Summary
The FE Exam in Structural Analysis and Structural Design essentially covers three college courses:
- Structural Steel Design (CE311)
- Reinforced Concrete Design (CE413)
- Structural Analysis - including indeterminate structural analysis (CE311 and CE412)
Along with some very basic:
- Structural Dynamics and Dynamics (CE414 and Physics)

Consequently, some of the material is truly a review, while much of it may be entirely new to you.
Therefore, your preparation realistically consists of three areas:
- Review material that you have actually covered in class (CE311, CE413, CE414)
- Learn enough about the other stuff to get some problems correct (CE412, CE413, CE414)
- Disregard material that you have no prayer of learning in a short period of time.

Key Notes:
LRFD – the structural design in both steel and concrete will be universally in LRFD.
Know your terms:
“Service” or “Working” loads – actual applied loads
“Ultimate” – factored loads: In general, $\phi R_n = R_u$. The $\phi$ factor is the resistance factor, which reduces the nominal (actual) resistance $R_n$. $R_u$ is the factored load.

Contents of this Review Package (Full version at https://sites.lafayette.edu/kurtzs/)
- The official NCEES Reference - equation sheets for Structural Analysis and Structural Design
  You will find that you can actually figure out a lot of problems, based on these sheets, even if you have never taken the subject (example: Reinforced Concrete, Indeterminate Structural Analysis)
- Sample Problems – Structural Steel Design
- Sample Problems – Reinforced Concrete Design
- Sample Problems – Indeterminate Structural Analysis
STRUCTURAL ANALYSIS

Influence Lines for Beams and Trusses
An influence line shows the variation of an effect (reaction, shear and moment in beams, bar force in a truss) caused by moving a unit load across the structure. An influence line is used to determine the position of a moveable set of loads that causes the maximum value of the effect.

Moving Concentrated Load Sets

The absolute maximum moment produced in a beam by a set of "n" moving loads occurs when the resultant "R" of the load set and an adjacent load are equal distance from the centerline of the beam. In general, two possible load set positions must be considered, one for each adjacent load.

Beam Stiffness and Moment Carryover

$$\theta = \frac{M L}{4 E I} \Rightarrow M = \left(\frac{4 E I}{L}\right) \theta = k_{a\theta} \theta$$

$$k_{a\theta} = \text{stiffness} \quad M_0 = M_a/2 = \text{carryover}$$

Truss Deflection by Unit Load Method
The displacement of a truss joint caused by external effects (truss loads, member temperature change, member misfit) is found by applying a unit load at the point that corresponds to the desired displacement.

$$\Delta_{\text{joint}} = \sum_{i=1}^{\text{members}} f_i / (\Delta L_i)$$

where:
- $\Delta_{\text{joint}} = \text{joint displacement at point of application of unit load (+ in direction of unit load)}$
- $f_i = \text{force in member } i \text{ caused by unit load (+ tension)}$
- $(\Delta L_i) = \text{change in length caused by external effect (+ for increase in member length)}$

- $f_i = \left(\frac{P L}{A E}\right)$ for bar force $F$ caused by external load
- $= \alpha L_i (\Delta T_i)$ for temperature change in member ($\alpha = \text{coefficient of thermal expansion}$)
- $= \text{member misfit}$

$M_e = M_4/2 = \text{carryover}$

$E = \text{member elastic modulus}$

Frame Deflection by Unit Load Method
The displacement of any point on a frame caused by external loads is found by applying a unit load at that point that corresponds to the desired displacement:

$$\Delta = \sum_{i=1}^{\text{members}} \int x - \frac{M_0 M}{E I} \ dx$$

where:
- $\Delta = \text{displacement at point of application of unit load (+ in direction of unit load)}$
- $M_0 = \text{moment equation in member } i \text{ caused by the unit load}$
- $M_f = \text{moment equation in member } i \text{ caused by loads applied to frame}$
- $L_i = \text{length of member } i$
- $I_i = \text{moment of inertia of member } i$

If either the real loads or the unit load cause no moment in a member, that member can be omitted from the summation.

Member Fixed-End Moments (Magnitudes)

$$FEM_{AB} = \frac{FEM_{BA}}{12} = \frac{w L^2}{12}$$

$$FEM_{AD} = \frac{P a b^2}{L^2} \quad FEM_{BD} = \frac{P a^2 b}{L^2}$$

152 CIVIL ENGINEERING
STABILITY, DETERMINACY, AND CLASSIFICATION OF STRUCTURES

\[ m = \text{number of members} \]
\[ r = \text{number of independent reaction components} \]
\[ j = \text{number of joints} \]
\[ c = \text{number of condition equations based on known internal moments or forces, such as internal moment of zero at a hinge} \]

**Plane Truss**

- **Static Analysis**
  - \( m + r = 2j \) : Unstable
  - \( m + r = 2j \) : Stable and statically determinate
  - \( m + r > 2j \) : Stable and statically indeterminate

**Plane Frame**

- **Static Analysis**
  - \( 3m + r < 3j + c \) : Unstable
  - \( 3m + r = 3j + c \) : Stable and statically determinate
  - \( 3m + r > 3j + c \) : Stable and statically indeterminate

Stability also requires an appropriate arrangement of members and reaction components.

STRUCTURAL DESIGN

**Live Load Reduction**

The effect on a building member of nominal occupancy live loads may often be reduced based on the loaded floor area supported by the member. A typical model used for computing reduced live load (as found in ASCE 7 and many building codes) is:

\[ L_{\text{reduced}} = L_{\text{nominal}} \left( 0.25 + \frac{15}{K_{LL} A_T} \right) \geq 0.4 L_{\text{nominal}} \]

where:

- \( L_{\text{nominal}} \) = nominal live load given in ASCE 7 or a building code
- \( A_T \) = the cumulative floor tributary area supported by the member
- \( K_{LL} A_T \) = area of influence supported by the member
- \( K_{LL} \) = ratio of area of influence to the tributary area supported by the member:
  - \( K_{LL} = 4 \) (typical columns)
  - \( K_{LL} = 2 \) (typical beams and girders)

Load Combinations using Strength Design (LRFD, USD)

Nominal loads used in following combinations:

- \( D \) = dead loads
- \( E \) = earthquake loads
- \( L \) = live loads (floor)
- \( L_L \) = live loads (roof)
- \( R \) = rain load
- \( S \) = snow load
- \( W \) = wind load

Load factors \( \lambda \): \( \lambda_D \) (dead load), \( \lambda_E \) (live load), etc.

Basic combinations:

\[ L_L/S/R = \text{largest of } L_L, S, R \]

\[ L \text{ or } 0.8W = \text{lager of } L, 0.8W \]

\[ \begin{align*}
1.4D \\
1.2D + 1.6L + 0.5 (L_L/S/R) \\
1.2D + 1.6(L_L/S/R) + (L \text{ or } 0.8W) \\
1.2D + 1.6W + L + 0.5(L_L/S/R) \\
1.2D + 1.0E + L + 0.2S \\
0.9D + 1.6W \\
0.9D + 1.0E
\end{align*} \]
DESIGN OF REINFORCED CONCRETE COMPONENTS (ACI 318-11)

U.S. Customary units

Definitions

\( a \) = depth of equivalent rectangular stress block, in.
\( A_e \) = gross area of column, in²
\( A_r \) = area of tension reinforcement, in²
\( A_s \) = total area of longitudinal reinforcement, in²
\( A_s' \) = area of shear reinforcement within a distance \( s \), in.
\( b \) = width of compression face of member, in.
\( \beta \) = ratio of depth of rectangular stress block, \( a \), to depth to neutral axis, \( c \)

\[
\beta = \frac{0.85 \geq 0.85 - 0.05 \left( \frac{L_s - 4,000}{1,000} \right) \geq 0.65}
\]

\( c \) = distance from extreme compression fiber to neutral axis, in.

\( d \) = distance from extreme compression fiber to centroid of nonpropped tension reinforcement, in.

\( d_s \) = distance from extreme compression fiber to extreme tension steel, in.

\( f_e' \) = compressive strength of concrete, psi

\( f_y \) = yield stress of steel reinforcement, psi

\( M_n \) = Nominal moment strength at section, in.-lb

\( M_{n,d} \) = design moment strength at section, in.-lb

\( P_n \) = Nominal axial load strength at given eccentricity, lb

\( P_{n,d} \) = design axial load strength at given eccentricity, lb

\( F_s \) = factored axial force at section, lb

\( \rho_s \) = ratio of total reinforcement area to cross-sectional area of column = \( A_s / A_e \)

\( s \) = spacing of shear ties measured along longitudinal axis of member, in.

\( V_e \) = nominal shear strength provided by concrete, lb

\( V_r \) = nominal shear strength at section, lb

\( V_{s,d} \) = design shear strength at section, lb

\( V_{s,s} \) = nominal shear strength provided by reinforcement, lb

\( V_{s,f} \) = factored shear force at section, lb

ASTM STANDARD REINFORCING BARS

<table>
<thead>
<tr>
<th>BAR SIZE</th>
<th>DIAMETER, IN.</th>
<th>AREA, IN²</th>
<th>WEIGHT, LB/FT</th>
</tr>
</thead>
<tbody>
<tr>
<td>#3</td>
<td>0.375</td>
<td>0.11</td>
<td>0.376</td>
</tr>
<tr>
<td>#4</td>
<td>0.500</td>
<td>0.20</td>
<td>0.568</td>
</tr>
<tr>
<td>#5</td>
<td>0.625</td>
<td>0.31</td>
<td>1.043</td>
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<tr>
<td>#6</td>
<td>0.750</td>
<td>0.44</td>
<td>1.502</td>
</tr>
<tr>
<td>#7</td>
<td>0.875</td>
<td>0.60</td>
<td>2.044</td>
</tr>
<tr>
<td>#8</td>
<td>1.000</td>
<td>0.79</td>
<td>2.570</td>
</tr>
<tr>
<td>#9</td>
<td>1.128</td>
<td>1.00</td>
<td>3.400</td>
</tr>
<tr>
<td>#10</td>
<td>1.270</td>
<td>1.27</td>
<td>4.303</td>
</tr>
<tr>
<td>#11</td>
<td>1.410</td>
<td>1.56</td>
<td>5.313</td>
</tr>
<tr>
<td>#12</td>
<td>1.690</td>
<td>2.25</td>
<td>7.650</td>
</tr>
</tbody>
</table>

UNIFIED DESIGN PROVISIONS

Internal Forces and Strains

Unbalanced forces:

\[ P_n = M_n / d \]

\[ V_e = A_e f_e' \]

\[ V_r = A_r f_y \]

\[ V_{s,d} = A_{s,d} f_y \]

\[ V_{s,s} = A_{s,s} f_y \]

\[ V_{s,f} = A_{s,f} f_y \]

Net tensile strain: \( \varepsilon_t \)

Strain Conditions

Tension-controlled section:

\[ \varepsilon_t \leq 0.005 \]

Compression-controlled section:

\[ \varepsilon_t \geq 0.005 \]

RESISTANCE FACTORS, \( \phi \)

Tension-controlled sections (\( \varepsilon_t \geq 0.005 \)):

\[ \phi = 0.9 \]

Compression-controlled sections (\( \varepsilon_t \leq 0.002 \)):

Members with tied reinforcement

\[ \phi = 0.65 \]

Transition sections (\( 0.002 < \varepsilon_t < 0.005 \)):

Members with tied reinforcement

\[ \phi = 0.48 + 0.83 \varepsilon_t \]

Shear and torsion

\[ \phi = 0.75 \]

Bearing on concrete

\[ \phi = 0.65 \]
BEAMS—FLEXURE
\[ \Phi M_u \geq M_u \]

For All Beams
Net tensile strain: \[ \epsilon_r = \frac{0.003(a - c)}{c} = \frac{0.003(b_d - a)}{a} \]

Design moment strength: \( \phi M_u \)
where: \[ \phi = 0.9 \begin{cases} & \epsilon_r \geq 0.005 \\ & 0.48 + 83\epsilon_r [0.0062 \leq \epsilon_r < 0.005] \\ & 0.65 \begin{cases} & \epsilon_r < 0.0062 \\ & \epsilon_r < 0.005 \end{cases} \end{cases} \]

Singly-Reinforced Beams
\[ a = \frac{A_s f_y}{0.85 \frac{f_c}{b}} \]
\[ M_u = 0.85 \frac{f_c a b (d - \frac{a}{2})}{2} = A_s f_y \left( d - \frac{a}{2} \right) \]

BEAMS—SHEAR
\[ \Phi V_u \geq V_u \]
Nominal shear strength:
\[ V_u = V_{cc} + V_{ct} \]
\[ V_c = 2 b_v d f_y \]
\[ V_t = \frac{A_s f_y d}{2} \text{ (may not exceed } 8 b_v d f_y) \]

Required and maximum-permitted stirrup spacing, \( s \)
\[ V_s \leq \frac{f_y s}{2} \text{. No stirrups required} \]
\[ V_s > \frac{f_y s}{2} \text{. Use the following table } (A_i \text{, given}) \]

<table>
<thead>
<tr>
<th>( \frac{f_y s}{2} )</th>
<th>( V_u &lt; V_s \leq V_{cc} )</th>
<th>( V_s &gt; V_{cc} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Required spacing</strong></td>
<td>Smaller of: [ s = \frac{A_s f_y}{50 f_c} ] [ s = \frac{A_s f_y}{0.75 b_v \sqrt{f_c}} ]</td>
<td>( V_s = \frac{V_u - V_c}{\phi} ) [ s = \frac{A_s f_y d}{V_s} ]</td>
</tr>
<tr>
<td><strong>Maximum permitted spacing</strong></td>
<td>Smaller of: ( s = \frac{d}{2} ) OR ( s = 24^{\circ} )</td>
<td>( V_s \leq 4 b_v d \sqrt{f_c} ) Smaller of: ( s = \frac{d}{2} ) OR ( s = 24^{\circ} ) Smaller of: ( s = \frac{d}{4} ) OR ( s = 12^{\circ} )</td>
</tr>
</tbody>
</table>
GRAPH A.11
Column strength interaction diagram for rectangular section with bars on end faces and $\gamma = 0.80$
(for instructional use only).

DESIGN OF STEEL COMPONENTS  
(ANSI/AISC 360-10)  
LRFD, $F_y = 29,000$ ksi  

BEAMS  
For doubly symmetric compact I-shaped members bent about their major axis, the design flexural strength $F_{d}$ is determined with $F_{y} = 0.90$ as follows:  

**Yielding**  
$$M_{y} = M_{p} = F_{y} Z_{p}$$  
where  
- $F_{y}$ = specified minimum yield stress  
- $Z_{p}$ = plastic section modulus about the x-axis  

**Lateral-Torsional Buckling**  
Based on bracing where $L_{b}$ is the length between points that are either braced against lateral displacement of the compression flange or braced against twist of the cross section with respect to the length limits $L_{b}$ and $L_{l}$:  

When $L_{b} \leq L_{b}$, the limit state of lateral-torsional buckling does not apply.  

