# Simulating Oligopoly to Enhance Student Learning 

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#### Abstract

How well do students understand price theory, market power, strategic interaction, and oligopoly theory? How can we help them get hands-on experience? Typical undergraduate exposure to these ideas may occur in Principles and Intermediate Microeconomics, followed by coverage in upper-level courses on Game Theory, Industrial Organization, and Regulation. We describe an extra-classroom activity called Virtual Corporate Reality (VCR), designed to engage students' contemplation of and experience with these ideas. Students compete in teams over the course of the semester in a price and location game based on Salop's (1979) circular city. Our experience is that VCR increases students' internalization of concepts such as sunk cost, best-response, Nash equilibrium, differentiated products, and even concepts as seemingly straightforward as pricing above marginal cost. They find it both entertaining and edifying, and come to class more prepared to understand the assumptions, structure, and predictions of oligopoly theory. Students receive significant feedback on their success at mastering and internalizing these concepts.


Keywords: experiential learning, problem-based learning

## Introduction

A simulation can provide students with important experiences that enhance their classroom learning and promote their long-term retention of the material. There are many simulations of business behavior available to instructors in business schools, and a few in economics (Borenstein 1998 . Such simulations should address several pedagogical goals. Bain's (2004, Ch. 4) analysis of exceptional teachers highlights the difficulty of breaking through students' existing mental models. His example of physics students studied by Halloun and Hestenes (1985) who arrive in class with an Aristotelean view of physical interaction and persistently maintain that perspective regardless of their grade in the class. We can imagine that students of economics also arrive with significant preconceptions about the way the world works and it will be difficult to get them to drop their existing mental models of producer-producer interaction. Bain's finding is that even when students want to learn, even when they know how to (and do) perform well on exams, the students need to first be shown how wrong their mental models are.

Another example of simulations in economics is Holt's (2007) VeconLab, a treasure trove of games, each of which can be run during a lecture period. VeconLab can be used Principles and Intermediate Microeconomics as an in-class exercise while teaching game theory (and in conjunction with VCR), but it appears to be used primarily in experimental economics classrooms (Chernomaz 2009), although this may be an artifact of the group surveyed.

There are three goals this game addresses: overcoming unschooled preconceptions, revising students misconceptions students may develop in introductory courses, and increasing students' familiarity with the producers' options. The first and third goals may be obvious, but that middle goal is also a challenge that students in upper-level economics courses face: How do they 'unlearn' the assumptions of perfect competition? Instructors of Principles and Intermediate Micro traditionally work hard to drill home the message that price equals marginal cost, for example. Given the time constraints faced in Principles and Intermediate courses, those instructors can be forgiven if they focus on inculcating the mantra "markets are good" to the detriment of understanding firms with market power.

We next describe the game, how it is presented in class, typical developments in students' play, and leveraging students experiences into 'teachable moments'. A companion paper (Ruebeck 2010) further explores the relationships between Bain's and others' taxonomies with regard to a problem-based learning activity.

## Introducing and explaining the game

Virtual Corporate Reality (VCR) is a price and location game for students to play over a period of days or weeks outside of class. Students work in teams to set prices for their products, decide on long-term investment in cost-reducing technologies, and introduce, remove, or relocate products. While interacting as teammates and competing with other teams, students how to think about cost functions, profit-making, and-most important-strategic interaction. The instructor combines this "learning by doing" with classroom lecture to help students achieve a deeper understanding of the theory and its use. The math and abstraction of mathematical modeling are less likely to be an impediment to learning. By using VCR, students develop patterns of thinking that allow them to quickly grasp those mathematical and modeling concepts in lecture.

The game is based on Salop's (1979) "circular city", a model of one-dimensional product differentiation. As compared to Hotelling's (1929) "linear city", there are no "edges" to the product space because one end wraps around to the other, allowing a symmetric structure in which student teams begin at parity.

Thus VCR does not ask students to make an initial entry or location decision. Instead, their teams are endowed with a single product at a given location. These initial products are spaced evenly about the circle, reflecting the principle of maximal differentiation (Carlton and Perloff (CH, YEAR); Church and Ware (CH, YEAR); Cabral (CH, YEAR)): when there are no relocation costs firms will choose for all products to be as different from each other as possible so that price competition will be less fierce. For example, with four teams of students the initial products are positioned at locations $0 / 1,0.25,0.5$, and 0.75 around the circle. As with many other aspects of the game, students' experience with maximal differentiation in VCR - both this initially imposed version and their own introduction decisions described later-can lead to a
more productive discussion later in class. There are always some students for whom (like Hotelling) the principle is not immediately obvious; they understand it better through experience.

## No stochastic element

Many of the other games mentioned above incorporate a noise level that obscures what is really going on. There may be parallels between this confusion and what happens in market interactions, but the theory in an industrial organization course is challenging enough without the noise. VCR is designed to help students learn economic concepts, not simulate real-world business environments. The complexity of descriptive realism is put aside to provide a rich enough structure to challenge, but a simple enough structure to learn. Although noise is a part of real life, we eliminate it because it can obscure learning.

Although unexpected developments can be entertaining for students, that entertainment may come with a cost to learning. "Virtual Corporate Reality" (VCR) cuts through the noise to help students understand strategic interaction and oligopoly theory. Like the classic games of Chess and Go, VCR has no stochastic element, although students perceive the uncertainty about other firms' intended decisions as an important part of the game. Thus we can characterize Borenstein (1998) and other games' focus as being on exogenous firm asymmetries, firms discovering the markets in which their advantages lie. VCR instead highlights endogenous asymmetries, those differences in choices that give firms a competitive advantage.

Students playing the game comment on this strategic uncertainty, and learn that taking into account others' actions can prove more useful than trying to guess what's going on in the noise. Students also develop misconceptions about the game (for example, thinking that demand is not uniform around the circle) that would be difficult to unlearn if there were a stochastic component.

## Introducing the game

Before playing the game, students receive a handout (available on the web Although the handout is quite detailed, it describes the mathematics of the game completely. Students may not use all of this information, simply skipping to the summary at the end of the document, but it the lack of a stochastic component is a more credible statement as a result. The first lecture can give them some pointers for reading through the document, in particular introducing them to the idea of the circular city.

Naturally, students will want to understand the grading. Although it could be played without a graded component, we have always included the team's profits as part of the students' grades. Two additional features temper students' concerns. The first is to include a writing component, as described below, that is part of their grade. It also helps to play the game twice, averaging the two outcomes (with writing in both runs) together. What remains, of course, is the manner in which profits are translated to grades.

We find that telling students they are being compared to past experience works well. Of course, this benchmark had to be developed over time, but we now have a fell for the range of outcomes. Setting this bar-but not specifying what it is-appears superior to an endogenously determined profit level because it allows students to focus more on making smart decisions than on figuring out what is "good enough". Another alternative would be available with multiple industries. With at least three industries playing simultaneously, each industry's grade could be
calculated relative to average profit in the other industries. A side-effect of this formulation is that it enhances the incentives for collusion. And because of the game currently has no limit on collusive profits, this side effect may be particularly deleterious. In fact, setting up the game this way is particularly unrealistic.

When following our recommended structure-a non-specific grading scale based on past experience - the key feature of the game to discuss with students before the game starts is that they are not being compared to each other. Although this reward structure loses some realism because investors in markets often reward the industry leader, it teaches lessons that students may not learn elsewhere. Every day the papers identify perceived industry leaders; students are likely to be fully aware of these metrics, and have little need to be taught to think that way. What VCR can provide is a lesson in focusing on their own performance and decisions with less emphasis on hurting others. It is especially useful to emphasize this because there will always be some students that ignore the advice. As described below, teams that focus on direct competition will not do well in the game-and as Michael Porter (1985) and others have shown us, such behavior is also likely to be punished in the marketplace as well. Emphasizing this advice at the beginning of the game also means students will remember it during debriefing discussions weeks later when the game is over. "I told you not to think that decreasing others' profits leads directly to increasing your profits! Wasn't that good advice? Did everyone follow it?" The answers are invariably, "Yes!" and "Unfortunately, no." We will return to this debriefing discussion later.

