The Batfox Assignment, and Other 3D Printing Adventures in Third-Semester Calculus

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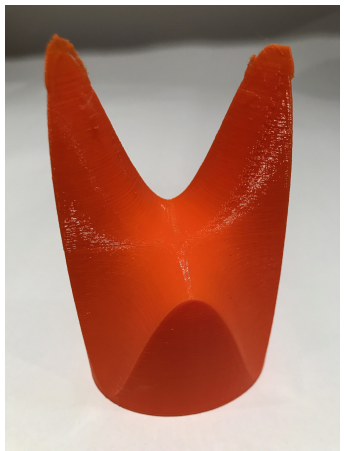
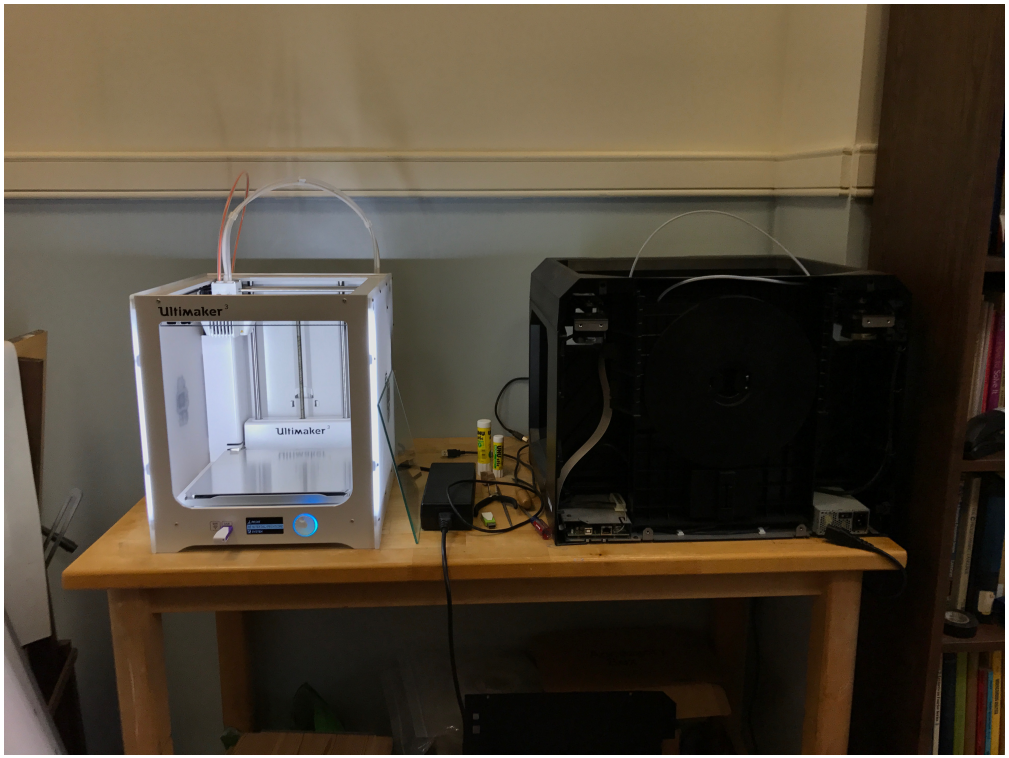