When $L_{l} < L_{b} \leq L_{l}$:  
$$M_{p} = C_{l} \left[ M_{y} - (M_{y} - 0.7 F_{y} Z_{p}) \left( \frac{L_{b} - L_{l}}{L_{l}} \right) \right] \leq M_{y}$$  
where  
- $C_{l} = \frac{12.5 M_{\max}}{2.5 M_{\max} + 3 M_{y} + 2 M_{u} + M_{l}}$  
- $M_{u}$ = absolute value of maximum moment in the unbraced segment  
- $M_{l}$ = absolute value of maximum moment at quarter point of the unbraced segment  
- $M_{b}$ = absolute value of maximum moment at centerline of the unbraced segment  
- $M_{c}$ = absolute value of maximum moment at three-quarter of the unbraced segment  

**Shear**  
The design shear strength $V_{d}$ is determined with $F_{y} = 1.00$ for webs of rolled I-shaped members and is determined as follows:  
$$V_{d} = 0.6 F_{y} \left( d I_{y} \right)$$  

**COLUMNS**  
The design compressive strength $F_{d}$ is determined with $F_{y} = 0.90$ for flexural buckling of members without slender elements and is determined as follows:  
$$F_{d} = F_{y} A_{w}$$  

where the critical stress $F_{c}$ is determined as follows:  
(a) When $\frac{KL}{r} \leq 4.71 \left( \frac{F_{c}}{F_{y}} \right)$,  
$$F_{c} = 0.658 F_{y}$$  

(b) When $\frac{KL}{r} > 4.71 \left( \frac{F_{c}}{F_{y}} \right)$,  
$$F_{c} = 0.877 F_{y}$$  

where  
- $KL/r$ is the effective slenderness ratio based on the column effective length ($KL$) and radius of gyration ($r$)  
- $KL$ is determined from AISC Table C-A-7.1 or AISC Figures C-A-7.1 and C-A-7.2 on p. 158.  
- $F_{y}$ is the elastic buckling stress $= \frac{E}{2(1+v^{2})}$

<table>
<thead>
<tr>
<th>VALUES OF $C_{b}$ FOR SIMPLY SUPPORTED BEAMS</th>
<th>LOAD</th>
<th>LATERAL BRACING ALONG SPAN</th>
<th>$C_{b}$</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image.png" alt="Diagram of Load Conditions" /></td>
<td>NONE</td>
<td>NO LOAD AT MIDPOINT</td>
<td>$C_{b}$</td>
</tr>
<tr>
<td></td>
<td>AT LOAD POINTS</td>
<td>1.07</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="Diagram of Load Conditions" /></td>
<td>NONE</td>
<td>NO LOAD AT THREE POINTS</td>
<td>1.14</td>
</tr>
<tr>
<td></td>
<td>AT LOAD POINTS</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="Diagram of Load Conditions" /></td>
<td>NONE</td>
<td>NO LOAD AT QUARTER POINTS</td>
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</tr>
<tr>
<td></td>
<td>AT LOAD POINTS</td>
<td>1.17</td>
<td></td>
</tr>
<tr>
<td><img src="image.png" alt="Diagram of Load Conditions" /></td>
<td>NONE</td>
<td>NO LOAD AT FIFTH POINTS</td>
<td>1.14</td>
</tr>
</tbody>
</table>

*NOTE: LATERAL BRACING BASED ON SUPPORT POINTS OF SUPPORT WHERE DEFORMATION CAPABLE.*  
TENSION MEMBERS
Flat bars or angles, bolted or welded
Definitions
Bolt diameter: \( d_b \)
Nominal hole diameter: \( d_h = d_b + 1/16" \)
Gross width of member: \( b_g \)
Member thickness: \( t \)
Connection eccentricity: \( x \)
Gross area: \( A_g = b_g t \) (use tabulated areas for angles)
Net area (parallel holes): \( A_n = \left[ b_g - \sum \left( d_h + \frac{1}{16}\right) \right] t / \sum \frac{t}{4} \)
Net area (staggered holes):
\[
A_n = \left[ b_g - \sum \left( d_h + \frac{1}{16}\right) \right] + \frac{s^2}{4g} t
\]
\( s = \) longitudinal spacing of consecutive holes
\( g = \) transverse spacing between lines of holes
Effective area (bolted members):
\[ A_e = U A_n \]
\( U = \begin{cases} 1.0 & \text{Flat bars:}\ U = 1.0 \\ 1 - \frac{t}{L} & \text{Angles:}\ U = 1 - \frac{t}{L} \end{cases} \)
Effective area (welded members):
\[ A_e = U A_n \]
\( U = \begin{cases} 1.0 & \text{Flat bars or angles with transverse welds:}\ U = 1.0 \\ \frac{U}{L} & \text{Flat bars of width } w, \text{ longitudinal welds of length } L \text{ only:}\ U = 1.0 (L \leq 2w) \\ 0.87 & (2w > L \geq 1.5w) \\ 0.75 & (1.5w > L > w) \end{cases} \)
\( U = 1 - \frac{t}{L} \)
\( U = \begin{cases} 0.65 & \text{Angles with longitudinal welds only:}\ U = 0.65 \end{cases} \)

Limit States and Available Strengths
Yielding:
\[ \Phi_y = 0.90 \]
\[ \Phi_T = \Phi_y F_y A_g \]
Fracture:
\[ \Phi_f = 0.75 \]
\[ \Phi_T = \Phi_f F_y A_g \]
Block shear:
\[ \Phi = 0.75 \]
\[ U_l = 1.0 \text{ (flat bars and angles)} \]
\[ A_{g,v} = \text{gross area for shear} \]
\[ A_{w,v} = \text{net area for shear} \]
\[ A_{w,t} = \text{net area for tension} \]
\[ \Phi_T = \begin{cases} 0.75 U_l & \text{in tension} \\ 0.75 U_l & \text{in shear} \end{cases} \]
### Table 1-1: W Shapes Dimensions and Properties

<table>
<thead>
<tr>
<th>Shape</th>
<th>Area A</th>
<th>Depth d</th>
<th>Web tw</th>
<th>Flange bf</th>
<th>Axis X-X</th>
<th>Axis Y-Y</th>
</tr>
</thead>
<tbody>
<tr>
<td>W24×68</td>
<td>20.1</td>
<td>23.7</td>
<td>0.415</td>
<td>8.97</td>
<td>0.680</td>
<td>1830</td>
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<td>W24×62</td>
<td>18.2</td>
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<td>0.435</td>
<td>7.04</td>
<td>0.590</td>
<td>1600</td>
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<td>W24×55</td>
<td>16.3</td>
<td>23.6</td>
<td>0.396</td>
<td>7.51</td>
<td>0.605</td>
<td>1300</td>
</tr>
<tr>
<td>W21×73</td>
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<td>21.2</td>
<td>0.495</td>
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<td>0.740</td>
<td>1600</td>
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<td>W21×68</td>
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<td>21.1</td>
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<tr>
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<td>0.335</td>
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Adapted from "Steel Construction Manual, 14th ed., AISC, 2011."
### AISC Table 3-2

**W Shapes – Selection by \( Z_x \)**

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<th>( Z_x )</th>
<th>( b_{M_N} ) kip-ft</th>
<th>( b_{M_M} ) kip-ft</th>
<th>( b_{M_F} ) kip-ft</th>
<th>( L_p ) ft.</th>
<th>( L_r ) ft.</th>
<th>( I_x ) in.(^4)</th>
<th>( \phi V_{PE} ) kips</th>
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\[
M_{FV} = (0.7F_v)S_x \quad BF = \frac{M_{PM} - M_{FV}}{L_r - L_d}
\]

Available Moment VS. Unbraced Length

TABLE C-A.7.1
APPROXIMATE VALUES OF EFFECTIVE LENGTH FACTOR, K

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<th>BUCKLED SHAPE OF COLUMN IS SHOWN BY DASHED LINE.</th>
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<th>(b)</th>
<th>(c)</th>
<th>(d)</th>
<th>(e)</th>
<th>(f)</th>
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FOR COLUMN ENDS SUPPORTED BY, BUT NOT RIGIDLY CONNECTED TO, A FOOTING OR FOUNDATION, G IS THEORETICALLY INFINITY BUT UNLESS DESIGNED AS A TRUE FRICITION-FREE PIN, MAY BE TAKEN AS 10. FOR PRACTICAL DESIGNS, IF THE COLUMN END IS RIGIDLY ATTACHED TO A PROPERLY DESIGNED FOOTING, G MAY BE TAKEN AS 1.0. SMALLER VALUES MAY BE USED IF JUSTIFIED BY ANALYSIS.

AISC Figure C-A.7.1
Alignment chart, side-way inhibited (braced frame)

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AISC Figure C-A.7.2
Alignment chart, side-way uninhibited (moment frame)

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**AISC Table 4-22**

Available Critical Stress \( \phi F_{cr} \) for Compression Members

\[ F_y = 50 \text{ ksi} \quad \phi_c = 0.90 \]

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Heavy line indicates K/L equal to or greater than 200

Adapted from Steel Construction Manual, 16th ed., AISC, 2011
### Simply Supported Beam Slopes and Deflections

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Cantilevered Beam Slopes and Deflections

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Piping Segment Slopes and Deflections

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Structural Steel Design Problems (from Lunenburger and Iqbal & Iqbal – FE Exam Review Materials)
42

Structural Steel: Beams

PRACTICE PROBLEMS

1. A 25 ft long steel beam is loaded uniformly (live) at 4 kips/ft. Loading due to self-weight is negligible, and there is adequate lateral support provided to the beam. The required plastic section modulus for a W12 shape using grade-50 steel is most nearly
   (A) 60 in³
   (B) 95 in³
   (C) 120 in³
   (D) 130 in³

2. A W-shaped beam has a warping constant of 3450 in⁶. The moment of inertia about the weak axis is 45.1 in⁴, and its elastic section modulus about the strong axis is 99.1 in³. The effective radius of gyration of the compression flange is most nearly
   (A) 2.0 in
   (B) 3.0 in
   (C) 4.0 in
   (D) 5.0 in

3. An 18 ft long custom manufactured and rolled continuous W14 x 68 shape beam is supported at each end and at the midpoint. The compression flange of the beam is restrained from moving laterally, and all forms of local buckling are prevented. The nominal shear strength as determined from LRFD principles is 146 kips. Determine the most likely steel variety used in the manufacture of the beam and whether or not this beam is compact.
   (A) probably noncompact; yield strength = 25 kips/in²
   (B) probably noncompact; yield strength = 36 kips/in²
   (C) probably compact; yield strength = 42 kips/in²
   (D) probably compact; yield strength = 50 kips/in²

SOLUTIONS

1. Determine the required moment strength.

   \[ M_p = 1.6M_L = 1.6 \times \frac{wL^2}{8} \]
   \[ = (1.6) \left( \frac{4 \text{ kips}}{\text{ft}} \right) \left( \frac{25 \text{ ft}}{2} \right)^2 \]
   \[ = 499 \text{ ft-kips} \]

   The required plastic moment is equal to the nominal moment.

   \[ M_p = M_n = \frac{M_x}{\phi} \]

   The required plastic section modulus is

   \[ M_p = F_yZ_x \]
   \[ M_n = F_yZ_x \]
   \[ Z_x = \frac{M_x}{F_y} \]

   \[ = \frac{(499 \text{ ft-kips}) \left( \frac{12 \text{ in}}{\text{ft}} \right)}{(0.90) \left( \frac{50 \text{ kips}}{\text{in}} \right)} \]
   \[ = 133.1 \text{ in}^3 \]

   The answer is (D).

2. The effective radius of gyration is

   \[ r_{xu} = \sqrt{\frac{I_x C_w}{S_x^2}} \]

   \[ = \sqrt{\frac{(45.1 \text{ in}^4)(3450 \text{ in}^6)}{(99.1 \text{ in}^3)^2}} \]
   \[ = 1.995 \text{ in} \]

   (The equation needed to solve this problem is no longer present in the NCEES Handbook.)

   The answer is (A).
3. The *NCEES Handbook* does not provide sufficient information to calculate the quantities involved in determining compactness. However, virtually all standard rolled W-shapes are compact. (The few exceptions are W40 x 174, W14 x 90, W14 x 90, W12 x 65, W10 x 12, W8 x 10, and W6 x 15 made from A992 steel.) Based on this fact, the beam is probably compact.

From the W-shapes dimensions and properties table, for a W14 x 68 shape, the beam depth is 14 in, and the web thickness is 0.415 in. The nominal shear strength is

\[ F_n = 0.6F_s d_w \]

\[ F_s = \frac{F_n}{0.6d_w} = \frac{146 \text{ kips}}{(0.6)(14 \text{ in})(0.415 \text{ in})} \]

\[ = 41.88 \text{ kips/in}^2 \quad (42 \text{ kips/in}^2) \]

The answer is (C).
43 Structural Steel: Columns

PRACTICE PROBLEMS

1. A steel compression member has a fixed support at one end and a frictionless ball joint support at the other as shown. The total applied design load consists of a dead load of 7 kips (which includes the weight of the member) and an unspecified live load. Design (not theoretical) effective lengths are to be used.

\[ \begin{align*}
    l_x &= 533 \text{ in}^4 \\
    I_y &= 174 \text{ in}^4 \\
    A &= 19.1 \text{ in}^2 \\
    F_y &= 50 \text{ kips/in}^2 \\
    \end{align*} \]

This compression member is controlled by which type of buckling?

(A) local  
(B) torsional  
(C) inelastic  
(D) elastic

2. A long column member has one end built-in and the other end pinned. The column is loaded in compression evenly until buckling occurs. Which statement about the column after buckling is true?

(A) The column experiences maximum deflection on its midpoint.  
(B) The maximum deflection point is closer to the pinned end than the built-in end.  
(C) The deflection curve is S-shaped.  
(D) The column experiences no deflection under buckling.

3. A solid steel column with a fixed bottom support and free upper end is concentrically loaded. Material and geometric properties are shown.

\[ \begin{align*}
    F_y &= 50 \text{ kips/in}^2 \\
    E &= 29,000 \text{ ksf/in}^2 \\
    \end{align*} \]

The available axial compressive design stress is most nearly

(A) 13 kips/in²  
(B) 18 kips/in²  
(C) 29 kips/in²  
(D) 39 kips/in²

4. A steel column is built-in at one end and free to translate and rotate at the other end. The column uses a 12 ft long W12 x 45 beam. If the yield strength of the steel is 50 kips/in², the available design stress in the column is most nearly

(A) 7.3 kips/in²  
(B) 9.0 kips/in²  
(C) 9.7 kips/in²  
(D) 10 kips/in²
SOLUTIONS

1. The effective column length factor for design use about both the x-axis and y-axis is 0.80. The unbraced length of the compression member is the same about the x-axis and the y-axis.

\[ L_x = L_y = (10 \text{ ft})(12 \text{ in/ft}) = 120 \text{ in} \]

The radius of gyration about the x-axis is

\[ r_x = \sqrt{\frac{I_x}{A}} = \sqrt{\frac{533 \text{ in}^4}{19.1 \text{ in}^2}} = 5.28 \text{ in} \]

The slenderness ratio about the x-axis is

\[ \text{slenderness ratio}_x = \frac{K_xL_x}{r_x} = \frac{(0.80)(120 \text{ in})}{5.28 \text{ in}} = 18.2 \]

The radius of gyration about the y-axis is

\[ r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{174 \text{ in}^4}{19.1 \text{ in}^2}} = 3.02 \text{ in} \]

The slenderness ratio about the y-axis is

\[ \text{slenderness ratio}_y = \frac{K_yL_y}{r_y} = \frac{(0.80)(120 \text{ in})}{3.02 \text{ in}} = 31.8 \]

The larger slenderness ratio controls, so use 31.8.

The modulus of elasticity, \( E \), is 29,000 kips/in\(^2\). The limiting slenderness ratio is

\[ 4.71\sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ kips in}^2}{50 \text{ kips in}^2}} = 113.4 \]

Since 31.8 is less than 113.4, the column fails by inelastic buckling.

The answer is (C).

2. The built-in end of the column is completely fixed against rotation and translation. The pinned end of the column is free to rotate but is fixed against translation. Therefore, the pinned end of the column experiences more rotational deflection, and the maximum deflection point is closer to the pinned end of the column.

The answer is (B).

3. Since the unsupported length of the column is the same about both the strong and weak axes, the largest slenderness ratio results from bending about the weak axis. Therefore, the least radius of gyration applies.

\[ I_{weak} = r^2A \]

\[ r = \sqrt{\frac{I_{weak}}{A}} = \sqrt{\frac{bh^3}{12}} \]

\[ = \sqrt{\frac{(9 \text{ in})(6 \text{ in})^3}{12}} = \frac{12}{(9 \text{ in})(6 \text{ in})} = 1.73 \text{ in} \]

The effective length factor, \( K \), is 2.10 for the given column end support conditions (fixed-free). Therefore, the slenderness ratio is

\[ \text{slenderness ratio} = \frac{KL}{r} = \frac{(2.10)(9 \text{ ft})(12 \text{ in/ft})}{1.73 \text{ in}} = \frac{130.94}{(131)} \]

From AISC Table 4-22, the available column strength for a compression member with a slenderness ratio of 131 is 13.2 kips/in\(^2\) (13 kips/in\(^2\)).