When initially handing them the description, it is also useful to spend some time explaining strategies for reading it. My advice is to start at the beginning, but not to be afraid to skip to the synopsis at the end and then pick back through the document for more detailed information. Recognizing that the mathematics in the document are intimidating and beyond their training at the beginning of the course, I also spend some time explaining and advocating the Scenario component of the online software. This component of the software is useful to them throughout the experience, but especially important at the beginning because there is no context for pricing, other than the math. I developed the first homework assignment described above after early experience with the game showed that students were not quick to use the Scenario component (which was also less user-friendly at the time). Students need practice with it before the game starts to get comfortable with its interface and to convince them that it can provide useful information. Using the Scenario functionality also simply helps students better understand the nature of differentiated products competition in price and location.

## A preparatory homework assignment

The key feature of the game to discuss with students after the first round will be pricing at, near, or below marginal cost. The first time students play the game, we can count on at least one team to choose a price at which positive profits are untenable-our first chance to satisfy Bain's (2004) requirement to challenge students' existing mental models. We will return to highlight topics that can be covered after that first period, but improving student's ability to learn from this first round of play will be enhanced by a homework assignment. The assignment is due before the first period of play, and it is discussed after students see the results of the first round.

To complete the assignment, students graph the cost curve described mathematically in the handout and shown here in Figure 0. They also spend some time running both prescribed and open-ended analyses with the Scenario tool online (see the discussion of Figure 6). So the order of events at the beginning of the semester is as follows.

Lecture A: Hand out the explanatory packet and assign homework with exercises on the simulator and graphing the average cost curve. Have a brief discussion of the game, focusing on how to read the document, cheating, and grading. Ask for questions.
Lecture B: Collect the homework at the following lecture, and ask for questions. Remind students of the game's structure and the initial period's coming deadline.
Outside class: Students enter decisions for the first period.
Lecture C: Hand back homework. Discuss the cost curve and pricing above marginal cost. (See the discussion in the section on initial play.)

Several goals are accomplished by this series of events. Pedagogically, students' brains are 'primed' with a context for the experiences they are about to have. They also are more likely to understand that cost curves can be a useful tool. Mechanically, it gets students 'over the hump' of learning how to use the simulator and imagining how to use it in the future. Combined with the results of the game's first round, this combination of a homework assignment, decision making, and discussion can make classroom participation more productive.

Although students might not ask questions due to the usual classroom dynamics-not wanting to look either stupid or like a know-it-all-students importantly have the strategic incentive to learn it for themselves rather than revealing useful ideas to other student teams. This is part of the reason for including the writing component we discuss below. It is also somewhat mixed with the students' uncertainty about what questions they are "allowed" to ask without being accused of cheating.

## Cheating

It is important spend some time on the first day emphasizing that explicit collusion is equivalent to cheating. At this point in their career as students of economics, they certainly have heard about the benefits of collusion and reasons why it is difficult to sustain, and they have probably been involved in classroom demonstrations that let them try to collude. Thus it is important to be clear that collusion is not the point of the game, just as it is not the point of commerce in the U.S. economy. Describing the various types of laws and resulting fines for collusion and market manipulation brings home the consequences of breaking anti-trust laws. By emphasizing that colluding in this class is equivalent to cheating, and pointing out the dire consequences to cheating as described in the syllabus (perhaps a failing grade and associated proceedings supervised by a Dean of the College), it is also placed in the context of the academic experience in which they are currently immersed.

This may lead to students' lack of questions for fear that they would be accused of trying to collude. More subtly, that discussion may emphasize to students that this is their firm, there are advantages to their private information, and a better understanding of the game will give them the upper hand. As with any classroom activity, it is important to address students' hesitance with a discussion that draws them out. This is part of the reason for assigning the homework above: it helps them develop enough familiarity with the game to feel that their questions are valid, relevant, or simply not "stupid". Allocating time for discussion in multiple classes-after describing the game, after handing in the homework, and after the first period's results are known-is also important because students may reach a sufficient comfort level at different
times or due to different information. It is also useful to have the results posted online so that students can review them at their own pace.

By including a writing assignment, students still process this information, with critiques from both the instructor and from their teammates.

## Numbers and words

Although this is a game about numbers (prices, quantitative locations, investments, profits) and the mathematics (game theory) that go with them, our students also enhance their learning by writing about the experience of making decisions and watching others make decisions. The writing should not be onerous, but it should be regular. It is convenient to use a class management electronic bulletin board (what some class management systems call a "forum"). Students can take turns explaining the most recent period's decision in a paragraph, so that typically each student explains one third of the decisions, and all students review the games' results in a short paper at the end of the game. An example is in Appendix X. Students have commented in their course evaluations that these reflections increase their ability to learn from the game. Not only can the instructor learn about their motivations (reflected in some of the discussion below), but the students comments provide fodder for classroom discussion, a way to selectively and confidentially share some of the private information either during the game play or at its end. Examples will be evident in the discussion below.

## Playing the game

Students use a web-based interface to make their decisions and then to view others' decisions and the results. The game starts in a simple state of perfect information with a simultaneous choice of price. From that point on, students have access to a mixture of public and private information about others' decisions. Each student on each team has a login and a password that allows him to enter decisions for the firm, analyze the options, and view results (Figure 1). The typical chronological time between periods that we have used is three or four days, two decisions each week.

Figure 1 about here
All team members have equal access to the game; it is immaterial which account is used to make changes. The main screen (Figure 2) provides a concise summary of the student's team and the game. Each screen's pull-down menu provides navigation to the other parts of the interface. Figure 3 shows the logout screen and the pull-down menu.

Figure 2 about here
Figure 3 about here
There are four categories of information that students have at their disposal: current decisions, hypothetical scenarios, public history, and private history. Students begin with no history other than their endowment of cash $(\$ 25,000,000)$ and a single product, each one located
evenly about the unit circle. Both the Current Decisions page (Figure 4) and the Scenarios (Figure 6) pages have the advantage that students see all the products currently available, ordered on the $[0,1)$ interval. Thus they can easily compute the distance between their products and others' products, they can easily compare prices of nearby products, and they can see their own investment and resulting marginal cost for each product.

Figure 4 about here
Students can directly enter price and new investment, the two actions they are most likely to perform each period, on the Current Decisions page. It also display's the team's current cash flow, a reminder of the decisions' timing, and restrictions on cash flow. The game does not support borrowing, so negative cash balances are not allowed. Students can also use the screen in Figure 4 to scrap, move, and auction off existing products, but these actions are seldom used-only by one or two teams in a typical game's time span.

The subscreen for new product introductions shown in Figure 5, is accessed with the 'Manage Introductions' button in Figure 4. Introductions occur frequently during the earlier rounds. As the text at the bottom of Figure 4 reminds the user, these introductions (and likewise the less-frequent scrap, move, and auction decisions) are announcements during the upcoming period. Because the action does not actually take place until the next period, the other teams can set prices that take these new product space developments into account.

All of the information entered in the screens shown in Figures 4 and 5 can be revised before the end of the current period with no effect on game play: prices can be changed, introductions can be removed, etc.

Figure 5 about here
The 'Scenarios' button in the upper left of the Current Decisions Screen displays the screen shown in Figure 5. Students use the Scenarios screen as a test bed changes in their product(s) and to anticipate the effects that other teams' changes will have. The interface allows them to include guesses about the investment that other teams have made in their products. It also calculates the resulting profits for all the firms even though in actual game play the profits are private information. A scenario defaults to the current product mix but can easily be modified to reflect anticipated introductions, withdrawals, and other changes.

Figure 6 about here
The displays of the game's history are divided into private and public information. An example of one team's private information is shown in Figure 7: it includes product investment and the resulting costs and profits. The resulting demand for each product (Figure 8) and each team's aggregate profits and cash balance (Figure 9) are public information. Teams cannot "back out" the investment information from changes in rivals' cash balances as long as those teams have made investments in multiple products in any given period. It is likewise difficult to infer these investments from profits (and the results of Scenario runs) if a team has multiple products because profits are not broken out by product. Teams almost always choose to have more than one product. All of these history screens can display either a single period (as shown in Figures 7 and 8) or the entire history (as shown in Figure 9).

Figure 7 about here
Figure 8 about here
Figure 9 about here

## Technology choice and its impact

The browser features are provided by Ajax in the Scenarios screen, and HTML for the other screens; both are generated with PHP. The advantage of Ajax is a more responsive and flexible form, but the drawback is that the data is not easily exported to a spreadsheet program. The ability to save other screens' HTML tables makes this less important; students can use a spreadsheet to create their own in-depth analyses inspired by their investigations with the Ajax tool, as discussed below.

