**Department of Mathematics**

**Lafayette College**

**MATH 371: Introduction to Bayesian Analysis**

**Spring 2021**

**Instructor**: Jeffrey Liebner

Pardee Hall 232

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**Class Time:** MWF 8:00-8:50 am

**Office Hours:** Tuesday/Thursday 8:30-10:30 am

**https://lafayette.zoom.us/j/94767601446**

Wednesday/Thursday 1:30-3:00 pm

**https://lafayette.zoom.us/j/96318154015**

**Texts:** Kruschke, J. (2015). Doing Bayesian Data Analysis, Second Edition, Academic Press. ISBN-13: 978-0124058880

*Optional:* Gelman, A., Carlin, J., Stern, H., Dunson, D, Vehtari, A., and Rubin, D. (2020) Bayesian Data Analysis, Third Edition, Chapman & Hall/CRC. ISBN-13: 978-1439840955

**Overview/Objectives:**

This course provides an option for students seeking an exposure to more specialized topics in probability and statistics. The Bayesian framework introduced in this class provides an approach to statistics that parallels the frequentist approach that is explored in MATH 336. One of the main differences between the two approaches is how the concept of probability itself is interpreted. For the frequentist, probabilities are defined in terms of long-term frequencies. (If I flip a fair coin a theoretically infinite number of times, half of the flips will land heads.) The Bayesian approach is sometimes referred to as subjective probability, where we can assign uncertainty to events and hypotheses, even those that are not repeatable and have no concept of long-term behavior. Instead, we consider the degree to which the idea is supported by the data. The topics of study will include Bayes’ Rule, prior and posterior distributions, credible intervals, assessing hypotheses, hierarchical models, and empirical Bayes. This course will also address many algorithms and computing techniques that have been created to address issues that arise in Bayesian analysis, including simulation studies, Gibbs samplers, and the Metropolis-Hastings algorithm. This course will seek to enhance the interaction between mathematics and other disciplines by showing how the use of Bayesian analysis is growing in a multitude of fields through practical application of the techniques covered in this class in real-world scenarios.

**Intended Student Learning Outcomes**

At the conclusion of this class, students will be able to:

* Understand and describe the differences between frequentist and Bayesian statistical analysis.
* Understand the concept of a prior and posterior distribution and be able to independently select appropriate priors and compute the associated posteriors using tools from calculus.
* Compute and interpret credible intervals and hypothesis analysis and compare these ideas to the parallel concepts in frequentist statistics.
* Build hierarchical models that incorporate multi-parameter situations.
* Understand the methodology behind computer algorithms that study Bayesian distributions and be able to independently construct appropriate computer code for Gibbs samplers and Metropolis-Hastings algorithms.
* Appreciate the importance and utility of the proper application of statistical ideas in real-world scenarios.

**Prerequisites:**

Students are expected to have successfully completed MATH 336 (Mathematical Statistics). Permission of the instructor is required if students wish to take this course while taking MATH 336 as a co-requisite. Students are also expected to be able to use techniques covered in the calculus sequence.

**Attendance:**

Participation in this class is an essential part of the learning environment. The opportunity to interact during the class period is important for building a proper understanding of the material. This is especially important for an advanced mathematics topic where independent study often will not suffice in replacing the classroom learning experience.

To encourage a productive class session, my aim is to have all students have the opportunity to participate through Zoom. As part of the effort to make this environment as fruitful as possible, I ask that all students have the cameras on their computers turned on and their microphones unmuted (within reason). Open class discussion and community building are beneficial to the learning goals of this class, and I hope these guidelines will encourage this behavior.

If you suspect you have COVID-19 and are seeking a Dean’s Excuse, please follow these steps:

Students learning remotely/from home: Please obtain documentation from a medical provider at home regarding your diagnosis and submit to Bailey Health Center. After review, and if symptoms are significant enough to interfere with remote learning/engagement with classes, Bailey Health Center will submit a Dean’s Excuse confirmation to the Office of Advising, who will process the Dean’s Excuse.

Students learning on campus: First contact Bailey Health Center for consultation and COVID-19 testing. If a positive test result is received, the student must follow the College’s protocols for clearance. If symptoms are significant enough to interfere with remote learning/engagement with classes, Bailey Health Center will submit a Dean’s Excuse confirmation to the Office of Advising, who will process the Dean’s Excuse.

In either case, since the course is being taught remotely, I will be using Zoom to record the lectures, which will be posted for the benefit of all students. Note that watching the recorded lectures does not replace the benefit of attending the virtual classes in synchrony with the other students and me.

**Course Information:**

The Moodle system will be used to maintain course information. Students should check here for assignments, course handouts, and other course related materials. A special note regarding Moodle privacy, via the Moodle homepage:

Moodle contains student information that is protected by the Family Educational Right to Privacy Act (FERPA).  Disclosure to unauthorized parties violates federal privacy laws.  Courses using Moodle will make student information visible to other students in this class.  Please remember that this information is protected by these federal privacy laws and must not be shared with anyone outside the class.  Questions can be referred to the Registrar's Office.