The answer is (A).

4. The y-axis has the smallest radius of gyration. The radius of gyration about the y-axis is 1.95 in for a W12 x 45 beam. The effective length factor is 2.10 when one end is fixed and one end is free. Use these values to determine the slenderness ratio.

\[ \text{slenderness ratio} = \frac{KL}{r} = \frac{(2.10)(12 \text{ ft})(12 \text{ in/ft})}{1.95 \text{ in}} = 155.1 \]

Determine the available design stress. The modulus of elasticity, \( E \), is 29,000 kips/in\(^2\).

\[ 4.71\sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ kips in}^2}{50 \text{ kips in}^2}} = 113.4 \]
The slenderness ratio is greater than 113.4. Therefore, the elastic buckling stress is

\[ F_e = \frac{\pi^2 E}{(KL)^2} \]

\[ = \frac{\pi^2 (29,000 \text{ kips/in}^2)}{155.1^2} \]

\[ = 11.90 \text{ kips/in}^2 \]

The available design stress is

\[ F_{ud} = 0.877F_e = (0.877)(11.90 \text{ kips/in}^2) \]

\[ = 10.44 \text{ kips/in}^2 \]

The answer is (D).
44 Structural Steel: Tension Members

PRACTICE PROBLEMS

1. A bolted steel tension member is shown.

   \[ F_p = 36 \text{ kips/in}^2 \]
   \[ F_u = 58 \text{ kips/in}^2 \]

   What is most nearly the effective net area in tension for this plate?
   
   (A) 2.3 in\(^2\)
   
   (B) 2.9 in\(^2\)
   
   (C) 3.4 in\(^2\)
   
   (D) 3.8 in\(^2\)

2. A steel tension member is 5 in long and \(1/2\) in thick. There are two holes in the bar. The holes are in parallel and have a diameter of \(1/4\) in each. The net area is most nearly

   (A) 2.0 in\(^2\)
   
   (B) 2.2 in\(^2\)
   
   (C) 2.5 in\(^2\)
   
   (D) 2.7 in\(^2\)

3. A W-shape member (yield strength of 36 ksi; ultimate strength of 58 ksi) carries an axial live tensile load of 420 kips. The member’s flanges are bolted to a connection bracket. The shear lag factor for the connection is 0.90. The required net area based on the fracture (rupture) criterion is most nearly

   (A) 12 in\(^2\)
   
   (B) 17 in\(^2\)
   
   (C) 22 in\(^2\)
   
   (D) 27 in\(^2\)

4. An \(L4 \times 4 \times 3/8\) angle made from A36 steel is used as a tension member as shown. The angle is connected to a gusset plate with \(5/8\) in diameter bolts. The center-to-center bolt spacing is 3 in. The distance from the centroid of the area connected to the plane of connection (the edge), \(z\), is 1.13 in.

   What is most nearly the shear lag factor?
   
   (A) 0.72
   
   (B) 0.78
   
   (C) 0.81
   
   (D) 0.86
3. Find the effective net area based on the fracture (rupture) criterion.

\[ \phi_t = 0.75 \]

\[ T_u = \phi_t T_n = \phi_t F_v A_e \]

\[ (1.6)(420 \text{ kips}) = (0.75)(58 \text{ kips/in}^2) A_e \]

\[ A_e = 15.45 \text{ in}^2 \]

The net area is

\[ A_n = U A_e \]

\[ A_n = \frac{A_e}{U} = \frac{15.45 \text{ in}^2}{0.90} = 17.2 \text{ in}^2 \]

The answer is (B).

4. The length of the connection for bolted connections is the distance between the outer holes.

\[ L = (2)(3 \text{ in}) = 6 \text{ in} \]

The shear lag factor is

\[ U = 1 - \frac{\pi}{L} \]

\[ = 1 - \frac{1.13 \text{ in}}{6 \text{ in}} = 0.812 \text{ (0.81)} \]

The answer is (C).
PRACTICE PROBLEMS

1. A W14 × 120, A992 steel beam has been chosen to carry an axial live compressive load of 140 kips and a factored 480 ft-kips live moment about the strong axis. The unsupported length is 20 ft. Sidesway is permitted in the direction of bending, K = 1.0. The compressive strength is 780 kips, and the bending strength is 485 ft-kips. The beam-column is subjected to

(A) small compression and is adequate
(B) large compression and is adequate
(C) small compression and is inadequate
(D) large compression and is inadequate

2. A W14 × 132 beam has been chosen to carry an axial live load of 160 kips and a maximum live moment of 320 ft-kips about the strong axis. The unsupported length is 32 ft. $C_n = 1, L = 1530$ in. Taking into account the second-order effects, what is most nearly the required flexural strength?

(A) 340 ft-kips
(B) 380 ft-kips
(C) 420 ft-kips
(D) 460 ft-kips

SOLUTIONS

1. To determine whether the member is subjected to small or large compression, find the ratio of compressive load to compressive strength.

$$\frac{P_t}{\phi P_n} = \frac{140 \text{ kips}}{780 \text{ kips}} = 0.18$$

Since $P_t/\phi P_n < 0.2$, the member is subjected to a small compression. Determine if the member is adequate.

$$\frac{P_t}{2(\phi P_n)} + \frac{M_t}{\phi M_{nt}} = \frac{140 \text{ kips}}{(2)(780 \text{ kips})} + \frac{480 \text{ ft-kips}}{495 \text{ ft-kips}} = 1.06$$

Since the computed value is greater than 1.0, the member is inadequate.

The answer is (C).

2. Find the Euler buckling load.

$$P_{el} = \frac{\pi^2 EI}{(KL)^2} = \frac{\pi^2 (29,000 \text{ kips/in}^2)(1530 \text{ in}^4)}{(1)(32 \text{ ft})(12 \text{ in/ft})^2}$$

$$= 2970 \text{ kips}$$

Find the $z-z$ axis flexural magnifier.

$$B_i = C_n \frac{P_t}{P_{el}} = \frac{1}{1 - \frac{160 \text{ kips}}{2970 \text{ kips}}} = 1.057$$

The required flexural strength is

$$M_t = B_i M_{nt} = (1.057)(320 \text{ ft-kips}) = 338.2 \text{ ft-kips}$$

The answer is (A).
Structural Steel: Connectors

PRACTICE PROBLEMS

1. The connection shown consists of 11 grade A307 
\( \frac{3}{8} \) in diameter bolts. Bolt hole sizes are standard. The 
ultimate strength of the connected member is 58 ksi. 
The connected member is 0.5 in thick. The edge clear 
distance is 2.5 in, and the center-to-center spacing of the 
holes is 3 in.

The available bearing strength per bolt per inch of 
thickness in the connection is most nearly

(A) 76 kips/in
(B) 83 kips/in
(C) 91 kips/in
(D) 110 kips/in

2. A connection is made from two \( \frac{3}{8} \) in bolts in parallel, 
placed on a 0.5 in wide steel bar. The bar is 1 in thick 
that has an ultimate strength of 65 ksi. The holes are 
3 in from their centers to the side of the bar and 2.25 in 
from center to center. Bolt hole sizes are standard. The 
-bearing resistance of the entire connection is most nearly

(A) 84 kips
(B) 95 kips
(C) 170 kips
(D) 210 kips

SOLUTIONS

1. Determine the bolt hole diameter.

\[ d_h = d_b + \frac{d}{b} \text{ in} \]
\[ = \frac{3}{8} \text{ in} + \frac{1}{16} \text{ in} \]
\[ = \frac{13}{16} \text{ in} \]

Determine which clear distance, \( L_c \), value controls for 
the bolts. Obtain the clear distance in between the 
interior holes.

\[ L_c = s - d_b \]
\[ = 3 \text{ in} - \frac{13}{16} \text{ in} \]
\[ = \frac{15}{16} \text{ in} < 2 \frac{1}{2} \text{ in provided} \]

The smaller value for the clear distance controls. Determine 
the available bearing strength.

\[ \phi r_n = \phi 1.2 L_c F_s \leq \phi 2.4 d_b F_s \]
\[ = (0.75)(1.2)(2 \frac{1}{16} \text{ in}) \left( \frac{58 \text{ kips}}{\text{in}^2} \right) \]
\[ \leq (0.75)(2.4)\left( \frac{3}{8} \text{ in} \right) \left( \frac{58 \text{ kips}}{\text{in}^2} \right) \]
\[ = 107.7 \text{ kips/in} \leq 91.35 \text{ kips/in} \]

Since the available bearing strength determined by the 
clear distance is larger than the available bearing strength 
determined by the bolt size, the smaller value 
controls, and the available bearing strength is 91 kips/in.

This value may also be determined using AISC tables 
for a center-to-center spacing of 3 in, a connected mem-
ber strength of 58 ksi, and a bolt size of \( \frac{3}{8} \) in.

The answer is (C).

2. Calculate the hole diameter using the bolt diameter 
and standard clearance.

\[ d_h = d_b + \frac{d}{b} \text{ in} \]
\[ = \frac{3}{8} \text{ in} + \frac{1}{16} \text{ in} \]
\[ = \frac{13}{16} \text{ in} \quad (0.8125 \text{ in}) \]
The center-to-center spacing of the interior holes is 3 in. Obtain the clear distance between the interior holes.

\[ L_c = s - d_h \]
\[ = 2.25 \text{ in} - 0.8125 \text{ in} \]
\[ = 1.4375 \text{ in} \]

Determine the clear distance between the hole and the edge of the steel bar.

\[ L_a = L_c - \frac{d_h}{2} \]
\[ = 3 \text{ in} - \frac{0.8125 \text{ in}}{2} \]
\[ = 2.594 \text{ in} \]

Use the smaller clear distance. Since each bolt has the same properties, calculate the available bearing strength for one bolt.

\[ r_n = \min \left\{ \phi 1.2 L_c F_u, \phi 2.4 d_h F_u \right\} \]
\[ = \min \left\{ \left( 0.75 \right) \left( 1.2 \right) \left( 1.435 \text{ in} \right) \left( 65 \frac{\text{kips}}{\text{in}^2} \right), \left( 0.75 \right) \left( 2.4 \right) \left( 0.75 \text{ in} \right) \left( 65 \frac{\text{kips}}{\text{in}^2} \right) \right\} \]
\[ = \min \left\{ 84.09 \frac{\text{kips}}{\text{in}}, 87.75 \frac{\text{kips}}{\text{in}} \right\} \]
\[ = 84.09 \frac{\text{kips}}{\text{in}} \]

The available bearing strength is

\[ \phi R_n = \sum \phi r_n t \]
\[ = (2 \text{ bolts}) \left( 84.09 \frac{\text{kips}}{\text{bolt-in}} \right) (1 \text{ in}) \]
\[ = 168.2 \text{ kips} \quad (170 \text{ kips}) \]

The answer is (C).
PRACTICE PROBLEMS

1. Which statement about the modulus of elasticity, $E$, is true?
   (A) It is the same as the rupture modulus.
   (B) It is the slope of the stress-strain curve in the linearly elastic region.
   (C) It is the ratio of stress to volumetric strain.
   (D) Its value depends only on the temperature of the material.

2. What modulus of elasticity is predicted by ACI 318 for normal weight concrete with a compressive strength of 3000 lb/in^2?
   (A) $2.0 \times 10^6$ psi
   (B) $1.9 \times 10^6$ psi
   (C) $2.8 \times 10^6$ psi
   (D) $3.2 \times 10^6$ psi

3. Which of the following criteria must be met in order for the compressive strength of concrete to be satisfied?
   I. No single strength test falls below the specified compressive strength, $f'_c$, by more than $0.10f'_c$.
   II. No single strength test falls below the specified compressive strength, $f'_c$, by more than $0.20f'_c$.
   III. The average of every three consecutive strength tests equals or exceeds the specified compressive strength.
   IV. The average of every three consecutive strength tests must not equal or exceed the specified compressive strength.
   (A) I and III
   (B) I and IV
   (C) II and III
   (D) II and IV

4. A waste product of coal-burning power-generation stations, fly ash, is the most common pozzolanic additive. Which of the following statements are true about fly ash?
   I. Fly ash reacts with calcium hydroxide to increase binding.
   II. Fly ash reacts with calcium silicate to form a binder.
   III. Fly ash acts as a microfiller between cement particles, increasing strength and durability while reducing permeability.
   IV. When used as a replacement for less than 45% of the portland cement, fly ash meeting ASTM C618 enhances resistance to scaling from road-deicing chemicals.
   (A) I and II
   (B) III and IV
   (C) I, III, and IV
   (D) II, III, and IV

5. Which category of steel contains 0.15–0.29% carbon?
   (A) low-carbon
   (B) mild-carbon
   (C) medium-carbon
   (D) high-carbon

6. Which property of steel allows it to undergo large inelastic deformations without fracture?
   (A) yield
   (B) elasticity
   (C) ductility
   (D) toughness
7. A simply supported reinforced concrete beam 12 in wide and 28 in deep spans 20 ft. The beam is subjected to a uniform service dead load equal to 2.0 kips/ft (exclusive of beam weight) and to a uniform service live load of 2.4 kips/ft. The factored uniform load is most nearly

(A) 6.7 kips/ft
(B) 7.4 kips/ft
(C) 8.0 kips/ft
(D) 9.2 kips/ft

8. A simply supported reinforced concrete beam 12 in wide, 24 in deep, and 30 ft long is subjected to a dead load of 1.2 kips/ft and a live load of 0.5 kip/ft in addition to its own dead weight. Most nearly, what moment should be used to determine the steel reinforcement at the center of the beam?

(A) 12 ft-kips
(B) 350 ft-kips
(C) 380 ft-kips
(D) 420 ft-kips

SOLUTIONS

1. The modulus of elasticity is the slope of the stress-strain diagram in the linearly elastic region.

The answer is (B).

2. The modulus of elasticity is

\[ E_c = 33 \left( \frac{154,000 \text{ psi}}{\text{ft}^2} \right)^{1.5} \sqrt{3000 \frac{\text{lb}}{\text{in}^2}} \]

= \[ 3.16 \times 10^6 \text{ psi} \]

Note that this equation is not dimensionally consistent.

The answer is (D).

3. The compressive strength of concrete is considered satisfactory if two criteria are met: (a) no single strength test falls below the specified compressive strength, \( f_{cu} \), by more than 0.10\( f_{cu} \), and (b) the average of every three consecutive strength tests equals or exceeds the specified compressive strength.

The answer is (A).

4. Fly ash reacts with calcium hydroxide to increase binding, acts as a microfiller between cement particles to increase strength and durability while decreasing permeability, and enhances resistance to scaling from road-deicing chemicals when it meets ASTM C618 and is used as a replacement for less than 45% of the portland cement.

Calcium silicate hydrate is a binder that holds concrete together on its own.

The answer is (C).

5. Carbon steels are divided into four categories based on the percentage of carbon: low-carbon (less than 0.15%), mild-carbon (0.15–0.29%), medium-carbon (0.30–0.59%), and high-carbon (0.60–1.70%).

The answer is (B).

6. Yield stress is the unit tensile stress at which the stress-strain curve exhibits a well-defined increase in strain without an increase in stress. The modulus of elasticity is the slope of the initial straight-line portion of the stress-strain diagram. Toughness is the ability of a specimen to absorb energy. Ductility is the ability of steel to undergo large inelastic deformations without fracture.

