

3-D CARTESIAN VECTOR OPERATIONS

Today's Learning Objectives:

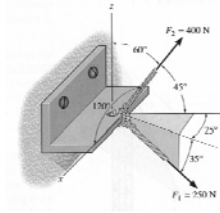
Write a vector in Cartesian form given information about the angles between the vector and the coordinate axes or coordinate planes

Determine the magnitude and coordinate direction angles (α, β, γ) of a force vector

Determine the magnitude and coordinate direction angles (α, β, γ) of the resultant force vector from two or more concurrent force vectors



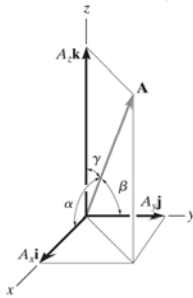
APPLICATIONS



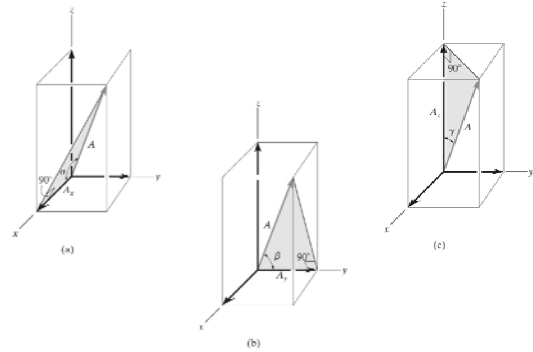
Given: Two concurrent 3-D forces acting on anchor.

Find: Express each force in Cartesian vector form and then calculate the magnitude and coordinate direction angles of the resultant force

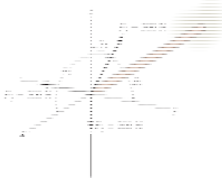
Cartesian Components - 3-D Vector Format



Coordinate Direction Angles (α, β, γ)



EXAMPLE



Given: The mast is loaded by three forces.

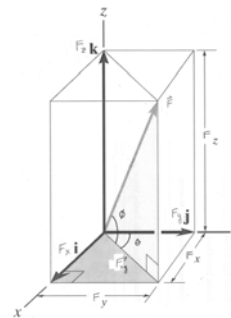
Find: The coordinate direction angles of \mathbf{F}_1 , so that the resultant force acting on the mast is zero

Plan:

- 1) Write \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 in Cartesian vector form ($\mathbf{i}, \mathbf{j}, \mathbf{k}$).
- 2) Impose the condition that the sum of the force vectors is zero

$$\mathbf{F}_R = \mathbf{0} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

Plane Angles (ϕ and θ)



Plane Angles (ϕ and θ)

1st Step

2nd Step

GROUP PROBLEM SOLVING

Given: Three forces act on hook. Magnitude and direction of the resultant force is given.

Find: The magnitude and direction of F_3 required to obtain the stated resultant force.

Plan:

- 1) Using the geometry and trigonometry, write F_1 , F_2 , & F_3 , and F_R in Cartesian vector form.
- 2) Set up equations to solve for components of F_3 .
- 3) Determine the magnitude of F_3 and its angles α , β , γ

GROUP PROBLEM SOLVING (CONT.)

Using the geometry and trigonometry, write F_1 , F_2 , & F_3 , and F_R in Cartesian vector form.

$F_1 = \{ (80)(4/5) i + (80)(0/5) j + (80)(3/5) k \}$ N

$F_1 = \{ 64 i + 0 j + 48 k \}$ N

$F_2 = \{ (110)(0) i + (110)(0) j - (110)(1) k \}$ N

$F_2 = \{ 0 i + 0 j - 110 k \}$ N

$F_3 = \{ F_{3x} i + F_{3y} j + F_{3z} k \}$ N

$F_R = \{ 120 \cos(45) \sin(30) i + 120 \cos(45) \cos(30) j + 120 \sin(45) k \}$ N

$F_R = \{ 42.4 i + 73.5 j + 84.9 k \}$ N

GROUP PROBLEM SOLVING (CONT.)

Set up relationships to solve for components of F_3 .