**Class Cancellations*:***

Lafayette College almost never cancels daytime classes. In the event that an unexpected problem requires that a class be cancelled, I will send an e-mail to all students prior to class. If you do not receive an e-mail, expect class to be in session. As I am originally from Buffalo, never assume that snow is a reason for class cancellation. (This last point makes less sense this semester, but still.)

**Incompletes:**

An incomplete will only be given if the majority of the work in the course is completed and an emergency situation exists that prevents the student from completing the course by the time of the final examination. Please notify me immediately if such an event occurs so that we may make arrangements.

**Special Needs:**

Lafayette is committed to providing support and reasonable accommodations for students with disabilities who self-identify with Accessibility Services. Students requesting accommodations to alleviate the impact of their disability should register their needs as soon as possible with the Accessibility Services Office, which is housed in the Academic Resource Hub (resourcehub@lafayette.edu). Once registered, students should request their accommodation letters to provide notification of their needs to their professors, on a semester-by-semester basis. If you have questions or concerns pertaining specifically to your accommodations within this course, please contact me to discuss them.

Eligibility for, and provision of accommodations may be influenced by the changes to the academic environment that are necessitated by the COVID-19 pandemic. If you are in need of accommodations, or adjustments to your accommodation plan, it is your responsibility to contact Accessibility Services immediately to discuss your needs.

Note that to be eligible for appropriate accommodations, you must provide me with the appropriate form from the Academic Resource Hub prior to any class event which would require these accommodations.

**Homework Assignments:**

Homework will be assigned on a weekly basis to be submitted for grading. Students may discuss the assignment with others in the class, but the submitted material must be each student’s own work. Failure to abide by these standards will result in disciplinary measures being taken. Homework submitted after the due date will not be graded.

**Course Project:**

Probability and statistics are not only a study of mathematics, but an exercise in communicating mathematical results to others. Thus, in addition to the weekly homework assignments, there will be a course group project involving analysis of a real-world dataset. Additional information regarding the project will be provided at a later time.

**Exams:**

There will be two exams presented during the course of the semester and a final exam scheduled at the end of the course. These exams will be a mix of in-class and take-home aspects. The take-home portion will often involve the use of computer software. Note that the take-home portion of the exam is an individual exam that is to be done without collaboration. In the event of an excused absence from an exam, the exam will be made-up within a week of the student’s return to class. Unexcused absences will receive a score of zero for the exam.

**Grading:**

Grades will be based on the following breakdown:

Assigned Homework: 20%

Course Project: 20%

Exams (2 at 20%): 40%

Final Exam: 20%

Students can expect that the following grade distribution will be roughly followed when assigning final grades: A: 90-100%, B: 80-90%, C: 70-80%, D: 60-70%, F: below 60%. Pluses and minuses will be distributed within each range.

**Academic Honesty**:

You are reminded of the college policy on Intellectual Honesty, as stated in the Student Code of Conduct, found in the Student Handbook.

To maintain the scholarly standards of the College and, equally important, the personal ethical standards of our students, it is essential that written assignments be a student’s own work, just as is expected in examinations and class participation. A student who commits academic dishonesty is subject to a range of penalties, including suspension or expulsion. Finally, the underlying principle is one of intellectual honesty. If a person is to have self-respect and the respect of others, all work must be his/her own.

Any case of academic dishonesty will be brought to the attention of the dean. If there are any questions on this matter please speak with me.

**Credit Hour Compliance:**

The student work in this course is in full compliance with the federal definition of a four credit hour course. Please see the Registrar’s Office web site (https://registrar.lafayette.edu/wp-content/uploads/sites/193/2013/04/Federal-Credit-Hour-Policy-Web-Statement.doc) for the full policy and practice statement.

**Course Outline**

**Week 1:** Introduction to the concept of Bayesian analysis and revisiting conditional probability

**Week 2:** Bayes’ Theorem, prior and posteriors

**Week 3:** Conjugate priors, informative and non-informative priors

**Week 4:** Interpreting the posterior distribution; credible intervals

**Week 5:** Multiparameter models

**Week 6:** Hierarchical models and Test 1

**Week 7:** Computer simulation of posterior distributions; introduction to R, JAGS, and/or Stan

**Week 8:** Markov chains

**Week 9:** Gibbs samplers

**Week 10:** Metropolis-Hastings algorithm

**Week 11:** Empirical Bayes and Test 2

**Week 12:** Model assessment

**Week 13:** Large sample theory, comparing Bayesian and frequentist results

**Week 14:** Additional application as time permits (e.g. Bayesian regression)