The answer is (C).
7. Although the unit weight of unreinforced normal weight concrete is taken as 145 lbf/ft³, the unit weight of reinforced normal weight concrete is usually assumed to be 150 lbf/ft³. The weight of the beam per unit length is

\[
W = bhw = \frac{(12 \text{ in})(28 \text{ in})}{(12 \text{ in})^2} \left( \frac{150 \text{ lbf}}{\text{ft}^2} \right) = 0.35 \text{ kip/ft}
\]

The factored uniform load is the maximum of

\[
U = 1.4D = (1.4) \left( \frac{2.0 \text{ kips}}{\text{ft}} + 0.35 \frac{\text{kip}}{\text{ft}} \right)
\]

\[
= 3.29 \text{ kips/ft} \quad \text{[does not control]}
\]

\[
U = 1.2D + 1.6L
\]

\[
= (1.2) \left( 2.0 \frac{\text{kips}}{\text{ft}} + 0.35 \frac{\text{kip}}{\text{ft}} \right) + (1.6) \left( 2.4 \frac{\text{kips}}{\text{ft}} \right)
\]

\[
= 6.66 \text{ kips/ft} \quad (6.7 \text{ kips/ft})
\]

The larger value controls, so use 6.7 kips/ft.

**The answer is (A).**

8. The specific weight of steel-reinforced concrete is assumed to be 150 lbf/ft³. The weight of the beam is

\[
W = bhw = \frac{(12 \text{ in})(24 \text{ in})}{(12 \text{ in})^2} \left( \frac{150 \text{ lbf}}{\text{ft}^2} \right) = 0.3 \text{ kip/ft}
\]

The factored uniform load is

\[
U = 1.2D + 1.6L
\]

\[
= (1.2) \left( 1.2 \frac{\text{kips}}{\text{ft}} + 0.3 \frac{\text{kip}}{\text{ft}} \right) + (1.6) \left( 0.8 \frac{\text{kip}}{\text{ft}} \right)
\]

\[
= 3.08 \text{ kips/ft}
\]

For a uniformly loaded beam, the factored moment is maximum at the center of the beam and is

\[
M_{f} = \frac{UL^2}{8} = \frac{(3.08 \frac{\text{kip}}{\text{ft}})(30 \text{ ft})^2}{8}
\]

\[
= 347 \text{ ft-kips} \quad (350 \text{ ft-kips})
\]

**The answer is (B).**
37  Reinforced Concrete: Beams

PRACTICE PROBLEMS

1. The span length and cross section of a reinforced concrete beam are shown. The beam is underreinforced. The concrete and reinforcing steel properties are $f' = 3000$ lb/ft$^2$, $f_y = 40,000$ psi, and $A_s = 3$ in$^2$.

   Neglecting beam self-weight and based only on the allowable moment capacity of the beam as determined using American Concrete Institute (ACI) strength design specifications, the maximum allowable live load is most nearly
   (A) 23,000 lbf
   (B) 29,000 lbf
   (C) 35,000 lbf
   (D) 50,000 lbf

2. A floor system consists of ten 30 ft long reinforced concrete beams and a continuous 5 in deck slab. (A typical section is shown for the deck and two of the beams.) Assume the beams are underreinforced.

   For each beam in the floor system, the ACI-specified effective top flange width is most nearly
   (A) 36 in
   (B) 50 in
   (C) 60 in
   (D) 90 in

3. The cross section of a reinforced concrete beam with tension reinforcement is shown. Assume that the beam is underreinforced.

   If the dead load shear force in the beam is 5 kips and the live load shear force in the beam is 15 kips, then the minimum amount of shear reinforcement needed for a center-to-center stirrup spacing of 8 in based on ACI strength design is most nearly
   (A) 0.10 in$^2$
   (B) 0.12 in$^2$
   (C) 0.14 in$^2$
   (D) 0.18 in$^2$
4. The span length and cross section of a reinforced concrete beam are shown. The beam is underreinforced. The concrete and reinforcing steel properties are $f'_c = 2500 \text{ lb/ft}^2$, $f_y = 50,000 \text{ lb/in}^2$, and $A_s = 3.8 \text{ in}^2$.

Assume the effective flange width for this beam is 40 in. If the area of reinforcing steel per beam is 6.00 in$^2$, the nominal moment capacity of each beam based on ACI strength design is most nearly

(A) 150 ft-kips 
(B) 160 ft-kips 
(C) 520 ft-kips 
(D) 650 ft-kips

5. A floor system consists of 30 reinforced concrete beams and a continuous 4 in deck slab. (A typical section is shown for the deck and two of the beams.) Assume the beams are underreinforced.

The beam supports a concentrated live load of 50,000 lb. Neglect beam self-weight. The minimum amount of shear reinforcement required for a center-to-center stirrup spacing of 12 in under ACI strength design specifications is most nearly

(A) 0.17 in$^2$ 
(B) 0.23 in$^2$ 
(C) 0.38 in$^2$ 
(D) 0.78 in$^2$

6. The span length and cross section of a reinforced concrete beam are shown. The beam is underreinforced. The concrete and reinforcing steel properties are $f'_c = 3100 \text{ lb/ft}^2$, $f_y = 35,000 \text{ lb/in}^2$, and $A_s = 2.5 \text{ in}^2$.

The balanced reinforcing steel ratio for this beam in accordance with ACI specifications is most nearly

(A) 0.037 
(B) 0.046 
(C) 0.051 
(D) 0.058

7. A floor system consists of 20 reinforced concrete beams and a continuous 3 in deck slab. (A typical section is shown for the deck and two of the beams.) Assume the beams are underreinforced.

$f'_c = 2800 \text{ lb/ft}^2$
$f_y = 42,000 \text{ lb/in}^2$
$L = 30 \text{ ft} \ [\text{simple span length}]$

$f'_c = 3000 \text{ lb/ft}^2$
$f_y = 60,000 \text{ lb/in}^2$
$L = 30 \text{ ft} \ [\text{simple span length}]$
Assume the effective flange width for this beam is 48 in. If the area of reinforcing steel per beam is 7.25 in², the nominal moment capacity of each beam based on ACI strength design is most nearly

(A) 680 ft-kips
(B) 770 ft-kips
(C) 800 ft-kips
(D) 880 ft-kips

8. The cross section of a reinforced concrete beam with compression reinforcement is shown.

\[ f'_c = 3000 \text{ lb/ft}^2 \]
\[ f_s = 40,000 \text{ lb/ft}^2 \]
\[ A_s = 3 \text{ in}^2 \]
\[ A'_s = 1 \text{ in}^2 \]

The nominal moment capacity of the beam is most nearly

(A) 130 ft-kips
(B) 150 ft-kips
(C) 170 ft-kips
(D) 190 ft-kips

9. A monolithic slab-beam floor system is supported on a column grid of 18 ft on centers as shown. The dimensions of the cross section for the beams running in the north-south direction have been determined.

What is most nearly the effective flange width?

(A) 45 in
(B) 54 in
(C) 63 in
(D) 72 in
10. A reinforced concrete T-beam with a 30 ft span and a 30 in effective width in a floor slab system is fixed at both ends and is reinforced as shown. $f'_c = 3000$ lb/ft$^2$, $A_s = 3$ in$^2$, and $f_y = 60,000$ lb/ft$^2$. The stress block is within the flange.

![Diagram of T-beam and stress block]

**SOLUTIONS**

1. The height of the stress block is

$$a = \frac{A_s f_y}{0.85 f'_c b} = \frac{(3 \text{ in}^2)(40,000 \text{ lb/ft}^2)}{(0.85)(3000 \text{ lb/ft}^2)(12 \text{ in})} = 3.92 \text{ in}$$

For flexure, the strength reduction factor, $\phi$, is 0.90.

$$M_s = \phi M_u = \phi A_f (d - a/2)$$

$$= (0.90)(3 \text{ in}^2)(40,000 \text{ lb/ft}^2)(15 \text{ in} - 3.92 \text{ in})$$

$$= 117,350 \text{ ft-lbf}$$

The maximum bending moment occurs at midspan.

$$M_s = 1.2 M_u + 1.6 M_L$$

$$= 1.2 \times 117,350 + 1.6 \frac{P_L L}{8}$$

This can be solved for the maximum allowable live load.

$$P_L = \frac{4}{1.6 L} \left( \frac{117,350 \text{ ft-lbf}}{8} \right)$$

$$= 29319 \text{ lb} / (29,000 \text{ lb})$$

The answer is (B).

2. The effective flange width is

$$b_e = \frac{1}{12} (\text{span length}) = \frac{1}{12} (30 \text{ ft})(12 \text{ in/ft})$$

$$= 90 \text{ in}$$

$$b_e + 16 h_f = 10 \text{ in} + (16)(5 \text{ in})$$

$$= 90 \text{ in}$$

Therefore, the effective flange width is 50 in.

The answer is (B).
3. For shear, the capacity reduction ratio is $\phi = 0.75$. The ultimate shear force in the beam is

$$V_u = 1.2V_D + 1.6V_L$$

$$= (1.2)(5 \text{ kips}) + (1.6)(15 \text{ kips})$$

$$= 30 \text{ kips}$$

The nominal concrete shear strength is

$$V_c = 2kh_d\sqrt{f_s} = (2)(12 \text{ in})(20 \text{ in})\sqrt{(2500 \text{ lbf in}^2)^{\frac{1}{2}}}$$

$$= 26,29 kips$$

$$\phi V_c = \frac{(0.75)(26.29 \text{ kips})}{2}$$

$$= 9.9 \text{ kips}$$

Since $V_u > \phi V_c$, shear reinforcement is required.

Since $V_u = 30 \text{ kips} > \phi V_c = 19.72 \text{ kips}$, the required shear strength provided by the steel is

$$V_s = \frac{V_u}{0.75} - 26.29 \text{ kips} = 13.71 \text{ kips}$$

The required steel area is

$$A_s = \frac{Vs}{f_y} = \frac{(8 \text{ in})(13.71 \text{ kips})\left(1000 \text{ lbf kip}^{-1}\right)}{(40,000 \text{ lbf in}^2)(20 \text{ in})}$$

$$= 0.1371 \text{ in}^2 (0.14 \text{ in}^2)$$

Although not required for this problem, in an actual design and analysis situation, a check should be made to ensure that $V_s$ does not exceed the ACI-allowed maximum shear reinforcement given by $V_{s,max} = 8\sqrt{f_s}bd$.

The answer is (C).

4. For shear, the strength reduction factor, $\phi$, is 0.75. The maximum factored shear force in the beam is at either one of the supports and is

$$V_u = 1.2V_D + 1.6V_L = 1.2 \times \frac{wL^2}{2} + 1.6 \times \frac{P_l}{2}$$

$$= (1.2)\left(\frac{12 \text{ lbf}}{\text{in}}\right)(10 \text{ ft}) + (1.6)(50,000 \text{ lbf})$$

$$= 40,072 \text{ lbf}$$

The nominal concrete shear strength is

$$V_c = 2kh_d\sqrt{f_s} = (2)(12 \text{ in})(18 \text{ in})\sqrt{2500 \text{ lbf in}^2}$$

$$= 25,200 \text{ lbf}$$

$$\phi V_c = \frac{(0.75)(25,200 \text{ lbf})}{2}$$

$$= 9450 \text{ lbf}$$

$$V_s > \phi V_c$$

Therefore, shear reinforcement is required.

In accordance with ACI specifications, the minimum required amount of shear reinforcement for a stirrup spacing of 12 in is

$$A_s = \frac{50b_s}{f_y} = \frac{(50)(14 \text{ in})(12 \text{ in})}{50,000 \text{ lbf in}^2}$$

$$= 0.168 \text{ in}^2$$

The amount of shear reinforcement based on factored loading can be determined as follows.

$$V_s = A_s f_y d$$

$$\phi(V_c + V_s) \geq V_u$$

$$A_s = \frac{V_u - \phi V_c}{\phi f_y d_s}$$

$$= \frac{40,072 \text{ lbf} - (0.75)(25,200 \text{ lbf})}{(0.75)(50,000 \text{ lbf in}^2)(18 \text{ in})}$$

$$= 0.370 \text{ in}^2 (0.38 \text{ in}^2)$$

The larger value for $A_s$ controls. Use $A_s = 0.38 \text{ in}^2$.

The answer is (C).

5. This problem asks for the nominal moment capacity, $M_n$, not the allowable moment capacity, $\phi M_e$. Therefore, the reduction factor, $\phi$, is not needed.

First, assume that each beam is rectangular with a width $b = b_s = 40$ in. The depth of the concrete compressive stress block is

$$a = \frac{A_s f_y}{0.85f_y' b}$$

$$= \frac{(6.00 \text{ in}^2)(42,000 \text{ lbf in}^2)}{(0.85)(2800 \text{ lbf in}^2)(40 \text{ in})}$$

$$= 2.65 \text{ in}$$
Since \( a < 4 \text{ in} \), the nominal moment capacity of the beam is the same as for a rectangular singly reinforced concrete beam. Using \( b = b_n = 40 \text{ in} \), the nominal moment capacity is

\[
M_n = 0.85f'\text{c}ab\left(d - \frac{a}{2}\right)
\]

\[
= (0.85)\left(\frac{2800 \text{ lb/ft}^2}{1000 \text{ lb/kip}}\right)\left(40 \text{ in} - \frac{40}{2} \text{ in}\right)
\]

\[
= \frac{12 \text{ in}}{1000 \text{ lb/kip}} \times 26 \text{ in} - 2.65 \text{ in}
\]

\[
= 520 \text{ ft-kips}
\]

This can also be calculated by using the following equation in accordance with rectangular singly reinforced concrete beam theory.

\[
M_n = A_f f_y \left(d - \frac{a}{2}\right)
\]

Even though the problem statement assumes that the beam is underreinforced, the actual reinforcing steel ratio and its limits should always be checked in real design and analysis problems.

The answer is (C).

6. The ratio of the rectangular stress block depth to the neutral axis depth is

\[
\beta_i = 0.85 \geq \left[0.85 - 0.05\left(\frac{f'_c - 4000}{1000}\right)\right] \geq 0.65
\]

Since \( f'_c < 4000 \text{ lb/in}^2 \), \( \beta_i = 0.85 \). The reinforcement ratio is

\[
\rho = \frac{A_s}{bd}\]

Expressions for \( A_s \) and \( d_i \) are needed.

From similar triangles,

\[
\frac{c}{d_i} = \frac{\varepsilon_f}{\varepsilon_y + \varepsilon_s}
\]

For a balanced condition, the concrete strain is 0.003, and the steel is at yield.

\[
\frac{c}{d_i} = \frac{\varepsilon_f}{\varepsilon_y + \varepsilon_s} = \frac{\varepsilon_f}{\varepsilon_y + \frac{f_y}{E_s}}
\]

The modulus of elasticity for steel is \( E_s = 29 \times 10^6 \text{ lb/ft}^2 \). Since \( \varepsilon_y = 0.003 \), for the balanced condition,

\[
\frac{1}{d_i} = \frac{87,000}{\varepsilon_y (87,000 + f_y)}
\]

The required steel area can be found from the nominal moment strength.

\[
M_n = 0.85f'\text{c}ab\left(d - \frac{a}{2}\right)
\]

\[
= A_f f_y \left(d - \frac{a}{2}\right)
\]

\[
A_s = \frac{0.85f'\text{c}ab}{f_y}
\]

And, since \( a = \beta_i c \),

\[
A_s = \frac{0.85f'\text{c}\beta_i c}{f_y}
\]

Combining the expressions for \( \rho \), \( 1/d_i \), and \( A_s \), the balanced reinforcement ratio is

\[
\rho_s = \left(\frac{0.85\beta_i f'_c}{f_y}\right) \left(\frac{87,000}{87,000 + f_y}\right)
\]

\[
= \left(\frac{0.85\beta_i f'_c}{f_y}\right) \left(\frac{87,000 \text{ lb/ft}^2}{87,000 \text{ lb/ft}^2 + f_y}\right)
\]

\[
= \left(0.85\beta_i\right) \left(\frac{87,000 \text{ lb/ft}^2}{35,000 \text{ lb/ft}^2}\right)
\]

\[
= \frac{87,000 \text{ lb/ft}^2}{35,000 \text{ lb/ft}^2}
\]

\[
= 0.0456 \quad (0.046)
\]

The answer is (B).