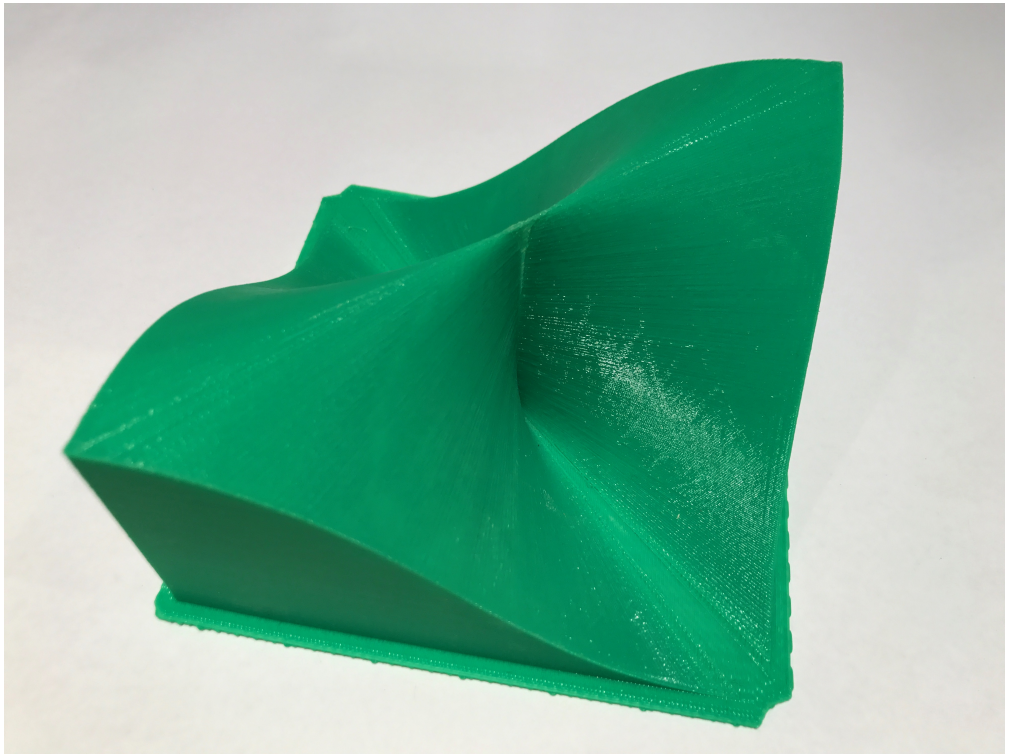
My "Lab"



Ultimaker 3 on the left. Expired Makerbot 5th Generation on the right. Each cost roughly $4000 with a few filament spools.

Other campus departments have labs with several machines.

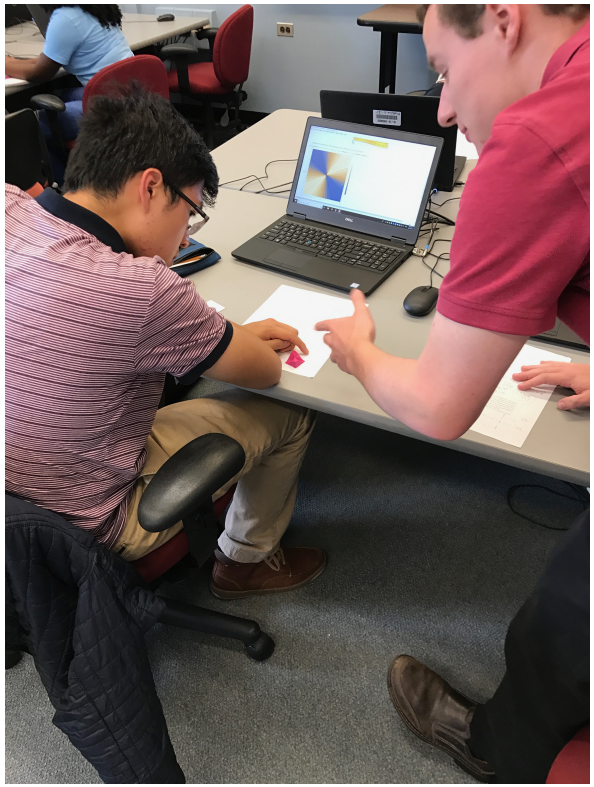
A Function That Can't Be Made Continuous at $(0, 0)$



The layer-by-layer printing process generates visible level curves, which are lines for this function:

$$f(x, y) = \frac{2xy}{x^2 + y^2}$$

Active Learning



Each student got a smaller version of the model for a group project. The first task is simply to place it correctly on a set of xy-axes.