## Students' use of the information

Because the display of the game's current state and history is segmented into separate screens and according to its status as private and public information, it is useful to let students know that they can get the "big picture" of the game by saving the web page (HTML, TXT) and opening it in a Microsoft Excel spreadsheet, or using copy-and-paste from the web page to Excel. Students are not required to turn in such quantitative analyses, but some are motivated to creatively design their own analysis methods rather than passively accepting the available basic information. Even if advanced displays that organize the information graphically, for example, were available, students might learn more if those were withheld until the second run of the game-after students have been pulled into an analysis mode of their own choosing. By pointing out to students that the data may be easily copied and pasted to a spreadsheet, some students are encouraged to take their analysis further than the Scenario interface can. Even students that simply use the Scenarios will get a feel for the advantage that data analysis can give them.

It is also apparent that students mull over much of the information even when not directly analyzing the data; they discuss it with each other and reflect on it between decision due dates. Course evaluations and stories told by the library staff and others around campus reveal that students are arguing and talking with each other about the game throughout the week.

## How students learn though VCR

The primary goal of VCR is to put the classroom's topics into an experiential setting. Although the focus of the game is on strategic interaction, ideas from earlier classes are also reinforcedcost functions and profit-maximizing decisions, for example. The repetition is pedagogically useful on its own, as well as providing a new context for those ideas-for example, considering firms' choice of price instead of quantity. Students also learn about the inter-connectedness of their decisions.

Two common mistakes arise often in students' play of the game. The first is pricing near, at, or even below marginal cost; the second is choosing new product introductions that are very close to or co-located with existing products. Although we could certainly construct equilibria in which these behaviors are rationalized, we characterize them as mistakes for a several reasons. Students typically reverse these behaviors over the course of the game, they don't use them the second time they play the game, they yield low long-run profits when used, and typically only one or two of the semester's six to eight teams use them. Undoubtedly the students that pursue these strategies begin by believing that they are rational-or at least smart-but those strategies have been shown to be suboptimal in the outcome of the game (with one possible exception described below). These experiences are useful pedagogically throughout the semester, from discussions of perfect competition, through market power and predatory pricing.

We describe students' experience with these topics and other strategic concepts below. The data are from outcomes over six semesters playing VCR. In each of these semesters there were two industries. In one semester there were five teams in both industries, in one semester one industry had two teams, and all other outcomes have four teams in each industry. In all but the first two semesters, students played the game a second time. In addition to these six semesters it was used in an industrial organization class at Lafayette College, it has also been used there as part of a course on simulation of economic behavior (in which two participants were playing a second time), and a colleague has used it in a corporate finance class.

Initial pricing ... where to start?
Students can be very uncertain of where to start the game. It's true that the participants in this abstract market have none of the conventional contexts that firms typically have for pricing: there are no markets for similar products in which we can observe prices that buyers are paying. Students do know cost, and they do have a simulator-something we could call 'market research'. Most students jump right in to try out prices using the simulator, and those students enjoy this new paradigm. Even accomplished students can feel it's too far out of their 'comfort zone' with too little information available; they may be accustomed to cut-and-dry assignments in previous classes. Thus VCR can immediately be a pedagogical device that asks students to think in a new way about their decision making.

## Initial pricing ... and marginal cost

The economics ideas in VCR start with something they've seen before, at least once (Intermediate Micro), if not twice (Principles, too): a graph of cost curves (Figure 0), prices, and the resulting profit rectangle. Generally one of the teams makes the mistake of pricing at marginal cost-or even below marginal cost-focusing on either the perfectly competitive result (when price is equal to marginal revenue) or on revenue maximization, or some combination of those ideas and the strategy of "when other teams do poorly our team does better." Pricing below marginal cost is consistent with a predatory strategy or a price war to establish collusion, and it is useful to refer in those lectures, later in the semester, to those early failed attempts.

Of course, with a declining average cost curve there is no quantity at which $p=M C$ is profitable. For some students it's enough to think about it, others are able to grasp it after the scenarios played in the homework assignment, and still others need to "learn from the school of hard knocks". For all of them the reinforcement in lecture will help them retain the concept.

Even when pricing at one dollar above marginal cost, breaking even requires demand far greater than market demand. Given that each firm's total cost function is $C(q)=500,000+50 q$ at the beginning of the game, an average cost of $\$ 51 /$ unit is reached when output is 500,000 . Given that market demand is $-110,000+120,000 n-1,500 n^{2}$, when there are four firms $(n=4)$ total market demand is only 346,000 . Even if the firm captures that entire market of 346,000 units, average cost has only fallen to $44.51 \notin /$ unit above the $\$ 51 /$ unit price and a total loss of $\$ 154,000$.

In the first four runs (2001-4) there has always been a firm that prices at 55 or less. Pricing at 50 occurred in both of those years, and there was even a price set at 20 in one case. One reason for students to focus on pricing at marginal cost is because marginal cost is the one solid number for them to grab hold on amongst the handout's equations. Not all students focus on marginal cost, only a minority do-providing evidence that students are in general using the Scenarios functionality. It's rather simple to recognize that $p=M C$ is not optimal: just enter that price for all firms in a scenario and then raise price for one of the firms to see that the best response. Evidently there are some students who fail to perform this exercise, either because they don't care, they don't understand how to use the Scenario software even after the homework assignment that gives them practice using scenarios, or are they so enraptured by their training with the model of perfect competition that they feel they don't need it.

Pricing at marginal cost, though, is useful pedagogically, and there's no question that these students that try it are surprised when it doesn't work. It's an excellent moment of epiphany for those that do it and an exemplar of peril for others. The timing of discussing marginal cost pricing-at the beginning of the semester-is also appropriate for models presented at that time: 1) the review of perfect competition and its extensions such as near-infinite elasticity of demand facing a single firm and 2) the monopolist's optimal choice where marginal revenue (not price) is equal to marginal cost. An industrial organization course is likely the students' first class where the focus is on firms with market power, and the initial period of VCR's play can help reinforce that focus.

## The mistaken principle of minimum differentiation

Every semester also has an instance of firms introducing a new product very close to a rival's existing product location. The modal location for these wars of attrition is at the "top" of the unit circle, where 0 and 1 meet. There are four cases near this location, three additional cases at 0.99 , and one at 0.1 . The other examples are introductions at 0.26 (within 0.01 of 0.25 ), at 0.48 (within 0.02 of 0.5 ), and at 0.63 (within 0.005 of 0.625 ). The extent to which a product is 'too close' obviously depends on the parameters of the model, in particular in this model: the parameter determining the disutility of distance from a consumer's ideal product characteristic. Regardless of the determinants of the model that determine when one model is too close to another, these outcomes occur regularly when playing the game.

There are many lessons for the instructor to emphasize after these entries occur, usually in the first five periods of the game. The first two lessons apply to game theory in general: players' payoffs depend on each other's decisions, and players are assumed to be rational. Students usually discuss the other teams' rationality at this point, especially those students that assumed others would act rationally. The interrelated payoffs in this game can also cause students can also have reservations about this project's contribution to their grade; we return to grading below.

Students' memories of these experiences also useful for topics covered later in the semester-predatory pricing, for example. It becomes much easier to convince students that predatory pricing is hard to justify theoretically when they can reflect on how poorly it paid off in VCR. We continue to discuss these long-lasting experiences in the sub-sections below.

## The value of a product

When students decide locate products exactly or nearly at the the same location as existing products, it is almost always the infringing product, not the original product, that is withdrawn, relocated, or sold, but in a few cases the two products coexisted. The reasoning behind this probably has to do with which product has lower costs, a lesson that we will return to below. The ultimate disposition of the product whose team 'blinks' is almost always that it is withdrawn.

When they decide to remove a product that's not very close to other products, students often try to sell the product to other teams, especially when it's not as close as some of the examples mentioned in the previous section. Students are surprised to learn that others don't value the product any more than they do; products offered for sale are seldom bought. This behavior is rational on the part of potential buyers, but the expectation of the team getting rid of the product is easier to characterize as 'hopeful' rather than rational. While discussing it in their written reports, students seldom take steps to calculate the value of the product. Of course, the students likely to introduce a product that later needs to be withdrawn are also less likely to be those students that take a quantitative approach to the game.