7. This problem asks for the nominal moment capacity, \( M_n \), not the allowable moment capacity, \( \phi M_n \). Therefore, the reduction factor, \( \phi \), is not needed.

The depth of the concrete compressive stress block must be checked to see whether or not it exceeds the 3 in deck thickness. If the depth of this compressive stress block exceeds the deck thickness, then each beam is a T-beam
and T-beam formulas apply for determination of the nominal moment capacity. If, however, the depth of the concrete compressive stress block does not exceed the deck thickness, then each beam is a rectangular beam and rectangular beam formulas apply for determination of the nominal moment capacity.

\[ b = b_y = 48 \text{ in} \]
\[ h_l = 3 \text{ in} \]

First, assume that each beam is rectangular with a width of \( b = b_y = 48 \text{ in} \). The depth of the concrete compressive stress block is

\[ a = \frac{A_y f_y}{0.85 f'_c b_y} = \frac{(7.25 \text{ in}^2)(60,000 \frac{\text{lb}}{\text{in}^2})}{(0.85)(3000 \frac{\text{lb}}{\text{in}^2})(48 \text{ in})} = 3.55 \text{ in} \]

Since \( a > \) slab depth of 3 in, the beam is a T-beam.

Since \( a = 3.55 \) in was found by assuming that the beam was a rectangular beam with width \( b = 48 \) in, this depth only indicates whether or not the beam is a T-beam and is not the correct depth for determining the nominal moment capacity. The correct depth is now found by applying the concepts of static equilibrium to the beam.

To find the correct depth, \( a \), sum horizontal forces in the T-beam to show that the upper (above the neutral axis) compressive concrete stress block force is equal to the lower (below the neutral axis) maximum tensile force sustained by the reinforcing bars. By dividing the entire concrete compressive stress block section into three parts (a rectangular part and two overhanging flanges), depth \( a \) can be found.

\[ A_f = (b_y - b_w) f_y \]
\[ = (48 \text{ in} - 12 \text{ in})(3 \text{ in}) \]
\[ = 108 \text{ in}^2 \]

From equilibrium of horizontal forces,

\[ 0.85 f'_c A_e = A_y f_y \]
\[ A_e = \frac{A_y f_y}{0.85 f'_c} \]
\[ = \frac{(7.25 \text{ in}^2)(60,000 \frac{\text{lb}}{\text{in}^2})}{(0.85)(3000 \frac{\text{lb}}{\text{in}^2})} \]
\[ = 170.59 \text{ in}^2 \]
\[ A_e = b_y a = A_x - A_f \]
\[ a = \frac{A_x - A_f}{b_w} \]
\[ = \frac{170.59 \text{ in}^2 - 108 \text{ in}^2}{12 \text{ in}} \]
\[ = 5.22 \text{ in} \]

Alternatively, a redefined stress block depth could be used.

\[ a = \frac{A_y f_y}{0.85 f'_c b_w} \cdot \frac{h_l (b_y - b_w)}{b_w} \]
\[ = \frac{(7.25 \text{ in}^2)(60,000 \frac{\text{lb}}{\text{in}^2})}{(0.85)(3000 \frac{\text{lb}}{\text{in}^2})(12 \text{ in})} \cdot \frac{3 \text{ in}}{(48 \text{ in} - 12 \text{ in})} \]
\[ = 5.22 \text{ in} \]

The nominal moment capacity of the T-beam is

\[ M_n = 0.85 f'_c h_f (b_y - b_w) \left( d - \frac{h_f}{2} \right) + 0.85 f'_c a b_w \left( d - \frac{3}{2} \right) \]
\[ + (0.85)(3000 \frac{\text{lb}}{\text{in}^2})(3 \text{ in}) \times (48 \text{ in} - 12 \text{ in}) \times (23 \text{ in} - 3 \text{ in} - 5.22 \text{ in}) \]
\[ = \frac{(12 \text{ in})(23 \text{ in} - 5.22 \text{ in})}{1000 \frac{\text{lb}}{\text{kip}}} \]
\[ = 765 \text{ ft-kips} \]

Even though the problem statement assumes that the beam is underreinforced, the actual reinforcing steel ratio and its limits should always be checked in real design and analysis problems.

The answer is (B).
8. Determine whether compression steel yields.

\[
A_s - A'_s = 3 \text{ in}^2 - 1 \text{ in}^2 = 2 \text{ in}^2
\]

\[
\frac{0.85 f_y d b}{f_y} = \frac{87,000}{87,000 - f_y}
\]

\[
= \frac{(0.85)(0.85)(3000 \frac{\text{lb}f}{\text{in}^2})(1 \text{ in})(12 \text{ in})}{40,000 \frac{\text{lb}f}{\text{in}^2}}
\]

\[
\times \left( \frac{87,000 \frac{\text{lb}f}{\text{in}^2}}{87,000 \frac{\text{lb}f}{\text{in}^2} - 40,000 \frac{\text{lb}f}{\text{in}^2}} \right)
\]

\[
= 1.2 \text{ in}^2
\]

Compare both sides of the equation.

\[
A_s - A'_s \geq 0.85 f_y d b \left( \frac{87,000}{87,000 - f_y} \right)
\]

\[
2 \text{ in}^2 \geq 1.2 \text{ in}^2
\]

Therefore, compression steel yields.

Calculate the depth of the concrete compressive stress block.

\[
a = \frac{A_s - A'_s}{0.85 f_y b} = \frac{(3 \text{ in}^2 - 1 \text{ in}^2)(40,000 \frac{\text{lb}f}{\text{in}^2})}{(0.85)(3000 \frac{\text{lb}f}{\text{in}^2})(12 \text{ in})}
\]

\[
= 2.61 \text{ in}
\]

The nominal moment strength is

\[
M_n = f_y \left( A_s - A'_s \right) \left( d - \frac{d}{2} \right) + A'_s \left( d - d' \right)
\]

\[
= \frac{(40,000 \frac{\text{lb}f}{\text{in}^2})(3 \text{ in}^2 - 1 \text{ in}^2)(20 \text{ in} - 2.61 \text{ in})}{(12 \text{ in})(1000 \frac{\text{lb}f}{\text{kip}})}
\]

\[
= 187.95 \text{ ft-kips} \quad (190 \text{ ft-kips})
\]

The answer is (D).

9. The effective width of the flange is determined as the smallest of

\[
\left\{(\frac{d}{2})(\text{span length}) = (\frac{d}{2})(18 \text{ ft})(12 \frac{\text{ in}}{\text{ ft}})\right\}
\]

\[
= 54 \text{ in}
\]

\[
b_w = \text{smallest} = 63 \text{ in}
\]

\[
\text{beam centerline spacing} = (6 \text{ ft})(12 \frac{\text{ in}}{\text{ ft}})
\]

\[
= 72 \text{ in}
\]

Therefore, the effective flange width is 54 in.

The answer is (B).

10. The positive moment region for slabs is the central region, so cross-section B-B is applicable. Since the stress block is within the flange, a < b_w and the T-beam can be analyzed like a rectangular beam. The distance from the extreme compression fibers to the extreme tensile steel is

\[
d = 4 \text{ in} + 26 \text{ in} - 2.5 \text{ in}
\]

\[
= 27.5 \text{ in}
\]

Determine the height of the stress block.

\[
a = \frac{A_s f_y}{0.85 f_y b_w} = \frac{(3 \text{ in}^2)(60 \frac{\text{kips}}{\text{in}^2})}{(0.85)(2 \frac{\text{kips}}{\text{in}^2}(30 \text{ in})}
\]

\[
= 2.35 \text{ in}
\]

The moment capacity can be calculated from either the steel properties (with b = b_w) or from the concrete properties. Using the steel properties,

\[
M_u = A_s f_y \left( d - \frac{d}{2} \right)
\]

\[
= \frac{(3 \text{ in}^2)(60 \frac{\text{kips}}{\text{in}^2})(27.5 \text{ in} - 2.35 \text{ in})}{2}
\]

\[
= 394.9 \text{ ft-kips} \quad (390 \text{ ft-kips})
\]
Using the concrete properties,

\[ M_n = 0.85f'_c a b_n \left( d - \frac{a}{2} \right) \]

\[ \left(0.85\right)\left(3 \text{ kips/} \text{in}^2\right)\left(2.35 \text{ in}\right)\left(30 \text{ in}\right) \]

\[ = \frac{x \left(27.5 \text{ in} \times \frac{2.35 \text{ in}}{2}\right)}{12 \text{ in/ft}} \]

\[ = 394.9 \text{ ft-kips} \]

The answer is (C).
### Reinforced Concrete: Columns

#### Practice Problems

1. For the short, concentrically loaded round tied column shown, the applied axial dead load is 150 kips, and the applied axial live load is 350 kips.

\[
\begin{align*}
\sigma_c &= 4000 \text{ lb/ft}^2 \\
\sigma_p &= 60,000 \text{ lb/ft}^2
\end{align*}
\]

Assuming that the longitudinal reinforcing bars are all the same size, the minimum required size of each longitudinal reinforcing bar is

(A) no. 3  
(B) no. 4  
(C) no. 5  
(D) no. 6

2. For the short, concentrically loaded square tied column shown, the applied axial dead load is 150 kips, and the applied axial live load is 250 kips.

\[
\begin{align*}
\sigma_c &= 4000 \text{ lb/ft}^2 \\
\sigma_p &= 60,000 \text{ lb/ft}^2
\end{align*}
\]

Assuming that the longitudinal reinforcing bars are all the same size, the minimum required size of each longitudinal reinforcing bar is

(A) no. 10  
(B) no. 11  
(C) no. 12  
(D) no. 14

3. A reinforced concrete tied column is subjected to a design axial compression force of 1090 kips that is concentrically applied. Slenderness effects are negligible, and the column is to be designed using ACI 318. Given a specified compressive strength of 5000 psi, grade 60 rebars, and a specified longitudinal steel ratio of 0.02, what is most nearly the width of the sides of the smallest square column that will support the load?

(A) 12 in  
(B) 16 in  
(C) 20 in  
(D) 24 in

4. A 16 in (gross dimension) square, tied column must carry 220 kip dead and 250 kip live loads. The dead load includes the column self-weight. The column is not exposed to any moments. Sideways is prevented at the top, and slenderness effects are to be disregarded. The concrete compressive strength is 4000 lb/ft\(^2\), and the steel tensile yield strength is 60,000 lb/ft\(^2\). The longitudinal reinforcement of this column is most nearly

(A) 0.028  
(B) 0.061  
(C) 0.092  
(D) 0.11

5. An 18 in square tied column is reinforced with 12 no. 9 grade 60 bars and has a concrete compressive strength of 4000 lb/ft\(^2\). The column, which is braced against side sway, has an unsupported height of 9 ft and supports axial load only without end moments. What is most nearly the design axial load capacity?

(A) 930 kips  
(B) 970 kips  
(C) 1800 kips  
(D) 1900 kips
6. The short tied column shown uses eight no. 8 bars. Assume the loading has a low eccentricity, $f_e = 3500$ lbf/in$^2$, and $f_y = 40,000$ lbf/in$^2$.

The design strength is most nearly:

(A) 710 kips
(B) 890 kips
(C) 1100 kips
(D) 7100 kips

---

**SOLUTIONS**

1. Determine the amount of reinforcing steel required by the minimum required reinforcement ratio, $\rho_y$, of 0.01. The minimum area of reinforcing steel required is

$$A_s = \rho_y A_y = (0.01)\left(\frac{\pi(18 \text{ in})^2}{4}\right) = 2.54 \text{ in}^2$$

Determine the required amount of reinforcing steel based on the factored axial load, $P_f$:

$$P_f = 1.2P_D + 1.6P_t$$
$$= (1.2)(150 \text{ kips}) + (1.6)(350 \text{ kips})$$
$$= 740 \text{ kips}$$

The nominal axial compressive load capacity is

$$P_n = 0.80P_e$$
$$= (0.80)(0.85f_y A_{concrete} + f_y A_y)$$
$$= (0.80)(0.85f_y (A_y - A_s) + f_y A_y)$$

It is required that $\phi P_n \geq P_f$. For axial compression with tied reinforcement, $\phi = 0.65$. Setting $\phi P_n = P_f$ and solving for the area of longitudinal reinforcing steel gives

$$A_y = \frac{P_f}{0.80\phi - 0.85f_y}$$
$$= \frac{740 \text{ kips} \left(\frac{1000 \text{ lbf}}{\text{kip}}\right)}{(0.80)(0.65)}$$
$$= 9.86 \text{ in}^2$$

$A_y = 9.86 \text{ in}^2$ is greater than $A_s = 2.54 \text{ in}^2$ based on the minimum allowed reinforcement ratio, $\rho_y = 0.01$. The minimum required area of reinforcement is $A_s = 2.54 \text{ in}^2$.

The column has six longitudinal reinforcing bars. The required area of each longitudinal reinforcing bar is

$$A = \frac{A_y}{n_{bars}} = \frac{9.86 \text{ in}^2}{6 \text{ bars}} = 1.64 \text{ in}^2/\text{bar}$$

A bar area of 1.64 in$^2$ is satisfied by a no. 14 bar, which has a nominal area of 2.25 in$^2$. 
In an actual design and analysis situation, a check should also be made to see that the actual longitudinal reinforcement ratio does not exceed the maximum allowable ratio of 0.08.

**The answer is (D).**

2. The minimum required reinforcement ratio, \( \rho_v \), is 0.01. The minimum area of reinforcing steel required is

\[
A_s = \rho_v A_y = (0.01)(18 \text{ in}^2) = 0.18 \text{ in}^2
\]

Determine the required amount of reinforcing steel based on the factored axial load, \( P_v \)

\[
P_v = 1.2P_D + 1.6P_L = (1.2)(150 \text{ kips}) + (1.6)(250 \text{ kips}) = 580 \text{ kips}
\]

The nominal axial compressive load capacity is

\[
P_n = 0.80P_v = (0.80)(0.85f'cA_{concrete} + f_y A_s) = (0.80)(0.85f'c(A_y - A_i) + f_y A_s)
\]

It is required that \( \phi P_n \geq P_v \). For axial compression with tied reinforcement, \( \phi = 0.65 \). Setting \( \phi P_n = P_v \) and solving for the area of longitudinal reinforcing steel gives

\[
A_s = \frac{P_v}{0.80\phi - 0.85f'c} = \frac{580 \text{ kips}}{0.80(0.85)} = \frac{1000 \text{ lb}}{\text{kip}} = 60,000 \text{ lb/in}^2
\]

\[
A_s = 0.244 \text{ in}^2
\]

\( A_s = 0.244 \text{ in}^2 \) is less than \( A_y = 3.24 \text{ in}^2 \) based on the minimum allowed reinforcement ratio, \( \rho_v = 0.01 \). Therefore, the minimum required area of reinforcement is \( A_s = 3.24 \text{ in}^2 \).

The square tied column has eight uniformly sized longitudinal reinforcing bars. The required area of each longitudinal reinforcing bar is

\[
A = \frac{A_s}{n_{bars}} = \frac{3.24 \text{ in}^2}{8 \text{ bars}} = 0.405 \text{ in}^2/\text{bar}
\]

A bar area of 0.405 in\(^2\) corresponds to a no. 6 bar, which has a nominal area of 0.44 in\(^2\).

In an actual design and analysis situation, a check should also be made to see that the actual longitudinal reinforcement ratio does not exceed the maximum allowable ratio of 0.08.