Easy As 1-2!

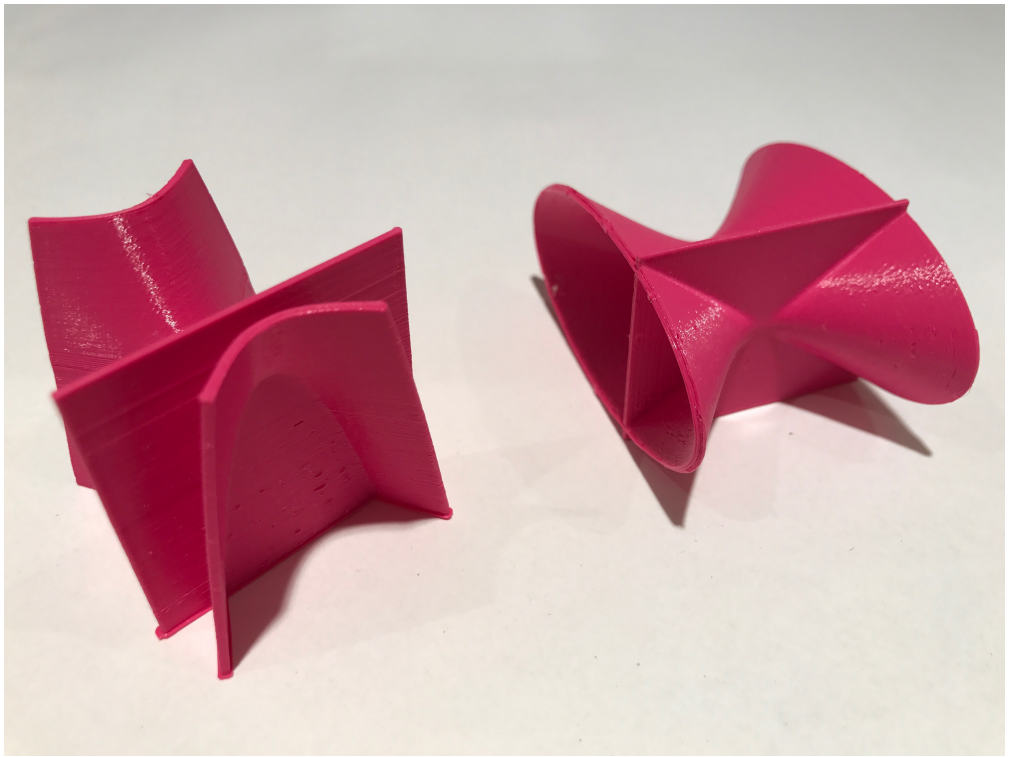
Two lines of Mathematica code and you've got an object to print:

$$\text{model} = \text{Show}[\ \text{Plot3D}[(2xy)/(x^2+y^2), \{x,-2,2\}, \{y,-2,2\},$$
$$\text{Filling} \to \text{Bottom}, \text{PlotPoints} \to 100]\ ];$$