Some of the best students take the material further because of VCR, analyzing the decision of investment, calculating the present value of investment and comparing it to the cost. The pitfall that they experience is assuming a static response (assuming no response) of others, or of ignoring the utility of a cost advantage, and thus feeling that no investment is useful. Most students use this analysis in conjunction with consistent conjectures about other students' behavior. The spreadsheets that they produce-without being required to do so-to calculate the net present values of investment are testament to the motivation and engagement this simple game provides.

## Industrial organization and strategic interaction

After some of the students make those early mistakes and the class learns from them, there remain many topics to be leveraged by experience with the game. Students have a hard time with best-response in class, perhaps because the graphical development moves from a single firm's $p-q$ graph to the both firms' best-response curves on a $q_{1}-q_{2}$ graph, and the best response axes are both independent and dependent variables. In any case, they grasp this material faster when they experience it in action before having it presented in class; students are more receptive to the idea of an optimal response. The are also more likely to understand the iteration involved in strategic interaction because they have not only tried to respond optimally, they have tried to anticipate others' actions, and they have tried to predict others' anticipations.

## Advanced topics in game theory

Surprisingly, the mistake of locating at or near an existing product continues to be useful as class progresses. Neither one of the teams involved in this war of attrition is likely to give up quickly, highlighting its costliness. Although the predatory team students didn't entirely think through the consequences, it also adds to the discussion of firms' forward-looking behavior when making decisions. Of course, this experience also brings home the irrationality of predatory pricing, as discussed above.

Although Baysian-Nash equilibria are not typically covered in undergraduate I.O., the experience of co-located firms highlights the importance of beliefs. As teams try to decide whether to exit or not, the team members typically observe (in their private, written descriptions of reasoning) that their firm will win if it's product has lower production cost than their own firm's product does. Of course, it's the belief of lower cost that matters, but it is useful to share with the students that the firm that typically 'wins' the war of attrition is the one that actually has lower costs!

## Lessons about the profit function

The less-frequent direction that advanced students' investigations take is into the nature of the profit function and even the nature of Nash equilibrium. The first case, the nature of the profit function, was motivated by my incentive for them to find the Nash equilibrium: extra points for all if they get within $\$ 1$ of the Nash equilibrium in the final three rounds. In analyzing this reaction, we should recognize that production cost is private information (although there are cases in which it can be backed out because spending can be inferred, with noise, by observing changes in cash balances). One student noticed that the profit function has a very gentle slope, so the improvements in profits are very small when changing price. He also argued that students are afraid of the downside they face when other teams decrease their prices below Nash-a clear connection to the loss aversion of Kahneman and Tversky's asymmetric utility function.

## Returning to the simplicity of the game

There are many ways for students to fall into a trap of feeling that the game is more complicated than it is. For example, they think that there are more consumers in a given part of the circle just because there are higher prices there or because there is more demand for the products that are there ... while they have failed to notice that the products are also spaced further apart, or even though two products are close together their neighbors on the other side are further away than at other locations on the circle.

These experiences help reinforce to students that finding simple representations is valuableperhaps because the world is simpler than we like to suppose it is! After getting caught up in more complex explanations for the way things are going in VCR, they can see the utility of Occam's Razor.

## Grades

Because students observe that one firm's mistake can lead to their own low payoffs, they can be concerned about grading. In particular, having another firm attempt to drive their own product out can make students pretty mad. It is important to emphasize with them-usually in class
discussion but if necessary also more privately outside of class-that they also made mistakes. Invariably they see that it was not only the other team's decisions that hurt their profit but their own as well. Addressing students concerns about the effect on their grades here can be accomplished through the following implementation features: a grading system that does not punish them too heavily for lower profits, including a writing component in the grading, and playing the game twice so that the students have a chance to average out an poor grade in the first run when all of them were less savvy to the consequences of their actions.

Students may complain about interrelated payoffs given that the exercise is part of their grades, but these concerns can be addressed by discussing the manner in which they are making their decisions: although entry hurts the poached team, it usually hurts the poachers moreespecially if the entry occurs at one of the initially endowed product locations. It is also useful to return to this discussion after grades are assigned so that students can see they were generally fair. These situations are also ones in which team members' anonymity is helpful.

## VCR can appeal to a variety of student learning styles

It may be appealing to focus on better students' outcomes and experiences, but we should consider the benefits that accrue to all types of students. We can also recognize the ways in which classroom or extra-classroom activities both add to the experience and potentially detract from our goals-although we may not be reaching those goals, anyway.

## Some learning styles and how they respond to VCR

Consider four kinds of students: those that care and care about grades; those that think carefully and ask insightful questions but don't care so much about grades; those that have a harder time with the abstraction and also have a harder time engaging the classroom; those that have disengaged from class. Risking generalities, take these as the four types of students.

We have already discussed benefits for the first two types of students. The first type is naturally stretched by this activity as they are by the class itself and by traditional homework, but this activity lets them be more creative. Why creative? Part of answering traditional homework is finding a way to figure out the designer's expected answer, but an activity like VCR allows students to creatively step outside that box.

The second type of student is very capable but either does not study as seriously for exams or have innate difficulties taking exams. In other words, there are students that don't get As, that get $\mathrm{B}+$ grades or maybe A - grades. They that are as strong as the top one or two students in their ability to think about the material-students that ask great questions in class but don't hammer the exams the way some of the quieter types do. These students are those that might respond most significantly to the game precisely because it looks more like 'reality' to them, it isn't the same old classroom grind. These are the students with the most practical view of their time in school, the ones that see diminishing marginal returns to getting that solid A. Whether their extra-curricular activities are on campus or in nascent business development, they don't feel as much need to be connected to the grades. VCR thus gives additional benefit to get over the opportunity cost they perceive in getting the best grades.

The third and fourth types of students gain as well. Their excitement about class increases, too. The third type's understanding does improve even if they may not be likely to transfer that
excitement to the course material when we're not talking about VCR itself, but when the subject comes up they're animated, demonstrative, and more likely to burst out with contributions to class. Those contributions may be provocative, but they're engagement.

Finally, consider the students that are less engaged with the class or students that just have a hard time with the class.

## Competing with video games?

Students are simply happy to be doing something outside of class, doing something that doesn't feel like studying. Yet this may also appear inappropriate to faculty because there is less focus on disciplinary knowledge. By spending time on such extra-classroom activities, students may also be missing out on the exciting, well-researched findings that we understand so well and are so well-prepared to teach them.

So students sometimes play just to play-actually a potential problem with immersive games. Students can be ignorant of the insights that they could have learned while playing the game. There are several ways to address these related problems. One is "baked in" to the design of the game: its simplicity. We're more likely to be able to pull apart the tangled strings of causality when there is no random element and there are only a few drivers of demand, prices, and profits. Other games achieve more "surprise" elements, more "action", evolution over time that is more "interesting"-but only surprising, active, and interesting because they can't be explained. It is unfortunate when students take away little more than the excitement of battle, unenlightening surprises, and something that they get to do other than studying the concepts that they can learn from the class. In those other games we're doing little more than helping them get out of the hard work of changing the way they look at the world.

And that's really it, isn't it? As Bain (2004) learned from observation and reading the literature, students don't really learn what we teach them unless they understand how it changes the way they view the world. We know they'll remember that they had fun with the games they played outside of class, but what we really want is for that excitement to translate into a changed view of the world.

## Conclusion

This game provides several benefits, not the least of which is generating excitement and enhancing students' mindshare between classes. There is variation in how VCR reaches students just as there is variation in how lecture and homework reaches students, but any negative comments we have received are minor compared to the general message that the game engages their attention and their learning centers. Students come to class prepared for the concepts we are to teach, with an intuition about best replies and the tradeoffs between minimal and maximal differentiation, for example.

When assigned to work together as a group, students learn about the dynamics that are part of group decision making, and they learn through discussion with each other. By assigning written follow-up to their decisions and the VCR experience as a whole, an additional conversation with themselves and with the professor increases the likelihood that their experiences will translate to application in the class and further learning.

In future work we plan to assess students' learning with VCR, as well as test hypotheses about how students learn using shocks to demand and cost and other aspects of the game. We will also use self-assessment methods to learn about how different student learning styles are affected by VCR. A companion paper also discusses VCR in relationship to Bloom's Taxonomy (Ruebeck 2010).