**The answer is (D).**

3. For a tied column, \( \phi = 0.65 \). For a concentrically loaded tied column, the design strength is given by

\[
\phi P_{n,\text{max}} = 0.80\phi(0.85f'c(A_y - A_i) + f_y A_{st})
\]

For a specified longitudinal steel ratio of 0.02,

\[
A_{st} = \rho_{st} A_y = 0.02A_y
\]

Substituting gives

\[
1090 \text{ kips} = \phi P_{n,\text{max}} = (0.80)(0.65)
\]

\[
A_y = 391 \text{ in}^2
\]

\[
b = \sqrt{A_y} = \sqrt{391 \text{ in}^2} = 19.8 \text{ in} (20 \text{ in})
\]

**The answer is (C).**

4. Determine the design load, \( P_v \)

\[
P_v = 1.2P_D + 1.6P_L = (1.2)(220 \text{ kips}) + (1.6)(250 \text{ kips}) = 664 \text{ kips}
\]

The gross column area is

\[
A_y = (16 \text{ in})(16 \text{ in}) = 256 \text{ in}^2
\]

For a tied column, \( \phi = 0.65 \). The capacity is

\[
\phi P_{n,\text{max}} = 0.80\phi(0.85f'c(A_y - A_i) + f_y A_{st})
\]

\[
= (0.80)(0.65)
\]

\[
A_{st} = 0.02A_y
\]

\[
= 452.6 \text{ kips} + (29.4 \text{ kips/in}^2)A_{st}
\]
The design criterion is

\[
\phi P_{n,\text{max}} \geq P_n
\]

\[
664 \text{ kips} = 452.6 \text{ kips} + \left( 29.4 \frac{\text{kips}}{\text{in}^2} \right) A_d
\]

\[
A_d = 7.19 \text{ in}^2
\]

\[
\rho = \frac{A_d}{A} = \frac{7.19 \text{ in}^2}{256 \text{ in}^2} = 0.028
\]

Check the limits.

\[
0.01 < \rho < 0.08 \quad [\text{OK}]
\]

The answer is (A).

5. The effective length factor for a column braced against sideways is

\[
K = 1.0
\]

The radius of gyration is

\[
r = 0.288 \bar{h} = (0.288)(18 \text{ in}) = 5.2 \text{ in}
\]

The slenderness ratio is

\[
\frac{KL}{r} = \frac{(1.0)(9 \text{ ft})(12 \text{ in/ft})}{5.2 \text{ in}} = 20.8
\]

The gross area of the column is

\[
A_p = \bar{b}^2 = (18 \text{ in})^2 = 324 \text{ in}^2
\]

The area of longitudinal steel reinforcement is

\[
A_{st} = N_A \bar{A}_s = (12)(1 \text{ in}^2) = 12 \text{ in}^2
\]

Since \( M_t = 0 \), the column is a short column. \( \phi = 0.65 \) for a tied column. The design axial load capacity is

\[
\phi P_n = 0.80 \phi \left[ (0.85_f_s (A_p - A_d) + A_{st} f_s) \right]
\]

\[
= (0.80)(0.65) \left[ 324 \text{ in}^2 - 12 \text{ in}^2 \right] + (12 \text{ in}^2) \left( 60 \frac{\text{kips}}{\text{in}^2} \right)
\]

\[
= 926 \text{ kips} \quad (930 \text{ kips})
\]

The answer is (A).

6. The gross area of the column is

\[
A_d = \frac{\pi D^2}{4} = \frac{\pi (22 \text{ in})^2}{4} = 380.1 \text{ in}^2
\]

Since there are eight no. 8 bars, the area of steel is

\[
A_{st} = 8(0.79 \text{ in}^2) = 6.32 \text{ in}^2
\]

\( \phi = 0.65 \) for tied columns. The design strength is then

\[
\phi P_n = 0.80 \phi \left[ (0.85_f_s (A_p - A_d) + A_{st} f_s) \right]
\]

\[
= (0.80)(0.65) \left[ 3500 \frac{\text{lb}}{\text{in}^2} (380.1 \text{ in}^2 - 6.32 \text{ in}^2) \right]
\]

\[
\times \frac{1000 \frac{\text{lb}}{\text{kip}}}{40,000 \frac{\text{lb}}{\text{in}^2}}
\]

\[
= 709.7 \text{ kips} \quad (710 \text{ kips})
\]

The answer is (A).
### Reinforced Concrete: Slabs

#### Practice Problems

1. A two-way slab supported on a column grid without the use of beams is known as a
   - (A) flat slab
   - (B) flat plate
   - (C) drop panel
   - (D) waffle slab

2. All of the following are conditions that must be satisfied in order to use the simplified method for computing shear and moments in one-way slabs, given in ACI 318 Sec. 8.3.3. EXCEPT
   - (A) There are two or more spans.
   - (B) Spans are approximately equal, with the longer of two adjacent spans not longer than the shorter by more than 20%.
   - (C) The loads are uniformly distributed.
   - (D) The ratio of live to dead loads is no more than 5.

#### Solutions

1. A two-way slab supported on a column grid without the use of beams is known as a flat plate.
   - The answer is (B).

2. Shear and moments in one-way slabs can be computed using the simplified method specified in ACI 318 Sec. 8.3.3 when the following conditions are satisfied:
   - (a) There are two or more spans.
   - (b) Spans are approximately equal, with the longer of two adjacent spans not longer than the shorter by more than 20%.
   - (c) The loads are uniformly distributed.
   - (d) The ratio of live to dead loads is no more than 3.
   - (e) The slab has a uniform thickness.
   - The answer is (D).
40 Reinforced Concrete: Walls

PRACTICE PROBLEMS

1. According to ACI 318 Sec. 14.3.3, the minimum horizontal reinforcement for nonbearing walls is
   (A) 0.0012 times the gross concrete area for deformed no. 5 bars or smaller and $f'_c \geq 60,000$ psi
   (B) 0.0015 times the gross concrete area for other deformed bars
   (C) 0.0020 times the gross concrete area for smooth or deformed welded wire reinforcement not larger than W31 or D31
   (D) 0.01 times the gross concrete area

SOLUTIONS

1. ACI 318 Sec. 14.3.3 gives the minimum horizontal reinforcement for nonbearing walls as
   (a) 0.0020 times the gross concrete area for deformed no. 5 bars or smaller and $f'_c \geq 60,000$ psi;
   (b) 0.0025 times the gross concrete area for other deformed bars, and
   (c) 0.0020 times the gross concrete area for smooth or deformed welded wire reinforcement not larger than W31 or D31.

The answer is (C).
41 Reinforced Concrete: Footings

PRACTICE PROBLEMS

1. The critical sections are a distance, \( d \), from the face of the column in
   (A) double-action shear
   (B) one-way shear
   (C) punching shear
   (D) two-way shear

SOLUTIONS

1. For one-way shear (also known as single-action shear and wide-beam shear), the critical sections are a distance, \( d \), from the face of the column.

   The answer is (B).
Mixed Steel and Concrete Problems (from Iqbal & Iqbal – FE Exam Problems)
Problem 12.37: A three-story building plan is 90 ft by 120 ft, as shown in Fig. 12.31. The building is 40 ft tall. The design wind pressure in the North-South direction is 20 psf. The total wind load (kips) on the building for the design wind is most nearly:

A. 72
B. 74
C. 96
D. 98

Solution: The total wind force on the building face is:

\[ F = (p)(A) \]

The wind pressure, \( p \), is constant over the building height. The tributary area for the wind, \( A \), is the area of the building façade perpendicular to the wind load.

The façade tributary area, \( A = (\text{Building width})(\text{Building height}) \)

For N-S wind, the building width = 1 ft + (30 ft)(3) + 1 ft = 92 ft

\[ A = (92)(40) = 3,680 \text{ sq. ft} \]

\[ F = (20)(3,680) = 73,600 \text{ lb} \]

The answer is B.

13. STRUCTURAL DESIGN

Problem 13.01: A W12 × 79 Grade 50 steel column is shown in Fig. 13.01. Using AISC standards, the buckling capacity (kips) of the column is most nearly:

A. 730
B. 660
C. 330
D. 28.5

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Solution: Of the two radii of gyration given for W12 × 79, use the smaller radius of gyration. For pinned-pinned end conditions, $K = 1$.

$L = 240$ in
$A = 23.2$ sq. in
$r_y = 3.05$ in

$KL = \frac{(1)(240)}{3.05} = 79$

From AISC table 4-22, $\theta F_{ty} = 28.5$ ksi

$\theta h_n = (\theta F_{ty})(A) = (28.5)(23.2) = 661.2$ kips

The answer is B.

Problem 13.02: Consider a W21 × 44, Grade 50 steel shape. If $L_b = 10$ ft and $C_b = 1.0$, its moment capacity about the major axis (kip-ft) is most nearly:

A. 252
B. 280
C. 308
D. 358

Solution: Use AISC Tables 1-1 and 3-2, as given in the Design of Steel Components section in the NCEES Handbook. For the W21 × 44 shape:

$S_x = 81.6$ in$^3$

$L_p = 4.45$ ft

$L_t = 13.0$ ft

$\theta_b M_{px} = 358$ kip − ft

$\theta_b M_{tx} = 214$ kip − ft

$\theta_b = 0.9$

Since $L_p < L_b \leq L_r$, use the limit state of lateral-torsional buckling equation given in the NCEES Handbook:

$M_n = C_b \left[ M_p - \left( M_p - 0.7 F_s S_x \right) \left( L_t - L_p \right) / L_r \right] \leq M_p$

In the $M_n$ equation,

$0.7 F_s S_x = (0.7)(50$ ksi$)(81.6$ in$^3) = 2,856$ kip − in = 238 kip − ft

Using kip-ft units in the $M_n$ equation,

$M_n = \left[ 358 - (358 - 238) \left( \frac{10 - 4.45}{13.0 - 4.45} \right) \right] = 252$ kip − ft

Thus, the moment capacity is

$\theta_b M_n = (0.9)(252) = 226.8$ kip − ft

The answer is A.

Problem 13.03: A concrete slab is cast on top of a steel beam as shown in Fig. 13.02.

Concrete Slab

Fig. 13.02

W - Section

Which of the following statements is correct?

A. There is no composite action, and the beam is unbraced.
B. There is no composite action, and the beam is fully braced.
C. The composite action is partial, and the beam is partially braced.
D. There is full composite action, and the beam is fully braced.

Solution: A composite action requires full resistance to sliding at the slab-beam interface. Casting a concrete slab over a steel beam creates a cold joint which is incapable of resisting sliding at the slab-beam interface. Therefore, there is no composite action between the slab and beam. For the beam to be braced, its compression flange should be restrained against lateral torsional rotation under vertical loads. A cold joint as shown does not provide the required restraint. To ascertain that the slab and beam act together compositely, shear connectors are needed to transfer the horizontal shear at the interface.

The answer is A.
Problem 13.04: A frame with a diagonal brace is subjected to a wind load, $P$, as shown in Fig. 13.03.

The brace consists of a $W10 \times 45$ steel section with $F_y = 50$ ksi, pinned at both ends. The maximum wind force (kips) the frame can resist at yield is most nearly:

A. 675  
B. 600  
C. 550  
D. 0

Solution: The frame is unstable without the brace. The force, $P$, the frame can resist depends on capacity of the diagonal member. Therefore, determine the member force, $F_{AC}$. Assume $F_{AC}$ is a tensile force. Use equilibrium at point D:

$\theta = \tan^{-1}\left(\frac{12}{30}\right) = \tan(0.4) = 21.8^\circ$

$F_{AC}\cos\theta = P$

Therefore, $F_{AC}(\cos 21.8^\circ) = 0.93 F_{AC} = P$

Thus, the ultimate load, $P$, depends on the tensile capacity of the diagonal member. The formulas for tension members are given in the NCEES Handbook.

For a $W10 \times 45$ section, the cross-sectional area, $A_g = 13.3 \text{ sq. in}$

For tension, $\Phi_y = 0.90$

Available tensile strength, $\Phi T_a = \Phi_y F_y A_g = (0.90)(50)(13.3) = 598.5 \text{ kips}$

At yield, $F_{AC} = \Phi T_a = 598.5 \text{ kips}$

Therefore, $P = 0.93 F_{AC} = (0.93)(598.5) = 557 \text{ kips}$

The answer is C.

Problem 13.05: A frame with a diagonal brace is subjected to a wind load, $P$, as shown in Fig. 13.04. The brace consists of a $W10 \times 54$ steel section with $F_y = 50$ ksi, pinned at both ends. Assuming that the diagonal brace has pinned-pinned end conditions and its capacity is the controlling factor for the frame stability, the critical force, $P$, (kips) the frame can resist is most nearly:

A. 550  
B. 155  
C. 144  
D. 0

Solution: The frame in this problem is identical to the frame in the previous problem. However, the direction of wind force is reversed. Therefore, the brace is in compression. Once it buckles, the frame becomes unstable. Therefore, check the buckling capacity of the brace.

For $W10 \times 54$:

$A = 15.8 \text{ in}^2$

Radius of gyration $r_y = 2.56 \text{ in}$

$L_{AC} =$Length of the brace

$L_{AC} = \sqrt{12^2 + 30^2} = 32.31 \text{ ft} = 388 \text{ in}$
Problem 13.06: Consider an interior column in a four-story building consisting of three floors and a roof, designed as shown in Fig. 13.05

The three floor levels have identical live load categories, which are different from the roof load category. The area of influence ($ft^2$) supported by the column at its base is most nearly:

A. 1,200  
B. 3,600  
C. 4,800  
D. 14,000

Solution: A member’s area of influence depends on the tributary floor area and the type of member. The area of influence is used to reduce the live load on a member. The area of influence depends on the cumulative floor tributary area of a single live load category supported by the member. See the Structural Design section of the NCEES Handbook.

Tributary area of an interior column per level = $(30)(40) = 1,200 \text{ ft}^2$

Column tributary area:

$A_T = (\text{No. of supported floors})(\text{Tributary area per floor})$

$A_T = (3)(1,200) = 3,600 \text{ ft}^2$

For columns, $K_{LL} = 4$

Area of influence, $K_{LL}A_T = (4)(3,600) = 14,400 \text{ ft}^2$

The answer is D.

Problem 13.07: An exterior column in a building with 10 supported floors and a roof carries no cantilever slabs. Its tributary area per floor is 400 sq. ft. The unreduced live load for each floor is 50 psf. The reduced live load (kips) at the base of the column is most nearly:

A. 200  
B. 120  
C. 80  
D. 75

Solution: The magnitude of the live load reduction is based on the area of influence, which depends on the tributary floor area and the type of member. It depends on the entire area of influence. See the Structural Design section of the NCEES Handbook for formulas.

Column tributary area:

$A_T = (\text{No. of supported floors})(\text{Tributary area/floor})$

$A_T = (10)(400) = 4,000 \text{ sq. ft}$

$K_{LL} = 4$

Area of influence, $K_{LL}A_T = (4)(4,000) = 16,000 \text{ sq. ft}$

The reduced live load can be determined as follows:
Components section of the NCEES Handbook. The bending moment induces tension in the top fibers. Therefore, the bars should be placed near the top face. If the beam width is insufficient to accommodate all the bars in one row with minimum spacing, then they can be placed in two or more layers.

**The answer is A.**

**Problem 13.09:** Consider the reinforced concrete column interaction diagram shown in Fig. 13.06 and the following statements about the diagram:

1. Point A is called the buckling point.
2. Point A is called the turning point.
3. Point A is called the balanced point.
4. The column with its loading pertaining to point X is adequate.
5. The column with its loading pertaining to point Y is adequate.
6. The column failure mode above point A is compressive.
7. The column failure mode below point A is compressive.
8. The column failure mode above point A is tensile.
9. The column failure mode below point A is tensile.

