The Batfox Assignment

Your task is to understand all of the local and global extrema for the function

$$z = f(x, y) = x^2y$$

on the closed and bounded region D in the xy-plane that is defined by

$$D = \{(x, y) \mid x^2 + 2y^2 \leq 6\}.$$

The orange model will guide you on your journey.

(a) First and foremost: Does the model look more like a fox or more like a bat? Discuss!

(b) Describe the region D in words. On a separate sheet of paper, draw a well-labelled set of axes for the xy-plane and place the model in position on these axes, tracing the boundary of D with your pencil. Be sure to specifically label the four points on the boundary of D where the boundary crosses the axes.

(c) How can you determine which regions of D should be under the "ears" of the model? Also, the sheet with your axes doesn't correspond to the plane $z = 0$. You'll figure out that value of z later.

(d) Carefully determine all of the critical points of f that lie in D. You might find that there are a lot of them!

(e) For each of the critical points you found in **(d)**, what is the value of the 2×2 Hessian determinant used in the Second Derivative Test? What can you conclude about each of the critical points using the Second Derivative Test?

(f) Regardless of your answer in **(e)**, use the model and insights about the function f to determine which critical points are local maxima and which are local minima.

The method on page 1006 of your textbook for finding absolute maximum and minimum values of f on the region D requires you to also find the extreme values of f on the boundary of D, which contains all of the points (x, y) such that $x^2 + 2y^2 = 6$. You'll find these extreme values in two different ways, which will be nice for double checking your answer ...

(g) The first way is to substitute $x^2 = 6 - 2y^2$ into the formula for f, which will convert f into a function g of the single variable y. Write down the formula for $g(y)$, and also determine the closed interval of y values that correspond to points on the boundary of D. Use Calc I techniques to find the values of y corresponding to the extreme values of g on that closed interval. How can you visualize the graph of g on the model?

As a final step, determine all of the x values corresponding to each of these y values.

You should have gotten a number of points (x, y) on the boundary of D which are the only possibilities for giving you the absolute maximum and minimum values for f on the boundary. Check that these points are correct by ...

(h) ... using the method of Lagrange multipliers to find these points as well!

Write down formulas for the functions $f(x, y)$ and $g(x, y)$. Then calculate ∇f and ∇g. Then write down the set of equations you need to solve, and solve! Take care to not miss any cases; sometimes it's helpful to make a case for when one of the variables equals 0, and another case for when it doesn't. You should get the same points you found in **(g)**.

(i) To wrap up, use your answers from parts **(d)**, **(g)**, and **(h)** to determine the point(s) that give the absolute maximum and absolute minimum values for f on the region D. Verify that your answers are correct by finding the points on the model. And you can now identify the sheet of paper as being part of a certain plane – label the sheet with that equation.

You may work with up to three classmates on this assignment; I only want one submission per group. Show all of your work. You can ask me questions and refer to your textbook, but you may not consult with any other sources. After you've provided **museum-quality** answers to all of the questions in this packet, scan or take an indisputably legible photo of each page, take a nice photo of your model in place on your well-labelled axes, and combine these into one .pdf document. **At least fifteen minutes before the beginning of class on Monday, email me your single .pdf document as an attachment.**