A final question remains: how have we affected our students' play by publishing this paper? If they read it before or while they play, will they have learned more or less? Once again we are left with the reality that different students learn in different ways and to different levels; we can use that to our advantage-and to each student's advantage.

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## Appendix $\mathbf{X}$

## ECON 331 - Industrial Organization

VCR Reports

These reports will make up $1 / 3$ of your VCR grade. So $5 \%$ of your final grade depends on them. This portion of the grade will be assigned separately to each group member. Consider it a "participation" grade. I want you to show that you were thinking and tell me, clearly and concisely, what you were thinking about-regardless of whether you "got it right" or not.
1.Reasoning for each period

Each period, one person in your group should write about two or three paragraphs describing the reasons for your group's decisions. Use the Group discussion board to enter your explanations. Describe the issues that entered your decisions: those that relate to your firm, particular products and/or particular firms, and the industry as a whole.

- During the first run of the game there will be 12 periods, so each team member should provide the reasoning for 4 periods. During the second run of the game there will be 9 periods, so each team member is responsible for describing 3 periods' reasoning.
2.Synopsis for the entire game (a total of two, one for each run of the game)

I would like every member of the team to turn in a report at the end of each run of the game, summarizing the team's decisions for that game and what that team member as an individual learned from the game. In about two to three pages, tell me what you learned and why you think that things turned out the way that they did.

## VCR firm value

The value of your firm at the end of the semester (see the VCR description for details) will determine $2 / 3$ of your VCR grade, which is $10 \%$ of your final grade. This part of your grade will be the same for all members of your group. This is a performance grade; it depends on how well your team builds the firm's value.


Figure 0: The cost curves, and a useful example.

## Welcome to Virtual Corporate Reality, it is Game Period 8



Please enter your username and password below.
USERNAME: Adamsmith PASSWORD: $\cdots \cdots$ Login

WARNING: Using an account other than your own in any way is considered cheating. After logging in, you will have 30 minutes before you will have to login again.

Please log off when you are finished to securely end your session.

Figure 1: The login screen.

## Welcome, AdamSmith, to Virtual Corporate Reality!

## Successfully authenticated!

The V.C.R. database server is up and running. You are on team 2. It is period 8.
Announcements this period (for older announcements, see message board):
No announcements were made.
Remember to check the Auctions, especially if your team is involved!
Navigator Menu: Select a destination... $\dagger$

Logout by clicking here
IMPORTANT NOTICE:

For security purposes, if you are still working after 30 minutes you will have to log on again and you will lose unsubmitted changes. Also, it is strongly recommended that you LOG OFF (from navigator menu) when finished.

Figure 2: Welcome screen.

# Virtual Corporate Reality 

## Confirm Logoff

## The current game period is: 8

If you are sure all your decisions have been submitted, click the button below to log off.
Otherwise, use the menu to navigate to the Current Decisions screen to verify all decisions are submitted.


Figure 3: The logout screen and Navigator Menu (common to other screens).

Current Screen: Current Decisions
Navigator Menu: Select a destination... $\hat{\boldsymbol{v}}$
Scenario
Current Decisions for Period: 8, Team: 2, Industry 1

| Location Pricing \& Investment |  |  |  |  |  | Other Actions* |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Location | Team | Product Price | Marginal Cost | Existing Investment | New Investment | Scrap | Move | Auction | None |
| 0 | 1 | - | - | - | - | - | - | - | - |
| 0.1665 | 3 | - | - | - | - | - | - | - | - |
| 0.333333 | 2 | \$72.5 | \$47.1698 | \$3000000 | \$0 | $\bigcirc$ | O | $\bigcirc$ | $\bigcirc$ |
| 0.495 | 1 | - | - | - | - | - | - | - | - |
| 0.666667 | 3 | - | - | - | - | - | - | - | - |
| 0.75 | 1 | - | - | - | - | - | - | - | - |
| 0.833345 | 2 | \$ 65 | \$49.0196 | \$1000000 | \$0 | 0 | O- | 0 | $\odot$ |

Submit Decisions Remember to submit decisions above BEFORE leaving this screen.
Manage Introductions
Scheduled Introductions

| Location | Initial Investment |
| :---: | :---: |
| 0.2 | $\$ 1000000$ |

CASH FLOW:
Cash balance at end of previous period:
$\$ 59910999$
Cash charges due to announcements and investment: $\$ 6000000$
Cash balance after costs due to stated announcements and new investment: \$53910999
*Notes about changes other than price and investment:

- You need to choose a price for the current location (in the first column).
- Changes in location are only announcements of your decision.
- You do not need to price the items marked with an asterisk until next period.

Figure 4: Screen for entering decisions about the current period.

## Introductions Management

Cash balance will be affected THIS period, new location comes into play NEXT period.
Location (a decimal between 0 \& 1, inclusively): 0.8
Initial Investment (0 if left blank): $\$ 2000000$

Click ONCE to add location introduction
Note: You MUST click the above button to make changes!
Here is a list of your submitted introductions for period 8:

| Location | Initial Investment | Delete? |
| :---: | :---: | :---: |
| 0.2 | $\$ 1000000$ | $\square$ |

Delete selected introductions
To change location and/or to modify investment to an introduction, delete the introduction in question and reintroduce with the correct location and/or correct investment

Click here to return to Current Decisions without taking any action

Figure 5: The product introduction interface.


Figure 6: Investigating hypothetical scenarios

Current Screen: Private History
Navigator Menu: Select a destination... $\Rightarrow$
Select Period: Period 7 :

| Period | Location | Price | Demand | Revenue | Existing <br> Investment | Marginal <br> Cost | Total Variable <br> Expense | Profit* |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 0.333333 | $\$ 84$ | 75005 | $\$ 6300420$ | 3000000 | $\$ 47.1698$ | $\$ 3537970.849$ | $\$ 2262449$ |
| 7 | 0.833345 | $\$ 68$ | 68933 | $\$ 4687444$ | 0 | $\$ 50$ | $\$ 3446650$ | $\$ 740794$ |

*Note: The calculation of profit includes only total production cost (total fixed cost and total variable cost) associated with locations. New investment and other decisions (scrapping, moving, etc.) are not included in the calculation. These items are reflected in the cash balance.

Figure 7: Privately available decisions history, for the most recent period.

Current Screen: Public History
Navigator Menu: Select a destination... :

Select Period: Period 7 :

| Period | Location | Owner | Price | Demand | Scrapped? | Moved to | Auctioned? |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 7 | 0 | 1 | $\$ 68$ | 129053 | No | No | No |
| 7 | 0.333333 | 2 | $\$ 84$ | 75005 | No | No | No |
| 7 | 0.666667 | 3 | $\$ 74$ | 50879 | No | No | No |
| 7 | 0.1665 | 3 | $\$ 80$ | 96287 | No | No | No |
| 7 | 0.833345 | 2 | $\$ 68$ | 68933 | No | No | No |
| 7 | 0.75 | 1 | $\$ 60$ | 90820 | No | No | No |
| 7 | 0.495 | 1 | $\$ 68$ | 145524 | No | No | No |

Figure 8: Publicly available decisions history, for the most recent period.

## Current Screen: Financial History

Navigator Menu: Select a destination...

Select Period: All Periods $\hat{*}$

| Period | Team | Profit* | Cash Balance |
| :---: | :---: | :---: | :---: |
| 7 | 1 | \$4350586 | \$45520155 |
| 7 | 2 | \$3003243 | \$59910999 |
| 7 | 3 | \$3109706 | \$65383243 |
| 6 | 1 | \$1541906 | \$39209113 |
| 6 | 2 | \$4324038 | \$55150244 |
| 6 | 3 | \$4017902 | \$59308130 |
| 5 | 1 | \$1843900 | \$40635435 |
| 5 | 2 | \$4370262 | \$48405910 |
| 5 | 3 | \$4091573 | \$52657360 |
| 4 | 1 | \$1931834 | \$36944319 |
| 4 | 2 | \$4104300 | \$41938712 |
| 4 | 3 | \$4174973 | \$46253130 |
| 3 | 1 | \$1709434 | \$33345224 |
| 3 | 2 | \$4012652 | \$38889916 |
| 3 | 3 | \$4316753 | \$40074435 |
| 2 | 1 | \$1074144 | \$30129324 |
| 2 | 2 | \$4745650 | \$33216442 |
| 2 | 3 | \$5337083 | \$34054935 |
| 1 | 1 | \$6421600 | \$27671600 |
| 1 | 2 | \$5865040 | \$27115040 |
| 1 | 3 | \$6100335 | \$27350335 |

*Note: The calculation of profit includes only fixed and marginal expenses associated with locations. New investment and other decisions (scrapping, moving, etc.) are not included in the calculation. These items are reflected in the cash balance.