**Solution:** The cantilever is subjected to a negative bending moment. The deflected shape is also shown in the Design of Reinforced Concrete.
To optimize the column capacity, longitudinal reinforcement should be added to:
A. Only one side: A, B, C or D
B. Two sides: A and B
C. Two sides: C and D
D. All four sides, equally distributed

Solution: See the Design of Reinforced Concrete Components section in the NCEES Handbook, which shows the interaction diagram of a rectangular column. A bending moment with \( e/h = 1.2 \) indicates that the column failure is close to the balanced point, and likely occurs by tension yielding of the longitudinal reinforcement. Therefore, the reinforcement should be placed on the side where the tension is greater. Since the applied moment is subject to reversal, as shown in Fig. 13.07, reinforcement is most effective at faces C and D.

The answer is C.

Problem 13.11: Which statement about the ACI code resistance factor (\( \phi \)) is NOT correct?
A. It accounts for the material quality control achievable for each member.
B. It accounts for the uncertainties in the strength of the member.
C. The higher the uncertainty, the smaller the value of \( \phi \).
D. The higher the uncertainty, the larger the value of \( \phi \).

Solution: The resistance factors, \( \phi \), are tabulated in the Design of Reinforced Concrete Components section of the NCEES Handbook. The ACI-318 building code calls the factors strength reduction factors. \( \phi \) varies depending on the type of loading and is always less than 1. Statements A, B and C are correct. Concrete is a heterogeneous material, and its strength may vary from one section to another. Tension-controlled strength can be predicted with more precision than compression-controlled concrete failure. The \( \phi \) factor takes this fact into account and reduces the load that can be placed. Higher uncertainties require more reduction in strength. Therefore, the \( \phi \) factor should be smaller for higher uncertainties. Statement D is incorrect.

The answer is D.
Problem 13.12: A rectangular singly-reinforced concrete beam shown in Fig. 13.08 has 3 #9, grade 60 as its bottom reinforcement.

![Diagram of a rectangular beam](image)

1' 10"

1' 0"

Fig. 13.08

The concrete strength is 5,000 psi. The section is tension-controlled. The ultimate moment capacity, \( \Delta M_u \) (ft-kips) of the section is most nearly:

A. 240
B. 270
C. 300
D. 3,000

**Solution:** The factored moment can be determined by the equation:

\[
M_n = A_s f_y \left( d - \frac{d}{2} \right)
\]

\[A_s = 3#9 = 3 \text{ in}^2\]

\[f_y = 60 \text{ ksi}\]

\[d = 22 - 2 = 20 \text{ in}\]

\[a = \frac{A_s f_y}{0.85 b_d} = \frac{(3)(60)}{(0.85)(12)(5)} = \frac{3.53 \text{ in}}{}
\]

\[M_n = (3)(60)(20 - \frac{3.53}{2}) = 3,280 \text{ in} - \text{kips}
\]

For flexure, \( \phi \) = 0.9

\[\Delta M_u = \text{Design moment strength} = (0.9)(3280) = 2,982 \text{ in} - \text{kips}
\]

\[= 246 \text{ ft} - \text{kips}
\]

Shortcut: For rectangular sections, assume lever arm = 0.9d

\[\Delta M_u = (\phi)(0.9d)(A_s)(f_y) = (0.9)(0.9)(20)(3)(60)
\]

\[= 2916 \text{ in} - \text{kips} = 243 \text{ ft} - \text{kips}
\]

The answer is A.

Problem 13.13: A 5,000-psi reinforced concrete beam is required to resist a dead load shear force of 10 kips and a live load shear force of 15 kips. The beam is 12-in wide and 23-in deep. The distance from the concrete cover to the center of the bottom steel is 3 in. Use \( f_d = 60 \text{ ksi} \) and \( f'_s = 5 \text{ ksi} \). The bar size needed for a 2-leg stirrup with a spacing of 10-in on center is most nearly:

A. \#3
B. \#4
C. \#5
D. \#6

**Solution:** The factored shear force, \( V_u \), can be determined using the following load combinations:

\[V_u = 2(0.9d) \text{ bd} = 2\sqrt{5000} (12)(20) = 33,941 \text{ lbs} = 33.94 \text{ kips}
\]

\[d = \text{Beam effective depth} = \text{total depth} - \text{concrete cover} = 23 - 3 = 20 \text{ in}
\]

The nominal strength provided by concrete, \( V_c \), can be determined as follows:

\[V_c = 2(0.75)(33.94) = 25.46 \text{ kips} < V_u\]

Since the applied shear is more than the available concrete capacity, shear steel is needed to resist \( V_u \).

\[V_s = \frac{V_d}{\phi} - V_c = \frac{36}{0.75} - 33.94 = 14.06 \text{ kips}
\]

Use maximum allowed stirrup spacing \( \frac{d}{2} = 10 \text{ in} \)

For stirrup design,

\[s = \frac{A_s f_d}{V_s}
\]

Therefore, \( A_v = \frac{(10)(14.06)}{(60)(20)} = 0.12 \text{ in}^2\)

Since the stirrup has two legs, the required area per leg is 0.06 in\(^2\).
Problem 13.14: A simply-supported concrete beam with 5 ksi concrete and grade 60 steel is used. The beam is 14 in wide and 32 in deep. The concrete cover to the center of the bottom steel is 3 in. The maximum steel area permitted by the ACI code is:

\[ f_y = 5 \text{ ksi} \]
\[ b = 14 \text{ in} \]
\[ d = 32 - 3 = 29 \text{ in} \]

The maximum longitudinal steel area permitted in the beam is nearly:

A. 7.11 in²
B. 9.10 in²
C. 10.9 in²
D. 9.9 in²

Solution:
\[ A_{s, \text{max}} = \frac{0.55(5)(b)(d)}{f_y} = \frac{0.55(5)(14)(29)}{5} = 107.5 \text{ in}^2 \]

The answer is B.

Problem 13.15: A simply-supported concrete beam is 14 in wide and 32 in deep. The concrete strength is 5 ksi. The beam has 3 in grade 60 steel in. The steel strain under ultimate condition is most nearly:

A. 0.003
B. 0.005
C. 0.003
D. 0.002

Solution:
\[ \epsilon_b = \frac{f_y}{E_y} = \frac{60}{5000} = 0.012 \text{ in}^{-1} \]

The steel strain under ultimate condition is most nearly:

A. 0.003
Problem 13.16: A reinforced beam is specified to be made with 6,000-psi concrete. However, it is instead made with 3,000-psi concrete. The concrete density remains unchanged at 150 lb/ft³. The ratio of the deflection of the as-built beam to the deflection of the designed beam is most nearly:

A. 2  
B. 1.5  
C. 1  
D. 0.5

Solution: The deflection of a flexural member is inversely proportional to the modulus of elasticity of its material. The larger the modulus, the smaller the deflection. The concrete modulus of elasticity given in the Design of Reinforced Concrete Components section of the NCEES Handbook is:

$E_c = 33w^{2.5} \sqrt{f'_c}$

Density for normal-weight concrete (NWC), $w = 150 \text{ lb/ft}^3$

The modulus of elasticity for $f'_c = 6,000 \text{ psi}:

$$E_c = (33)(150)^{1.5} \sqrt{6,000} = 4,696,000 \text{ psi} = 4,696 \text{ ksi}$$

Similarly, the modulus of elasticity for $f'_c = 3,000 \text{ psi}:

$$E_c = (33)(150)^{1.5} \sqrt{3,000} = 3,321,000 \text{ psi} = 3,321 \text{ ksi}$$

In this case, the beams have identical size and moment of inertia. Therefore, the deflection ratio is:

$\text{Deflection ratio} = \frac{\text{Deflection of as-built beam}}{\text{Deflection of designed beam}} = \frac{E_{design}}{E_{as-built}}$

$$\text{Deflection ratio} = \frac{4,696}{3,321} = 1.41$$

The answer is B.

Problem 13.17: A short tied column is reinforced using a bar pattern shown in Fig. 13.10.

The applied axial DL = 150 kips, and the LL = 550 kips. $f'_c = 7 \text{ ksi}$, and $f_y = 60 \text{ ksi}$. For a tied column, $\phi = 0.65$. The minimum bar size needed for the column to carry the load is most nearly:

A. #8  
B. #9  
C. #10  
D. #11

Solution: A factored load combination is required. The governing combination is:

$$R_e = 1.2D + 1.6L = (1.2)(150) + (1.6)(550) = 1,060 \text{ kips}$$

The available column strength, $\phi P_n$, can be determined using the tied column formula given in the Design of Reinforced Concrete Components section of the NCEES Handbook. The available strength, $\phi P_n$, should be more than the ultimate load the column is being designed for.

Strength reduction factor, $\phi = 0.65$

Column gross area, $A_g = \frac{\pi D^2}{4} = \frac{\pi(18)^2}{4} = 254 \text{ in}^2$

$\phi P_n = 0.80\phi f'_c (A_g - A_{st}) + A_{st} f_y$

$A_{st} = \phi P_n - 0.80\phi f'_c (A_g - A_{st})$

$$A_{st} = \frac{1060 \text{ kips} - (0.80)(0.65)(0.85)(7 \text{ ksi})(254 \text{ in}^2)}{(0.80)(0.65)(60 \text{ ksi} - (0.85)(7 \text{ ksi})}$$

$$= 9.70 \text{ in}^2$$
$A_{st,min} = 1\%$ of $A_g = 2.54$ in$^2$

Therefore, $A_{st} > A_{st,min}$, $A_{st}$ controls

The column has 8 longitudinal bars. The required minimum area of one bar is:

$A_{bar} = \frac{A_{st}}{8} = \frac{9.70}{8} = 1.21$ in$^2$

A #10 bar has an area of 1.27 in$^2$. Therefore, use 8 #10 bars.

The answer is C.

**Problem 13.18:** A rectangular concrete beam was cast without any web reinforcement, as shown in Fig. 13.11.

![Diagram of a concrete beam](image)

If $f'_c = 4$ ksi and $f_y = 60$ ksi, the ultimate shear force (kips) the beam can carry is most nearly

A. 0
B. 9.6
C. 12.9
D. 25.8

**Solution:** If no web reinforcement is used, the ultimate applied shear should not be larger than one-half the design shear strength $\Phi V_c$. It can be determined by using the equations given in the Design of Reinforced Concrete Components section of the NCEES Handbook:

$V_c = 2b_wd\sqrt{f'_c}$

For shear, $\Phi = 0.75$

$f'_c = 4$ ksi, $f_y = 4,000$ psi

Total beam height, $h = 20$ in

Effective depth, $d = h - cover = 20 - 3 = 17$ in

Beam width, $b_w = 12$ in

$V_c = 0.75 \left( \frac{2(12)(17)}{4,000} \right) = 9.677 \text{ lb} = 9.677 \text{ kips}$

The answer is B.

**Problem 13.19:** A rectangular wood beam is required to support a uniformly distributed load (UDL) of 480 lb/ft over its span of 15 ft. The UDL includes the beam’s self-weight. The modulus of elasticity of the wood material is $2 \times 10^6$ psi. The maximum allowable short-term deflection is 1 in. If the beam is 3.5 in wide, its minimum depth (in) is most nearly:

A. 7.5
B. 9.5
C. 11.5
D. 13.5

**Solution:** From the equation for the deflection of a beam carrying a UDL in the Mechanics of Materials section of the NCEES Handbook:

$\frac{5wl^4}{384EI}$

Convert the measurements to inches and pounds and determine the moment of inertia, $I$:

$w = 480 \frac{\text{lb}}{\text{ft}}$

$\frac{1}{12} \frac{\text{ft}}{\text{lb}}$

Span, $L = 15 \text{ ft} = 180 \text{ in}$

$E = 2 \times 10^6 \text{ psi}$

Allowable $\gamma_{max} = 1$ in

Therefore, $\gamma_{max} = \frac{5(40)(180)^4}{384(2)(10^6)L} = 1 \text{ in}$
Problem 13.20: An 18-in × 24-in rectangular short tied reinforced concrete column is required to carry a factored axial load of 700 kips and an eccentricity of 10-in. The compressive strength of concrete is 4 ksi, the tensile yield strength of steel is 60 ksi, and the ratio of the core diameter to the column diameter is 0.8. Using ACI 318, the total longitudinal steel area using bars at the end faces is most nearly:

A. 4  
B. 8  
C. 16  
D. 32

Solution: Use the load-moment strength (interaction) diagram. The non-dimensional parameters, $K_n$ and $R_n$, are:

\[
K_n = \frac{P_u}{\phi f'c A_g} \\
R_n = \frac{P_u e}{\phi f'c A_g h}
\]

$\phi$ for tied reinforcement = 0.65  
$p_u = 700$ kips  
$f'c$ = 4 ksi  
$A_g = (18)(24) = 432 \text{ in}^2$  
h = 24 in  
e = 10 in  

\[
K_n = \frac{700}{(0.65)(4)(432)} = 0.62
\]

$R_n = \frac{(350)(10)}{(0.65)(4)(432)(24)} = 0.26$

Using the interaction diagram, the steel ratio is obtained by interpolation between the curves: $\rho_g = 0.037 = 3.7\%$

Column reinforcement, $A_{st} = \rho_g A_g = (0.037)(432) = 16 \text{ in}^2$

The answer is C.

Problem 13.21: The controlling radius of gyration (in) of the concrete rectangular column shown in Fig. 13.12 is most nearly:

A. 24  
B. 18  
C. 10  
D. 7

Solution: Considering the x- and y-axis, the radius of gyration is:

\[
r_x = \sqrt{\frac{I_x}{A}}
\]

\[
r_y = \sqrt{\frac{I_y}{A}}
\]

For a rectangular shape,

Area, $A = bh$

For column bucking about the major axis, the moment of inertia is:
Problem 13.22: The term “nominal strength” of a member denotes:
A. The theoretical strength with no safety factors (Ω) or resistance factors (Θ) applied.
B. The theoretical strength in which either a safety factor (Ω) or resistance factor (Θ) is applied.
C. The theoretical strength in which a resistance factor (Θ) is applied.
D. The service load level maximum strength for which a member should be subjected.

Solution: The nominal strength is defined as the theoretical strength with no safety factors (Ω) or resistance factors (Θ) applied. The resistance factor (Θ) is multiplied with the nominal strength of a member to account for the material strength variations, member dimensions, and workmanship.

The answer is A.
Solution: Statements A and B are correct. Both ASD and LRFD are design methods and use the same methods of structural analysis. Performance-based design criteria involve non-linear analysis, which is not required in either method.

The answer is D.

Problem 13.26: A steel bar is connected to a gusset plate as shown in Fig. 13.13.

![Diagram showing a gusset plate and bolt connection]

The yield strength of the steel is 50 ksi, and its ultimate strength is 65 ksi. The bolts are adequate to carry the connection load. Using the LRFD method of design, the design capacity of the bar in tension (kips) is most nearly:

A. 195
B. 150
C. 135
D. 125

Solution: The design of a member in tension using LRFD requires a three-step process:

1. Determine the yield strength based on gross area
2. Determine the fracture strength based on net area
3. Determine the design strength by selecting the smaller strength of the first two steps.

Gross area, $A_g = (6 \times \frac{1}{2}) = 3 \text{ in}^2$

Bolt diameter, $d_b = \frac{3}{4} \text{ in}$

Hole diameter, $d_h = d_b + \frac{1}{16} \text{ in}$

For 3/4-in diameter bolts, $d_h = \frac{3}{4} + \frac{1}{16} = \frac{13}{16} \text{ in}$

Net area, $A_n = (b_g - \sum d_h) \cdot t$

$A_n = \left( \frac{6 - 13}{16} \right) \cdot (0.5) = 2.59 \text{ in}^2$

For yielding, $\phi_y = 0.90$

$\phi T_n = \phi_y F_y A_y = (0.9)(50)(3) = 135 \text{ kips}$

For fracture, $\phi_f = 0.75$

$F_n = 65 \text{ ksi}$

$\phi T_n = \phi_f F_n A_n = (0.75)(65)(2.59) = 126.3 \text{ kips}$ (controls)

The answer is D.

