**LVAIC Mathematics Contest – November 12, 2017**

*Do as many of these problems as you can.*

*No calculators of any sort, notes, or reference materials are allowed.*

*Your solutions must be complete and your work justified to receive full credit.*

*Each solution must be written on a separate piece of paper with your team’s code on it.*

1. Here’s the first problem on the test. The test only takes 3 hours. Afterwards we’ll meet for pizza so I’ll see you soon. Which reminds me. Suppose you observe the following equation:

 **SEE**

 **+ YOU**

 **SOON**

where each different letter represents a unique digit and neither S nor Y are equal to 0. Find the value of  **U – N.**

2. Three friends (Jasmine, Kelly, and Leah) are in a room. Their ages are 20, 21, and 22 (in some order) and their friend Mark is trying to figure out how old each of them are. To “help” him out, they share the following information:

a) Kelly is 21, b) Jasmine is the youngest, c) Jasmine is 22, d) Kelly is younger than one of the friends but older than the other, e) Kelly is not the youngest of the three, f) Jasmine is not two years older than Kelly, g) Kelly is the oldest, h) Leah is older than Jasmine.

Mark is very confused by these contradictory hints until Leah states that only one of the hints they gave is actually true. How old are the three friends?

3. Suppose a function *f* exists that has the following properties:



Find a positive value *k* such that  or show that no such *k* exists.

4. Show that there exist no positive integer solutions to the equation 

5. Suppose Alice, Bob, and Cathy are given the following challenge. In another room, each of their names has been randomly written on one of three index cards (one name per card) and flipped over, with the other side simply numbering the cards “1”, “2”, or “3”. Alice will enter the room first, and she can flip over up to two of the cards. She will then leave, the cards will be replaced in their original spot, and Bob will enter. Bob also can flip over up to two cards, he then leaves, the cards are replaced, and then Cathy enters the room to repeat the process. Alice, Bob, and Cathy win the challenge together if they are each able to find the card that has their name on it. That is, they all must find their name in order to win. They cannot communicate with each other between their attempts, but they can plan their strategy before the challenge starts. Find a strategy that maximizes their probability of winning the challenge.

6. Given a positive rational number  written in lowest possible terms, you may transform *r* by adding 1 to either the numerator or the denominator of *r*, and then reducing the result to lowest possible terms. You are allowed to iterate this procedure as many times as you like, but your fraction must remain less than 1. For example, here is a sequence of steps that transforms  to 



a) Find a sequence that transforms  to 

b) Let  be a rational number with  written in lowest terms. Show it is possible to transform  to  in a finite number of steps.

7. Show that there exists a positive integer *n* such that the leftmost 10 digits of  are 1234567890.

8. Four friends are standing in a circle, holding hands. Ignoring leap years and assuming all birthdays are equally likely, what is the probability that no two people with the same birthday are holding hands?

9. Yesterday, you made a purchase at the grocery store for $4.01, paid $5 and received 99 cents in change: a half-dollar piece, a quarter, 2 dimes, and 4 pennies – 8 total coins. This seems like a waste. It would be easier if you didn’t get so many coins back in change. Obviously you can’t expect the government to print a coin for each value from 1 cent to 99 cents, but could you develop a system of coinage that allows you to get any amount of change from 1 cent to 99 cents using only 2 coins at most? The answer is clearly yes, but the group that finds a solution requiring the fewest number of unique coin values will receive full credit.

10. Xavier and Zoe are playing the following game with one checker on a 3×2017 checkerboard. The checker is initially in the lower left square, as in the picture, which shows a simpler 3×7 grid. Xavier and Zoe alternate moving the checker one square at a time with Xavier moving first. On any move, the checker may move exactly one square up, down, or right, but not left. Further, the checker can never occupy the same square twice. The winner is the person who moves the checker into the last column on the right for the first time.



Note: the sequence of moves RRUURDR is legal, but RUD is not (same square occupied twice) and UUU is not (the checker would leave the board).

Does either player have a winning strategy? If so, who and what is that strategy?