Figure 9: Publicly available financial history, chosen to display all past periods.

# Virtual Corporate Reality (VCR) Project 

Econ 331 - Industrial Organization

Business is a good game - lots of competition and a minimum of rules. You keep score with money. Atari founder Nolan Bushnell

An economist is someone who sees something happen [in reality] and wonders whether it would work in theory. Ronald Reagan

You will participate in a game this semester as a member of a team and your team will be charged with running a firm in an abstract market. Twice a week, your firm will make decisions on the pricing of its product(s), product introductions, product withdrawals, product relocations, and cost-reducing investment. As a result of the decisions made by your team and the other teams that produce in the same industry, demand for the products will generate revenues and incur costs of production. Your goal will be to maximize the value of your firm.

To better understand the abstract industry in which you compete, we need to specify the nature of demand, costs, entry \& exit, some financial details, and communication. A word at the outset: the model of this market is described in detail conceptually and mathematically in the next few sections. The "scenarios" area of the web site (described in Section 5) helps you get a better intuitive feel for the market by performing "what if" scenarios.

## 1 Demand

First, think of yourself as a customer in this market. The customers that buy this product only care about one product feature. Some examples are a cereal's sweetness, Chinese food's spiciness, the tightness in fit of a pair of jeans, or the location of a gas station. We can talk about customers' "location in product space" by, in the Chinese food example, assigning 0 to very bland food and 1 to the most spicy food imaginable. A customer "located at 0.45 " means that he prefers his food to have spiciness level 0.45 . In our simple model, there is no other way in which customers differentiate Chinese food.

We will also assume that customers are "evenly distributed" among these values so that, continuing the example, if there are 15 people that like foods of spiciness levels from 0.2 to 0.3 then there must also be 15 people that like spiciness in the range 0.8 to 0.9 (or
any other interval of length 0.1 ), and the 15 people in each of these two ranges are evenly distributed in their ranges.

Customers prefer one particular type of product: a customer located at $x$ thus would most prefer a product also located $x$. But if there is no product where the customer is, the customer will consider buying another product not too far away. The further away the product is, though, the less the customer will be willing to pay, as detailed below. Note that "further away" means "less similar," so it makes sense that the consumer will be willing to pay less for the product. Also, moving left and right are equivalent; all that matters (other than the products' prices) is how far from the customer's location the alternative products are.

For technical reasons, we will assume that the points 0 and 1 are located right on top of each other. ${ }^{1}$ Thus it's a circle - not a line - model of demand. See Figure 1, and remember that there is only one dimension to product space, so you can't get to the interior of the circle. You can only move around the circle. It might be helpful to think of this as modeling demand for gas stations located on a beltway around a city like Washington, D.C. (But don't take for granted that the prices you set should match those that you see at gas stations in D.C. And ignore the negative externality created for a consumer from having a gasoline station located right around the corner from his home.)

Note that the locations 0 and 1 occupy the same point. If two products at $w$ and $y$ have the same price, then a customer at $x$ prefers the one located at $w$ because it is closer to $x: d(w, x)=1.01-0.9=0.11$ is smaller than $d(y, x)=0.2-0.01=0.19$.
If the prices of $w$ and $y$ are not the same, then the customer's choice takes this into account. In this example, if the price of $y$ is enough lower than the price of $w$, then the customer at $x$ will instead prefer $y$.


Figure 1: Some points in the circle model of demand.
Figure 1 considers a specific example in the circular model with a customer, located at $x=0.01$, comparing two identically priced products, located at $w=0.99$ and $y=0.20$.

[^0]If the two products are the same price, the customer at $x$ prefers $w$ because it is closer ( 0.11 units away from $x$ ) than $y$ ( 0.19 units away from $x$ ).

How does the consumer take price into account? Read on.

### 1.1 Consumers' utility and their consumer surplus

Consumers place value $V$ on the good that is exactly what they want. If someone located at $x$ buys a product located at $w$ and pays price $p_{w}$, she receives consumer surplus

$$
\begin{equation*}
V-200 d(w, x)-p_{w} \tag{1}
\end{equation*}
$$

The net benefit (utility) that the consumer receives from the product is equal to $V$ $200 d(w, x)$, where the function $d(w, x)$ is the distance between the product's location, $w$, and the consumer's location, $x$. The negative sign ensures that the value the customer receives decreases as the distance to the product located at $w$ grows. The consumer's surplus from purchasing a product is the net benefit she receives minus the price she pays. (Hopefully that sounds familiar from Principles and Micro!) Price is the easy part: the lower the better. The consumer's net benefit (utility), though, depends on how close the product is to her location (her preferred product). The distance function $d(w, x)$ is always a positive number, so it doesn't matter to which side the product is, just how far. The formula for distance, $d(w, x)$, is simple if the shortest way to get from $x$ to $w$ doesn't pass over the 0 and 1 point: it's just $|w-x|$. If the shortest path around the circle from the consumer to the product crosses the 0 and 1 point, add 1 to the smaller of the two locations and subtract the larger. (See the caption to Figure 1 for examples.) $V$ is the same for every product. ${ }^{2}$

You can also think of $d(w, x)$ as the "utility cost" to the customer of not buying her most ideal product. In order to get the benefit $V$ from a product, the consumer pays a price and incurs a utility cost due to the product not being exactly what she prefers.

### 1.2 A consumer's decision between two products

Consumers only buy one product each period. If a consumer's location is between two products, how does he decide which one he prefers? He buys the one that gives him the most consumer surplus. Thus we can see from equation (1) that the decision depends on both the distance from the consumer to each product and on the price of each product. There are three possible cases. If a customer located at $x$ is comparing a product located at $w$ with price $p_{w}$ to a product located at $y$ with price $p_{y}$, then

[^1]he prefers $w$ if $V-200 d(w, x)-p_{w}>V-200 d(y, x)-p_{y}$;
he prefers $y$ if $V-200 d(w, x)-p_{w}<V-200 d(y, x)-p_{y}$;
and he is indifferent between $w$ and $y$ if
$$
V-200 d(w, x)-p_{w}=V-200 d(y, x)-p_{y}
$$

Rearranging this last case (indifference between $w$ and $y$ ) gives

$$
\begin{equation*}
200 d(w, x)+p_{w}=200 d(y, x)+p_{y} . \tag{2}
\end{equation*}
$$

Thinking of $200 d(w, x)+p_{w}$ as combining both the price of $w$ and the utility cost of buying a less than ideal product, these equations say that the customer located at $x$ buys the product which has the lowest of these combined costs. He is indifferent between the two products if equation (2) is satisfied.

Some important features to remember about the model of demand are the following.

- Consumers differ only in their ideal product (represented by the consumer's location on the circle).
- Customers are always located evenly (uniformly distributed) around the circle.
- Consumers don't change their positions - although firms can change products' positions.


### 1.3 Which consumers buy a product

Now shift perspective to the one which you will have for the rest of the semester.
You are choosing the price of a product located at $y$, and you want to know how much demand $y$ will receive. The "scenarios" area of the web site you have is going to tell you this, but the description that follows gives the details of how the computer program calculates it.

Say the two products closest to $y$ are located at $x$ and $z$, as pictured in Figure 2. If the prices of the three products are not too different from each other, the only other two products that affect the demand for $y$ are these two nearest ones. ${ }^{3}$

For example, say that there are six products in the industry, located at points 0.1, $0.3,0.5,0.65,0.8$, and 0.95 around the circle. This is not shown in Figure 2, but you can draw it yourself. First consider the product at 0.65: its nearest neighbors are those (going

[^2]The two nearest products are located at $x$ and $z$, and they may be your firm's or another's. Given these product locations and the prices of the products $\left(p_{x}, p_{y}, p_{z}\right)$, we can calculate where the locations $u$ and $v$ are such that all customers between $u$ and $v$ (the bold portion of the circle) decide to purchase $y$. From the locations of $u$ and $v$ in this particular graph, we can see that $p_{y}<p_{x}$ (because more customers between $y$ and $x$ buy $y$ ) and $p_{y}>p_{z}$ (because more customers between $z$ and $y$ buy $z$ ).


