EQUILIBRIUM OF 3-D FORCE SYSTEMS

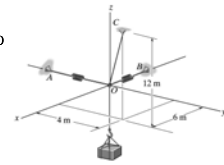
Today's Objectives:

a) Draw a 3-D free body diagram
b) Solve for the unknowns (forces, angles, distances) in 3-D particle equilibrium problems using the equations of equilibrium and possibly other equations related to spring deformation or frictionless pulleys
c) Solve for the unknowns in 3-D particle equilibrium problems using the equations of equilibrium when multiple free body diagrams are required.

3-D EXAMPLE #1

Given: A 20-kg (196.2 N) crate is supported by three cords. Two of the cords are springs.

Find: Tension in cords *OA*, *OB* and *OC* and the stretch of each spring. ($k_{OA} = k_{OB} = 300$ N/m)



Plan:

1) Draw a free body diagram of Point O. Label the unknown force magnitudes as $\mathbf{F_{OA}}$, $\mathbf{F_{OB}}$, $\mathbf{F_{OC}}$

2) Write out each force in Cartesian vector form (*i*, *j*, *k*)

3) Apply equilibrium equations to solve for up to three unknowns

4) Find the stretch of each spring using $F = k\,(l - l_0)$

3-D CONCEPT QUESTIONS

1. In 3-D, when you know the direction of a force but not its magnitude, how many unknowns corresponding to that force remain?

   A) One B) Two C) Three D) Four

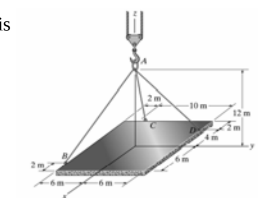
2. In 3-D, when you don't know the direction or the magnitude of a force, how many unknowns do you have corresponding to that force?

   A) One B) Two C) Three D) Four

IN-CLASS PROBLEM SOLVING

Given: A 150-kg (1472 N) plate is supported by three cables and is in equilibrium.

Find: Force in each of the three cables.



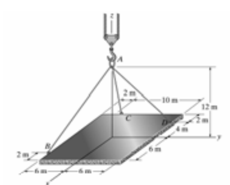
Plan:

1) Draw a free body diagram of Point A. Let the unknown force magnitudes be F_{AB}, F_{AC}, F_{AD}

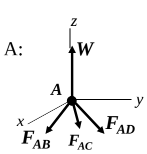
2) Represent each force in the Cartesian vector form (*i*, *j*, *k*).

3) Apply equilibrium equations to solve for the three unknowns.

IN-CLASS PROBLEM SOLVING



FBD of Point A:



$\boldsymbol{W}$ = load or weight of plate = (150 kg)(9.81m/s^2) $\boldsymbol{k}$

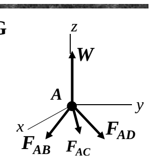
$W = \{0\,\boldsymbol{i} + 0\,\boldsymbol{j} + 1472\,\boldsymbol{k}\}$ N

$\boldsymbol{F_{AB}} = \|F_{AB}\|(\boldsymbol{r_{AB}}/\|r_{AB}\|) = \{(4/14)\,F_{AB}\,\boldsymbol{i} - (6/14)\,F_{AB}\,\boldsymbol{j} - (12/14)\,F_{AB}\,\boldsymbol{k}\}$ N

$\boldsymbol{F_{AC}} = \|F_{AC}\|(\boldsymbol{r_{AC}}/\|r_{AC}\|) = \{-(6/14)\,F_{AC}\,\boldsymbol{i} - (4/14)\,F_{AC}\,\boldsymbol{j} - (12/14)\,F_{AC}\,\boldsymbol{k}\}$ N

$\boldsymbol{F_{AD}} = \|F_{AD}\|(\boldsymbol{r_{AD}}/\|r_{AD}\|) = \{-(4/14)\,F_{AD}\,\boldsymbol{i} + (6/14)\,F_{AD}\,\boldsymbol{j} - (12/14)\,F_{AD}\,\boldsymbol{k}\}$ N

IN-CLASS PROBLEM SOLVING



The particle A is in equilibrium, hence

$\boldsymbol{F_{AB}} + \boldsymbol{F_{AC}} + \boldsymbol{F_{AD}} + \boldsymbol{W} = 0$

Now equate the respective ***i***, ***j***, ***k*** components to zero (i.e., apply the three scalar equations of equilibrium).

$\sum F_x = (4/14)F_{AB} - (6/14)F_{AC} - (4/14)F_{AD} = 0$

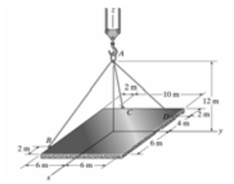
$\sum F_y = -(6/14)F_{AB} - (4/14)F_{AC} + (6/14)F_{AD} = 0$

$\sum F_z = -(12/14)F_{AB} - (12/14)F_{AC} - (12/14)F_{AD} + 1472 = 0$

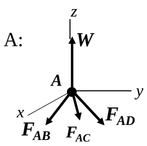
Solving the three simultaneous equations gives

$F_{AB} = 858$ N, $F_{AC} = 0$ N, $F_{AD} = 858$ N

CHANGE-UP QUESTIONS…



FBD of Point A:

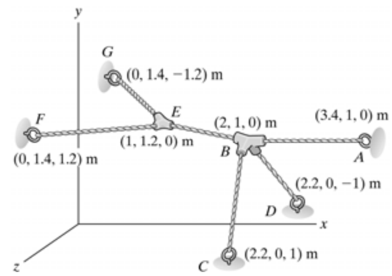


Cable AB is attached to the corner of the plate instead of at 2 m.

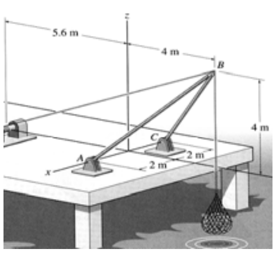
Knowing the tension force in Cable AD, solve for the weight that can be carried on the plate.

If the maximum tension force in any of the cables is 9 kN, what is the maximum weight that can be carried on the plate.

WHAT? We Have to Solve This???



3-D or 2-D APPROACH?



The shear leg derrick is to be designed to lift a maximum of 500 kg of fish.

What is the effect of different offset distances on the forces in the cable and the derrick legs?

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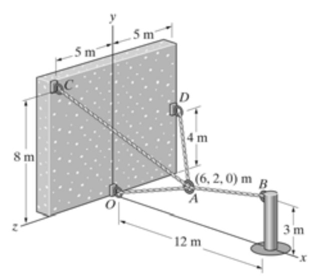

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