$$\text{Export}[\text{“PinchedSurface.stl”}, \text{model}];$$

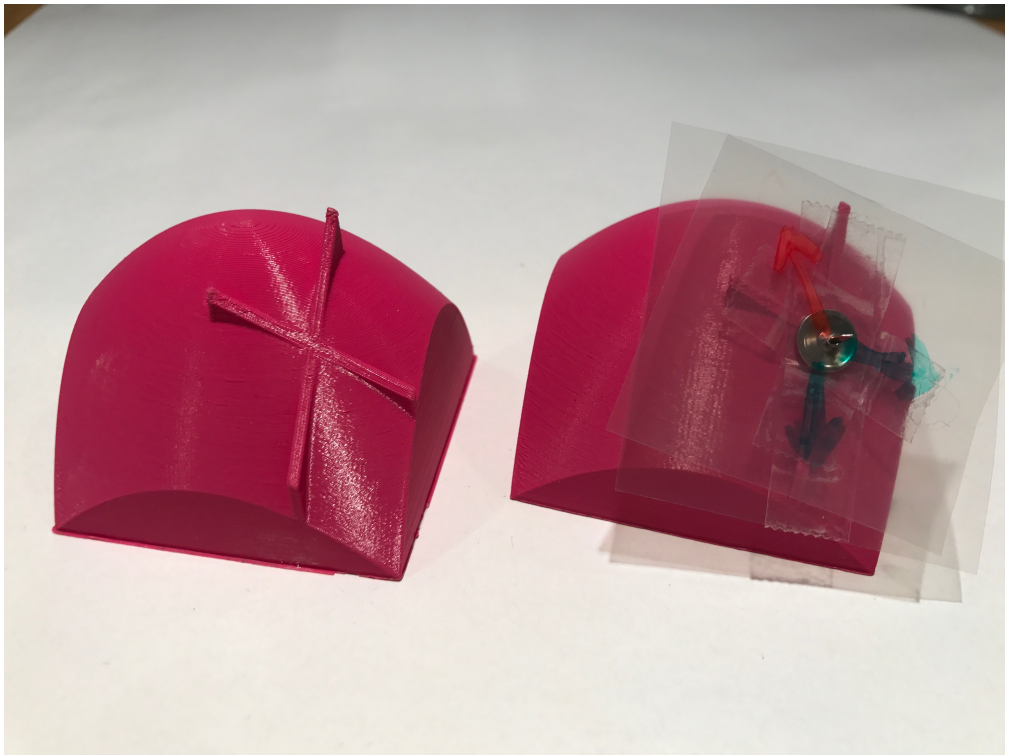
The .stl file can be sent to any 3D printing software. That's it!

Two Quadric Surfaces



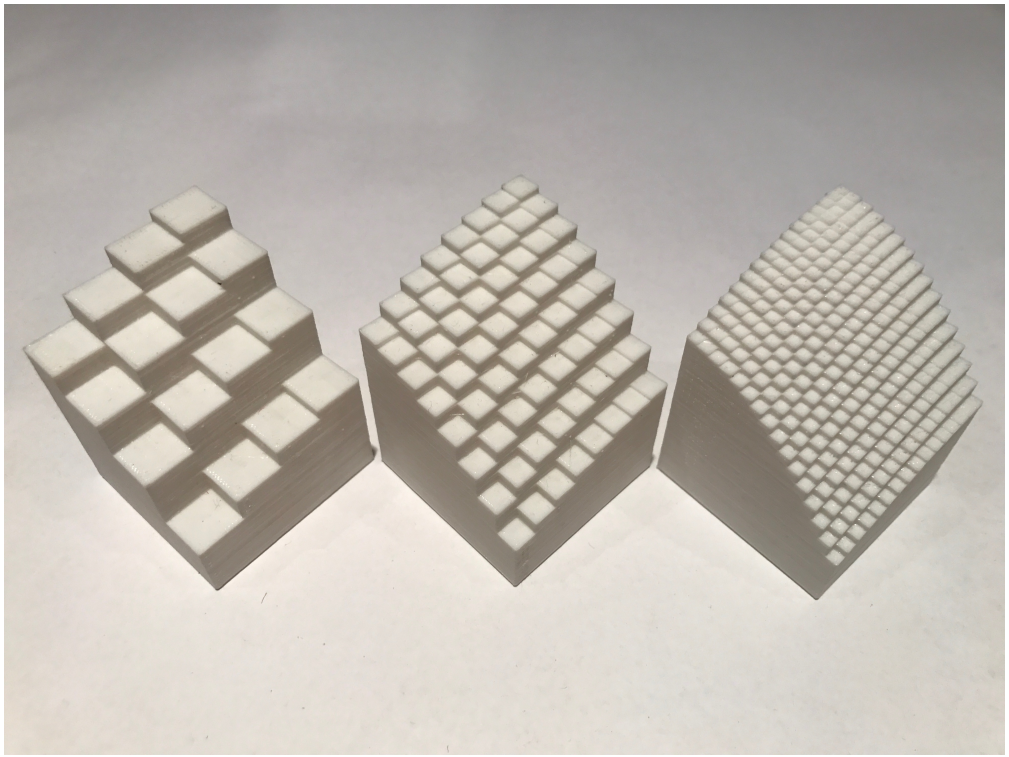
The planes intersect the surfaces along traces. Yes, you do get two intersecting lines!