Figure 2: How much demand does a product located at $y$ receive?
counter-clockwise) at 0.5 and (going clockwise) at 0.8 . If we consider the product located at 0.1 , then its nearest neighbors are located (going counter clockwise) at 0.95 and (going clockwise) at 0.3.

Returning to Figure 2, let $p_{x}, p_{y}$, and $p_{z}$ be the prices associated with the products at $x, y$, and $z$, respectively.

Let's see first how many of the customers between $x$ and $y$ decide to buy $y$. If $p_{x}$ and $p_{y}$ are very different from each other, then all the customers between the two products will buy the lower priced one, but if the prices are not too different from each other, then some customers will buy $x$ while some buy $y$. Assume that the prices are not too different. Here is the key: we want to find the location $u$ (see Figure 2) at which a customer (located at $u$ ) is indifferent between products $x$ and $y$ because then we know that the customers closer to $x$ than $u$ will buy $x$ and the ones closer to $y$ than $u$ will buy $y$. So in Figure 2, we're finding the bold portion of the circle between $y$ and $u$ : the demand for $y$ that's counterclockwise from $y$. (Then we will find the rest of of the bold portion, that which is clockwise from $y$, between $y$ and $v$.)

We know from equation (2) above that when products are located at $x$ and $y$, then the location $u$ where a customer is indifferent between them is described by

$$
200 d(x, u)+p_{x}=200 d(y, u)+p_{y},
$$

so solve this for the distance from $y$ to the location of the customer who is indifferent between products $x$ and $y$ :

$$
\begin{equation*}
d(y, u)=d(x, u)+\frac{p_{x}-p_{y}}{200} . \tag{3}
\end{equation*}
$$

That's the portion of demand counter-clockwise from product $y$ that it receives (the bold area between $y$ and $u$ in Figure 2), but it's still got $u$ in the equation. We can get rid of
that by using the fact that the distance from $x$ to $y$ is equal to the distance from $x$ to $u$ plus the distance from $u$ to $y$, or $d(x, y)=d(x, u)+d(y, u)$. Rearrange that to be

$$
\begin{equation*}
d(x, u)=d(x, y)-d(y, u) \tag{4}
\end{equation*}
$$

Now substitute equation (4) into (3) and solve for

$$
\begin{equation*}
d(y, u)=\frac{d(x, y)}{2}+\frac{p_{x}-p_{y}}{400} \tag{5}
\end{equation*}
$$

which is the distance from $y$ to the consumer indifferent between products $x$ and $y$. And it's in terms of the things we know: prices and locations of the two products.

We can do the same thing for demand between $x$ and $z$ by talking about a point $v$ that divides the demand between the two products. All consumers clockwise from $y$ to $v$ (the bold area between $y$ and $v$ in Figure 2) will buy $y$ instead of $z$. Using a derivation similar to that for equation (5), this formula for the distance between $y$ and $v$ is

$$
\begin{equation*}
d(y, v)=\frac{d(y, z)}{2}+\frac{p_{z}-p_{y}}{400} \tag{6}
\end{equation*}
$$

Finally, add together equations (5) and (6) to get the distance around the circle within which consumers all buy $y$ (all of the bold line in Figure 2):

$$
\begin{equation*}
d(u, v)=\frac{d(x, y)+d(y, z)}{2}+\frac{p_{x}+p_{z}-2 p_{y}}{400} \tag{7}
\end{equation*}
$$

In other words, this is the market share of product $y$.

### 1.4 The number of consumers in the total market

We will assume that the total number of consumers in the market, $N$, increases as the total number of products available, $n$, increases, but at a decreasing rate. This is based on the idea that more consumers will enter this market when there is a greater variety of products, but that there is some degree of saturation. The following function increases as $n$ increases, for values of $n$ that are not too large.

$$
\begin{equation*}
N=-110,000+120,000 \cdot n-1,500 \cdot n^{2} \tag{8}
\end{equation*}
$$

### 1.5 The demand function

So we're there! We now know the quantity demanded of product $y$ when its nearest neighbors are $x$ and $z$, with prices $p_{y}, p_{x}$, and $p_{z}$, respectively. It's the fraction of the
total consumers (market share) captured by $y$, multiplied by the total market size, using equations (8) and (7):

$$
\begin{aligned}
& D\left(y, p_{y} ; x, z, p_{x}, p_{x}\right)=N \times d(u, v) \\
& \quad=\left(-110,000+120,000 n-1,500 n^{2}\right) \times\left(\frac{d(x, y)+d(y, z)}{2}+\frac{p_{x}+p_{z}-2 p_{y}}{400}\right) .
\end{aligned}
$$

Let's check to make sure that this demand function has the properties that we expect all demand functions to satisfy.

- As the price of the product $p_{y}$ increases, demand for it decreases (because the coefficient on $p_{y}$ is negative).
- As either price $p_{x}$ or $p_{z}$ of the similar products (substitutes) increases, though, the demand for $y$ increases (because the coefficient on $p_{x}$ or $p_{z}$ is positive).
- As the substitute $x$ or $z$ gets further from $y$ (becomes less of a substitute), demand for $y$ increases. It can also be shown that $y$ 's price elasticity of demand decreases as the distance to other products increases.

No surprises here.
As was mentioned in footnote 3, this demand function is only true when prices are not too different. If $p_{y}$ is very high compared to $p_{x}$ and/or $p_{z}$, there may be no demand for $y$. If $p_{y}$ is very low, there may be customers beyond locations $x$ and/or $z$ that purchase $y$.

## 2 Costs

The firm cares about not only demand for its products, but also about the costs incurred in production as well. (In the discussion below, the terms "location" and "plant" are synonymous.)

Cost function: Each product is produced at constant marginal cost, and there is a fixed cost incurred each period per product. If a product is withdrawn ("srapped") from the market (see below), it no longer incurs the fixed cost. The fixed cost is $\$ 500,000$ per period, but the marginal cost depends on the level of investment spent on the product, as described next.

Cost-reducing investment: Marginal cost does not depend on $q$, the quantity produced, so it is constant marginal cost. But it does depend on, $X$, the total investment made in this product's manufacturing plant in previous periods. Marginal cost is given by

$$
\$ 50 \cdot\left(\frac{50,000,000}{X+50,000,000}\right)
$$

so you can see that increased investment, $X$, decreases the marginal cost of production. To repeat, $X$ is total investment made in all previous periods.
For example, if $\$ 0$ dollars were invested in the product during periods 1 and 2 and $\$ 1,000,000$ was invested during period 3 and again in period 4 , then marginal cost in periods 1 , 2 , and 3 would be $\$ 50$. In period 4 , marginal cost would be $50 \cdot \frac{50}{51}$ dollars. In period 5 , marginal cost would be $50 \cdot \frac{50}{52}$ dollars.

Thus, the cost function for a product of which $q$ is produced and in which a total of $X$ has been invested in previous periods is its fixed cost plus its variable cost

$$
C(q ; X)=\$ 500,000+\$ 50 \cdot\left(\frac{50,000,000}{X+50,000,000}\right) \cdot q
$$

Make sure the following facts are clear.

- Investment is specific to a particular product, but you can invest in as many of your products as you like.
- There is a one-period delay between when you invest and when marginal cost falls.
- Investment is "private" information (see below), so you only announce it to me, not to the industry in general.
- Investment is irreversible and sunk, so once you spend the funds on investment they cannot be recovered.
- If a product is relocated (see below), all investment in that product is lost.

The initial products start with zero investment ( $X=0$ in period 1 ), but you may invest in period 1 to bring down marginal cost in period 2 . Of course, you may also invest more in later periods to further decrease marginal cost.

## 3 Product entry, exit, relocation, and auction sales

Initially, each firm in the industry will have one product, and these products will be spaced equally around the circle. So if there are five firms in the industry there will be products located at $0,0.2,0.4,0.6$, and 0.8 . If there are four firms, the products will be at $0,0.25$, 0.50 , and 0.75 . Firms may also introduce, withdraw ("scrap"), and trade products during all but the final three periods.