$\Sigma F_{ix} = F_{Rx} \Rightarrow 64 + 0 + F_{3x} = 42.4$

$\Sigma F_{iy} = F_{Ry} \Rightarrow 0 + 0 + F_{3y} = 73.5$

$\Sigma F_{iz} = F_{Rz} \Rightarrow 48 - 110 + F_{3z} = 84.9$

Solve

$F_{3x} = -21.6$ N

$F_{3y} = 73.5$ N

$F_{3z} = 146.9$ N

GROUP PROBLEM SOLVING (CONT.)

Determine the magnitude of F_3 and its angles α , β , γ

$\|F_3\| = \sqrt{(F_{3x})^2 + (F_{3y})^2 + (F_{3z})^2}$

$\|F_3\| = \sqrt{(-21.6)^2 + (73.5)^2 + (146.9)^2}$

$\|F_3\| = 165.7$ N

$\alpha = \cos^{-1} \left(\frac{F_{3x}}{\|F_3\|} \right) = \cos^{-1} \left(\frac{-21.6}{165.7} \right) \Rightarrow \alpha = 97.5^\circ$

$\beta = \cos^{-1} \left(\frac{F_{3y}}{\|F_3\|} \right) = \cos^{-1} \left(\frac{73.5}{165.7} \right) \Rightarrow \beta = 63.7^\circ$

$\gamma = \cos^{-1} \left(\frac{F_{3z}}{\|F_3\|} \right) = \cos^{-1} \left(\frac{146.9}{165.7} \right) \Rightarrow \gamma = 27.5^\circ$

CONCEPT QUESTION (& HW PROBLEM TOO!)

Given $F_x = 1.5$ kN and $F_z = 1.25$ kN and $\beta = 75$ degrees.

Determine the magnitudes of F and F_y and also find α and γ .

IMPORTANT NOTES

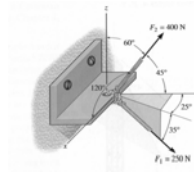
Sometimes 3-D vector information is given as:

- Magnitude and the coordinate direction angles, or
- Magnitude and projection angles.

You should be able to use both these types of information to change the representation of the vector into the Cartesian form, i.e.,

$$\mathbf{F} = \{10\mathbf{i} - 20\mathbf{j} + 30\mathbf{k}\} \text{ N}$$

EXAMPLE



Given: Two concurrent 3-D forces acting on anchor.

Find: Express each force in Cartesian vector form and then calculate the magnitude and coordinate direction angles of the resultant force

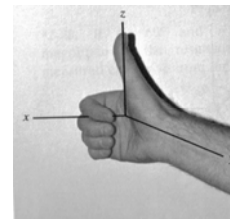
Plan:

- Using geometry and trigonometry, write F_1 and F_2 in Cartesian vector form.
- Add the i , j , and k components of the two forces to find i , j , and k components of the resultant, F_R .
- Calculate the magnitude and coord. direction angles of F_R

CONCEPT QUESTIONS

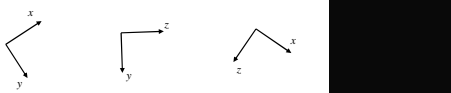
- If you know just u_F , you can determine the _____ of F uniquely.
 - magnitude
 - angles (α , β and γ)
 - components (F_x , F_y , & F_z)
 - All of the above.
- For an arbitrary force vector, the following parameters are randomly generated. Magnitude is 0.9 N, $\alpha = 30^\circ$, $\beta = 70^\circ$, $\gamma = 100^\circ$. What is wrong with this 3-D vector?
 - Magnitude is too small.
 - Angle γ is too large, should be less than 90° .
 - All three angles do not add up to 180° .
 - One or more of the angles have incorrect values.

DEF^N RIGHT-HANDED COORDINATE SYSTEM

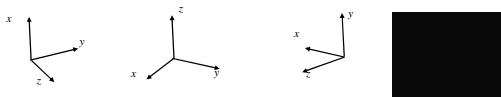


CONCEPT QUIZ

- Determine if the missing axis is into or out of the screen for a right-handed coordinate system



- Determine which coordinate systems are right-handed

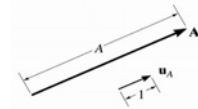


UNIT VECTORS

For a vector A with a magnitude of A , an unit vector is defined as $u_A = A / \|A\|$

Characteristics of a unit vector:

- Its magnitude is 1.
- It is dimensionless.
- It points in the same direction as the original vector, A .



The unit vectors in the Cartesian axis system are i , j , and k . They are unit vectors along the positive x , y , and z axes respectively.