Tangent Planes and Directional Derivatives



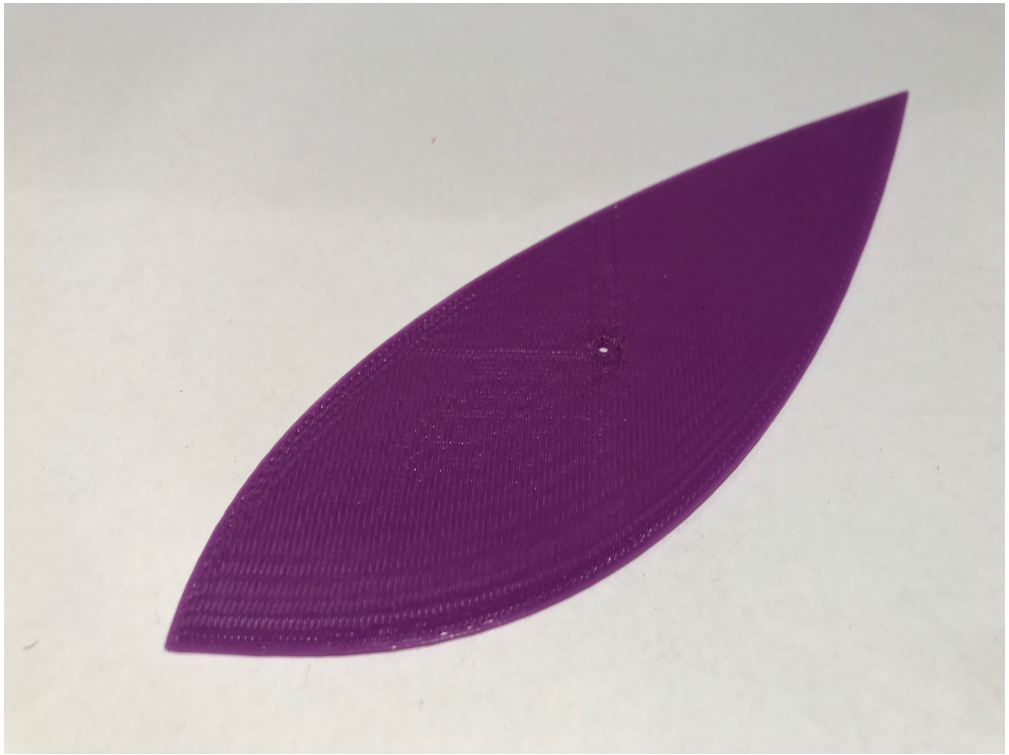
The slopes of the tangent lines in the x and y directions are now easy to see. Later we rested a tangent plane on top of these lines, with a normal thumbtack – careful! And then we added a unit vector that could rotate in any direction.

