphi \to \mathsf{R}_\emptyset\varphi$

Jan Broersen, Andreas Herzig, and Nicolas Troquard. What groups do, can do, and know they can do: an analysis in normal modal logics.
Journal of Applied Non-Classical Logics, 19(3):261–289, 2009

Responsibility and Knowledge

NEW YORK The New York Times SUBSCRIBE NOW Account

Woman Fatally Shot by Hunter Who Mistook Her for a Deer, Officials Say

Jamie and Rosemary Billquist. Ms. Billquist was shot by a hunter near their home in western New York on Nov. 22.

By **Matt Stevens**

Nov. 26, 2017

Rosemary Billquist had worked a little late on Wednesday, so by the time she returned to her home in western New York it was already getting dark.

She took her yellow Labrador retrievers, Sugar and Stella, for a walk around 5:30 p.m., while her husband, Jamie, stayed behind at their home in Sherman, N.Y., about 65 miles southwest of Buffalo.

But 15 or 20 minutes later, Sugar and Stella came bounding back without her. They were peering backward, barking with unusual urgency.

“They obviously knew something happened,” Mr. Billquist said in an interview on Saturday night. “And I’m thinking to myself, ‘This isn’t right.’”

Blameworthiness

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L.A. NOW LOCAL

4-year-old accidentally shoots and kills toddler cousin in San Bernardino County

By **BENJAMIN ORESKES** JUL 20, 2018 | 8:45 PM

A 4-year-old boy accidentally shot and killed his 2-year-old cousin Friday morning in a small town in San Bernardino County, according authorities.

A 4-year-old boy accidentally shot and killed his 2-year-old cousin in a small San Bernardino County town Friday, and authorities have arrested the victim's grandfather on suspicion of leaving his gun in a spot accessible to the children, the Sheriff's Department said.

Cesar Lopez, 53, the victim's grandfather, was arrested on suspicion of child endangerment.

San Bernardino County sheriff's deputies responded to the shooting, which occurred in the unincorporated town of Muscoy a little after 9 a.m. When they arrived to the 2700 block of Duffy Street, they found a young girl with gunshot wounds.

She was taken to a hospital and died just after 10 a.m., according to Cindy Bachman, a Sheriff's Department spokeswoman.

There were several adults in the house when the shooting occurred, Bachman said. Both

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1h

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3h

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4:00 PM

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1:35 PM

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12:10 PM

Thank you!