### 3.1 Entry/Introduction

In any period, a firm can announce the introduction of one or more new products and specify the locations at which they will be available. In the period the announcement is made, these products have no demand (and do not affect other products' demand), so they do not generate revenue, but the firm does incur the cost of introducing them in that period. That cost, $\$ 5,000,000$, is half due to the cost of constructing a new plant and half due to advertising costs. The advertising cost is sunk (none of it is recoverable), but the manufacturing plant is assumed to have a resale value (see below) and to never have decreased production capacity.

The firm can make cost-reducing investment along with the introduction announcement, or it can wait until later periods. Thus, new products may have $X>0$ in their first period of generating revenue (the period after the introduction is announced).

To reiterate:

- Production begins and revenues begin being generated the period after the announcement of a new product. So if a new product is announced in period $t$, the cost of introduction and any initial cost-reducing investment is incurred during period $t$, but production costs $C(q ; X)$ and revenue do not start occurring until period $t+1$.


### 3.2 Exit/Withdrawal ("Scrap")

A firm can also choose to withdraw any of its products at any time. Like introduction, a withdrawal announced in period $t$ affects production and demand only starting at $t+1$.

The manufacturing plant for the withdrawn product can be sold on the open market for $\$ 1,000,000$ ("scrapped"), or to another firm in an "auction" on the VCR site for as much as that firm is willing to pay. A general announcement will be made that the plant is available for purchase, and then other teams can make bids. As emphasized below, you can talk about this, but you cannot talk about product prices and other decisions. Other teams make bids for the location using the VCR site. The announcment area can also be used to justify proposed bids and an asking price.

The plant does not need to be bought by any other firm. If it is not bought by another firm the firm that is withdrawing the plant receives the $\$ 1,000,000$ market value.

Remember:

- Although the firm announces in period $t$ that it will withdraw a product in $t+1$, the firm must still also set a price in period $t$. Of course, in period $t+1$ and all following periods, the product is no longer sold and no longer priced.
- If you want to ask for buyers of a plant, make the announcement (to me) before the period ends. Another firm must commit to buying the plant during the same
period you announce the withdrawal. If the plant is purchased, then in period $t+1$ the new firm must set price for this location.


### 3.3 Relocation

A product can also be relocated. This costs $\$ 2,500,000$ and all investment in cost-reduction is lost ( $X$ goes to zero). Think of it as the need to advertise the new location and to relearn or reinvest in any production techniques that reduce marginal cost. The relocated product, like a newly introduced product, may receive cost-reducing investment in the same period as the announcement of the relocation.

Note again:

- Like entry and exit, relocation occurs with a one-period delay. So along with announcing in period $t$ the new location of a product for period $t+1$ and all future periods, the firm must also set the price at the old location in period $t$. In period $t+1$ and later periods the firm will set price at the new location.


## 4 Financial details

Your team is endowed with $\$ 25,000,000$ cash (and one product, as stated above) at the beginning of the game. If the firm ever has a negative cash balance, you are declared insolvent and will be forced to restructure. Details will provided if this becomes necessary. There is no market for borrowing, so your only sources of cash are your initial assets, $5 \%$ interest received on cash, and your profits. Each period, your cash balance changes according to the rules outlined above and summarized here as the following credits $[+]$ and charges $[-]$.

+ interest of $5 \%$ on beginning of period cash balance
+ the revenue from all products
- the cost of production for all products' output
- cost-reducing investment for all products
- the number of product introductions $\times \$ 5,000,000$
+ the number of products withdrawn $\times \$ 1,000,000$ (or revenues received from their sale)
- the number of product relocations $\times \$ 2,500,000$
- the amount paid for any manufacturing plants purchased from other teams
$+/-$ any other charges or credits as appropriate.


## 5 Procedures, software, and grading

So now you know all there is to know about the industry in which you will be participating and the financial structure of revenues and costs. This final section describes how to report decisions, what you can talk about and with whom, and the incentive to do well.

Communication The antitrust laws apply to your industry and firm. That means that you are not allowed to talk or otherwise communicate with any member of another team about price, investment, and/or product decisions. Any communication about these subjects are the same as cheating on an exam (see the course syllabus).
Some communication, though, is allowed. One example is given above in the discussion of selling a manufacturing plant. Note that you must already be committed to selling the plant before discussing it with another firm. Your (legal) discussions with other firms can take place either directly or through me.
I will also consider any other type of proposal that you wish to make to me.
You are not allowed to ask questions of me or anyone outside your group about strategy or any particular decision. Asking me for clarification of rules or procedure is allowed.

Scenarios You can perform "what if" analyses about the industry using the "scenarios" area of the VCR web site. It embodies all of the definitions given in this document about demand and cost. By specifying each firm's product location(s) and pricing, you can see what demand and revenues would be generated for your firm and all others. By specifying investment, you can predict your firm's profits. You can also save set-ups for reference and modification later.

Decisions You will be responsible for reporting your decisions on the VCR web site. You can change those decisions at any time before the deadline for each period.
You will need to specify the following each period.

- The prices of your products (which default to the last period's prices).
- The level of any additional cost-reducing investment for each product (which defaults to zero-but $X$ is still the sum of all previous investment in each particular product).
- The location of any new products, the withdrawal of any existing products, and any relocations.
- Any other notes about changes that you would like to make (which can also be communicated via email before the deadline).

Important: If you fail to submit any of the information by the required deadline, the "default" values will be used. "Defaults" are your past pricing information; product entry, exit, relocation, and investment will not occur.

If there is no past pricing information (during the very first period of the game or in the first period after a product is introduced), demand will be zero but fixed costs will still be incurred.

Auctions During any period, some team may put a location up for auction. If your team does so, please send me an email message notifying me so that I can let the rest of the class know. Once you have decided to put a location up for auction, you cannot remove it.

If you wish to bid on an a location being auctioned, you must do so before the end of the period, and you are responsible for monitoring others bids that may be higher than yours.

Public Results The following information about your industry will be available on the VCR web site as soon as I increment the period. The full history of this information will be available, so you don't have to keep track of it.

- The location and ownership of all products. This includes any announcements of new products that will generate revenue in the next period.
- A list of products that will be withdrawn and will generate no more revenue in the next period.
- Change in ownership will be evident from reviewing the past history of existing products but not indicated directly.
- Prices of all products in production that period.
- The profits and cash holdings of all firms

Private Results The following information is made available on the VCR site at the same time as the public results, but other firms cannot seem it. The full private history remains available to you throughout the game.

- Your firms' investment decisions, production costs, demand, revenues, and profits broken out separately by product.

The final three decision rounds During the last three periods, no changes in products other than their prices will be allowed. No introductions, withdrawals, relocations, changes in investment, etc. Only prices may be changed in the last four periods. This allows future expected profits to be calculated, as described below.

Grading Your grade will be based on your final financial assets compared to levels which I expect you to reach based on the structure of the game.

Your final assets consist of both your cash and the present value of each manufacturing plant's future profit stream.

To estimate the future profit stream of each plant, I will assume that in every future period each product will earn the average of its profit over the final three periods, and I will apply a discount rate of $10 \%$ to these future profits.

For example, suppose that during the last three periods a product generated $\$ 400,000$, $\$ 500,000$, and $\$ 600,000$ in profits, respectively. The average is then $(\$ 400,000+$ $\$ 500,000+\$ 600,000) / 3=\$ 500,000$, and the present value of the plant producing this average forever in the future is $\$ 500,000 / 0.1=\$ 5,000,000$.

If the present value of any plant's future profit stream is less than its $\$ 1,000,000$ market value, I will use the market value (so the value of each plant will be at least $\$ 1,000,000)$.

One final note:

- Your grade does NOT depend on the size of your financial assets relative to the other teams in the industry in which you compete. Causing other firms to earn lower profits is not inherently better for your firm.


[^0]:    ${ }^{1}$ The technical reason for using a circle model instead of a line model is that if the two ends are not attached then the points 0 and 1 are very different from all other points. By attaching the ends of the line to make a circle (and assuming that customers are evenly distributed along the circle), all points are exactly the same as each other.

[^1]:    ${ }^{2}$ We don't need to assign a number to $V$, as you'll see later in equation (2). The details of why $V$ is in equation (1) aren't too important, but note that its size determines whether or not consumer surplus is positive.

[^2]:    ${ }^{3}$ If one price is much lower than the others, then one or both of the other products can receive zero demand. The math is somewhat different from that presented below, but still follows the principal that each consumer buys the product that provides him or her with the most consumer surplus. The "scenarios" area of the web site takes all of this into account.