Riemann Sums and Multiple Integration



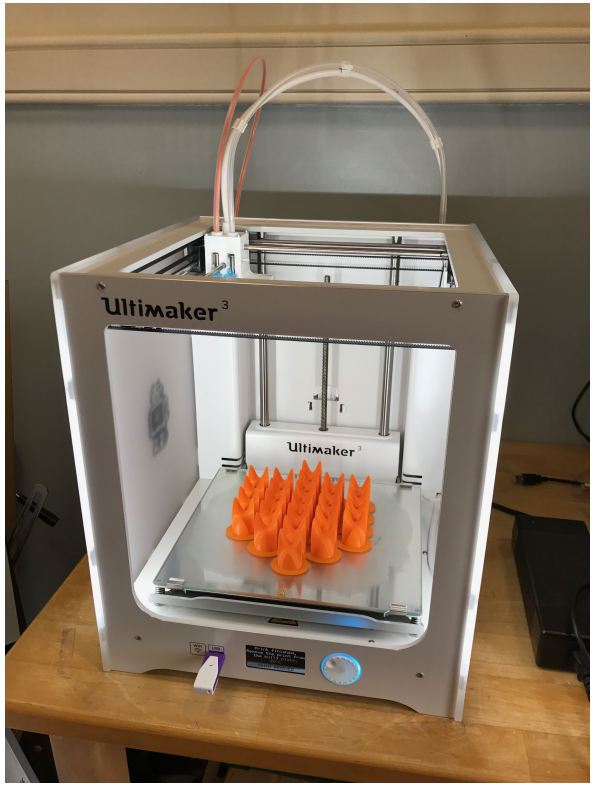
The underestimates get better as you move to the right.

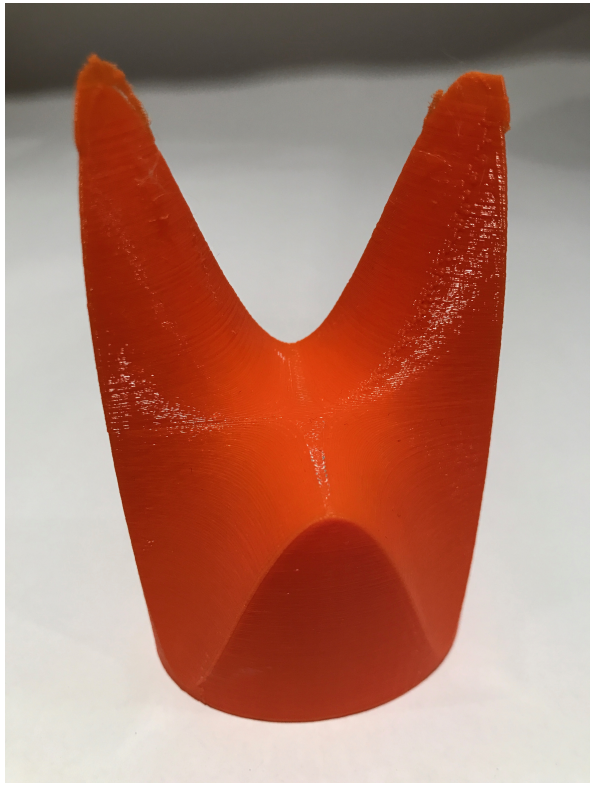
A Balancing Act



A student wanted to see if the center of mass of a lamina bounded by $y = \sqrt{x}$ and $y = x^3$ really is $(3/7, 12/25)$. It is.

The Batfox!





Is it a fox? A bat? No, it's

$$z = x^2 y \quad \text{subject to} \quad x^2 + 2y^2 \leq 6.$$

The Batfox!

Parts **(d)**, **(e)**, and **(f)** force students to deal with a full line segment of critical points, and the Second Derivative Test is inconclusive for each point.

Parts **(g)** and **(h)** offer two ways to handle the boundary ellipse: substitute $x^2 = 6 - 2y^2$ into x^2y to get a cubic function of y, or use Lagrange multipliers. There are six points to find on the boundary, but it's very easy for students to only find the same four points using each method.

The Batfox!

Did you know that batfoxes come in pairs?

Can you explain why?

Thank You!

Thank you!

Thanks to the conference organizers!
And to Lafayette College for two $5000 grants.
And to many colleagues for suggesting nice 3D printing projects.
And to future technology for disposing of all of this plastic appropriately, and for making printers that will affordably print in biodegradable material.

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