Problem 13.27: A 1-in diameter, A-325 bolt is used to connect a clevis (Fig. 13.14). The shear capacity of the bolt is 35.3 kips. The maximum load P (kips) the bolt can carry is most nearly:

A. 20
B. 30
C. 35
D. 70
The steel yield stress is 50 ksi. The available strength in axial compression (kips) of the column is most nearly:

A. 90
B. 270
C. 360
D. 690

**Solution:**
The column base is fixed against both rotation and translation. The top end is fixed against rotation but free to translate. The recommended design value of the effective length factor, \(k\), is 1.2.

Use AISC Table 4-1.

\[ kL = 1.2 \times 15 = 18 \text{ ft} \]

\[ kF_p = 270 \text{ kips} \]

The answer is B.

---

**Problem 13.28:** A W12\(\times\)50 steel section is used as a column as shown in Fig. 13.15.

The bolt is in double shear. In double shear, the shear force required to shear the bolt is twice the shear force required in single shear. Therefore, the available bolt strength, \(V_n\), is:

\[ V_n = 2 \times 35 = 70.6 \text{ kips} \]

The answer is D.
Problem 13.29: A W10×45 steel column is 20-ft tall. Its weak axis is braced at its mid-length, as shown in Fig. 13.16. Its controlling slenderness ratio is most nearly:
A. 5  
B. 60  
C. 65  
D. 120  

Solution: The column has pinned-pinned end conditions so that its effective length factor, k, is 1.0.  
Column weak (y-) axis is braced at 10-ft intervals. Therefore, its effective length in the y-direction is  
KL_y = (1.0)(10) = 10 ft = 120 in  
The column is unbraced along its major (x-) axis. Therefore, the effective length in the x-direction is:  
KL_x = (1.0)(20) = 20 ft = 240 in  
The radii of gyration of W10×45 given in the W Shapes Dimensions and Properties table in the NCEES Handbook are:  
\( r_x = 4.32 \) in  
\( r_y = 2.01 \) in  
The slenderness ratios for x- and y-axes are  
\[ \frac{KL_y}{r_y} = \frac{240}{4.32} = 55.6 \]  
\[ \frac{KL_x}{r_x} = \frac{240}{2.01} = 59.7 \]  
The larger ratio controls.  
The answer is B.

Problem 13.30: A W10×49 steel column carries 100 kips of service dead load. Its controlling slenderness ratio is 120, and the yield stress is 50 ksi. The service live load capacity of the column (kips) is most nearly:  
A. 225  
B. 150  
C. 100  
D. 65

Solution: The area of W10×49 is:  
\[ A = 14.4 \text{ in}^2 \]  
Its slenderness ratio is:  
\[ \frac{KL}{r} = 120 \]  
Using the critical stress table, the available stress is:  
\[ \frac{\sigma_c}{F_{y}} = 15.7 \frac{ksi}{kips} \]  
Compressive strength of column:  
\[ \frac{\sigma_c}{F_{y}, A} = (15.7)(14.4) = 226 \text{ kips} \]  
The factored load combination is:  
\[ U = 1.2D + 1.6L \]  
226 = (1.2)(100) + 1.6L  
L = 66 kips  
The answer is D.

Problem 13.31: A steel beam is required to carry a service point load of 50 kips at its mid-span. The beam span is 20 ft. The beam has a maximum depth of 18.5 in. Using the AISC allowable stress design method and a yield strength of 50 ksi, the lightest wide flange section to carry the load is:  
A. W21×55  
B. W18×65  
C. W18×55  
D. W18×46

Solution: Determine the maximum bending moment in the beam. As shown in Fig. 13.17, the maximum moment is:  
\[ M_{max} = \frac{PL}{4} = \frac{(50)(20)}{4} = 250 \text{ kip} \cdot \text{ft} \]  

Use the W-shape selection table and select a beam section that has its allowable moment capacity, \( \phi_{p} M_{yx} \), slightly greater than the applied moment and which meets the depth limitation. The selected section is W18×55 with a depth of 18.1 in. and a flexural strength of:  
\[ \phi_{p} M_{yx} = 258 \text{ ft} \cdot \text{kips} \]  
The answer is C.
Problem 13.32: A beam carries two equal factored loads of 30 kips at its third points along the span. Its span is 36 ft. The beam is braced every 6-ft along its span. The lightest wide-flange section with the yield strength of 50 ksi that can carry the load is most nearly:

A. \( W21 \times 44 \)
B. \( W21 \times 48 \)
C. \( W18 \times 46 \)
D. \( W16 \times 45 \)

Solution: See Fig. 13.18. The maximum bending moment in the beam is:

\[ M_{\text{max}} = (30)(12) = 360 \text{ ft kips} \]

Since the loads are factored, the required bending strength is:

\[ \phi M_{\text{p}} = 360 \text{ ft kips} \]

Since the unbraced length is 6 ft, the selected section should be able to carry the moment with a larger unbraced length. Use the Available Moment vs. Unbraced Length diagram in the Civil Engineering section of the NCEES Handbook. Several sections qualify. The lightest section that qualifies is a \( W21 \times 48 \). Use the W-shape beam table to verify the selection. The selected section is fully compact.

\[ l_p = 6.09 \text{ ft} > 6 \text{ ft provided} \]
\[ \phi_b M_{ps} = 398 > 360 \text{ ft kips} \]

The answer is B.

Problem 13.33: A single bay plane moment frame shown in Fig. 13.19 consists of A50 steel. The column ends connected to the footings are frictionless pins.
Side-way is permitted in the plane of the frame. The relative column-girder stiffness, \( G \), used in the Alignment chart is defined as
\[
G = \frac{\left( \frac{E}{L} \right)_{\text{column}}}{\sum \left( \frac{E}{L} \right)_{\text{girders}}}
\]
From the Alignment Chart provided in the NCEES Handbook, the K-factor for the columns in the plane of the frame is most nearly:
A. 0.77
B. 1.7
C. 2.2
D. 10

Solution: Since both columns are identical, consider one column. Calculate the \( G \) values at both ends of the column and use the alignment monogram (Jackson and Moreland chart). It is given that the column ends connected to the footings are frictionless pins. Therefore,
\( G_{\text{bot}} = \infty \)
Since the modules of elasticity, \( E \), for the girder and columns is the same and only one girder is connected to a column, the relationship is simplified to
\[
G_{\text{top}} = \frac{\left( \frac{L}{E} \right)_{\text{column}}}{\left( \frac{L}{E} \right)_{\text{girder}}}
\]
Column is \( W \) 12 \( \times \) 50, \( I_x = 391 \text{ in}^4 \)
Beam is \( W \) 21 \( \times \) 44, \( I_x = 843 \text{ in}^4 \)
\( L_{\text{col}} = 15 \text{ ft} = 180 \text{ in} \)
\( L_{\text{girder}} = 25 \text{ ft} = 300 \text{ in} \)
\[
G_{\text{top}} = \frac{\left( \frac{391}{180} \right)}{\left( \frac{843}{300} \right)} = 2.17 \approx 0.77
\]
Use the alignment monogram for uninhibited side-way. By connecting \( G_{\text{top}} \) and \( G_{\text{bot}} \), the effective length factor, \( K = 2.2 \)
The answer is C.

Problem 13.34: A simply-supported steel W-section is carrying a uniformly distributed load over its span. If the beam is braced at its supports only, its \( C_b \) factor is most nearly:
A. 1.0
B. 1.12
C. 1.14
D. 1.56

Solution: The moment capacity modification factor, \( C_b \), can be found by using the AISC formulas or the table. For the beam loading and bracing configuration, \( C_b = 1.14 \).
The answer is C.

Problem 13.35: A simply-supported W24x68 beam has an unbraced length of 20 ft, and its \( C_b \) factor is 1.67. The beam design strength is most nearly:
A. 375
B. 611
C. 650
D. 678

Solution: Use the AISC table 3-10 (Available Moment vs. Unbraced length).
\[
\varphi M_n, \text{Available moment} = 366 \text{ ft} - \text{kips}
\]
Since the table is based on \( C_b = 1 \), the design strength is multiplied by 1.67.
\[
\varphi M_n, \text{Available moment (for } C_b = 1.67) = 1.67(366) = 611 \text{ ft} - \text{kips}
\]
The answer is C.

Problem 13.36: A straight steel bar is fastened to a gusset plate with three \( \frac{3}{8} \)-in diameter ASTM A-307 bolts, as shown in Fig. 13.13. The bolt capacity in single shear is 7.95 kip. Using standard size holes, the design
Problem 14.01: Which of the following statements regarding the phase diagram of soils is NOT correct?

A. It shows three parts: solids at the bottom, water above it, and then the weight-volume relationship of a soil mass.
B. The three parts in the diagram depict the conditions at the bottom of a water channel where the soil is at the bed, water above it, and the weight-volume relationship of a soil mass.
C. The weight of the air is taken as zero.

Solution:

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Shear strength of the connection (kips) is most nearly:

A. 126.3
B. 47.7
C. 23.85
D. 15.9

\[ \phi_n = (3)(7.99) = 23.85 \text{ kips} \]

Since the capacity of the bolts is less than the plate capacity of 126.3 kips, the bolt capacity controls. The answer is D.
12. STRUCTURAL ANALYSIS

Problem 12.01: The arch shown in Fig. 12.01 is:
A. unstable
B. stable and statically determinate
C. stable and statically indeterminate
D. redundant

![Diagram of an arch with loads and supports]

Solution: This is a 3-hinge arch with two supports at points A and B and a crown hinge at C. The moment is zero at point A, point B, and point C. The arch has the following properties:

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Number of members, \( m = 2 \)
Number of independent reaction components, \( r \)
\[
2 \text{ reactions} \times 2 \text{ supports} = 4
\]
Number of points, \( j = 3 \)
Number of condition equations based on known internal moments or forces, such as internal moment at zero at a hinge, \( c = 1 \)

As described in the Structural Analysis section of the NCEES Handbook, the stability and determinacy of an arch depends on whether
\[
3m + r < c = or > 3j + c
\]
In this case,
Left hand side = \( 3m + r = 3(2) + 4 = 10 \)
Right hand side = \( 3j + c = 3(3) + 1 = 10 \)
Because \( 3m + r = 3j + c \), the arch ABC is stable and statically determinate.

**The answer is B.**

Problem 12.02: The arch shown in Fig. 12.01 is subjected to both uniformly distributed horizontal and vertical loads. The vertical reaction (kN) at support A is:
- A. 25 \( \uparrow \)
- B. 40 \( \uparrow \)
- C. 55 \( \uparrow \)
- D. 20 \( \uparrow \)

Solution: This is a 3-hinge arch with two supports at points A and B and a crown hinge at C. The moment is zero at A, B, and C points. The resultants of vertical and horizontal loads, \( V \) and \( H \), respectively, are shown in Fig. 12.02:
\[
V = \left( \frac{2 \times 10^3 \text{ kN}}{m} \right) (40 \text{ m}) = 80 \text{ kN}
\]
\[
H = \left( \frac{3 \times 10^3 \text{ kN}}{m} \right) (20 \text{ m}) = 60 \text{ kN}
\]

To determine the vertical reaction at A, \( R_{VA} \), let clockwise moments be positive. Taking the moment about support B:
\[
\sum M_B = 0
\]
\[
(R_{VA})(40 \text{ m}) + (60 \text{ kN})(10 \text{ m}) - (80 \text{ kN})(20 \text{ m}) = 0
\]
\[
R_{VA} = 25 \text{ kN}
\]
The positive sign of \( R_{VA} \) shows that the assumed direction of reaction, shown in Fig. 12.02, is correct.

**The answer is A.**

Problem 12.03: The arch shown in Fig. 12.01 is subjected to both uniformly distributed horizontal and vertical loads. The horizontal reaction (kN) at support A is:
- A. 25 \( \leftarrow \)
- B. 30 \( \leftarrow \)
- C. 60 \( \leftarrow \)
- D. 60 \( \leftarrow \)

Solution: This structure is a 3-hinge arch with two supports at point A and point B, and a crown hinge at point C. The moment is zero at point A, point B, and point C. To determine the horizontal reaction at point A, \( R_{HA} \), take the moment about point C for the arch segment AC, as shown
Problem 12.04: A steel beam is carrying a uniformly distributed load (UDL) of 2 kips/ft, as shown in Fig. 12.04.

For steel, $E = 29,000$ ksi, $I = 500$ in$^4$, $L = 20$ ft $= 12 \frac{ft}{in} = 240$ in

The answer is $A$. 

\[ \sum M_c = 0 \]

\[ V = (\frac{V}{2}) = 80 \text{ kips} \]

\[ \sum M_c = -(R_{ext}) \cdot (20) - (25 \text{ kips})(20 \text{ ft}) + (R_{ext})(10 \text{ ft}) \]

From the previous solution, $R_{ext} = 25$ kips

\[ \sum M_c = 0 \]

\[ R_{ext} = 25 \text{ kips} \]

Let counterclockwise moments be positive.

\[ \delta_B = \frac{W L^3}{24EI} \]

There is no load applied over segment AB. Therefore, segment AB remains a straight line with point A deflecting up distance $\Delta_A$, as shown in Fig. 12.05.

Deflection at $A, \Delta_A = \text{(slope at B) - (distance AB)}$
Problem 12.06: A truss is shown in Fig. 12.06.

\[ n' = \frac{k}{ft} = \frac{2 \left( \frac{1 ft}{12 \text{ in}} \right)}{ft} = 0.167 \frac{k}{\text{in}} \]

\[ \delta_B = \frac{wL^3}{24EI} = \frac{(0.167 \text{ kips/in})(240 \text{ in})^3}{24(29,000 \text{ kips/ft}^2)(500 \text{ in}^4)} = 0.0066 \text{ rad} \]

Distance AB = 5 ft = 60 in

\[ \Delta_{AB} = \theta_B (60 \text{ in}) = (0.0066 \text{ rad})(60 \text{ in}) = 0.40 \text{ in} \]

The answer is B.

Solution: The truss has 11 joints and 19 members. Each member has an unknown axial force. The support at F is a roller. Therefore, it is capable of developing only a vertical reaction. The support at A is a hinge, so it can develop both a horizontal and a vertical reaction. Therefore, the truss has 3 unknown reactions.

For a statically determinate truss, the relationship between bars, reactions, and joints must satisfy the following criteria:

\[ r + b = 2j \]

\[ r = \text{the number of reactions} = 3 \]

\[ b = \text{unknown bar forces} = 19 \]

\[ j = \text{total number of joints} = 11 \]

Substituting the values in the criteria equation:

\[ \text{LHS: } r + b = 3 + 19 = 22 \]

\[ \text{RHS: } 2j = (2)(11) = 22 \]

Since the criterion is satisfied, the truss is statically determinate.

The answer is B.

Problem 12.07: A frame is shown in Fig. 12.07.
This frame is:
A. unstable
B. statically determinate
C. statically indeterminate to the first degree
D. statically indeterminate to the second degree

Solution: The frame has 6 joints and 6 members. One truss support is a roller, which is capable of developing only a vertical reaction. The other support is a hinge, which can develop both a horizontal and a vertical reaction. Therefore, the frame has 3 unknown reactions. The frame also has an additional 4 hinges at the internal truss joints. Therefore, the frame has the following properties:
Number of members, \( m = 6 \)
Number of independent reaction components, \( r = 3 \)
Number of joints, \( j = 6 \)
Number of condition equations based on known internal moments or forces, such as internal moment of zero at a hinge, \( c = 4 \)

As described in the Structural Analysis section of the NCEES Handbook, the stability and determinacy of an arch depends on whether 
\( 3m + r < c \), or \( 3j + c \)

In this case,
Left hand side = \( 3m + r = 3(6) + 3 = 21 \)
Right hand side = \( 3j + c = 3(6) + 4 = 22 \)
Because \( 3m + r < 3j + c \), the frame ABC is unstable.

The answer is A.

Problem 12.08: A set of six wheel loads is moving across a 60-ft span girder from A to B. The relative positions of the wheels are shown in Fig. 12.08.

![Diagram of wheel loads on a girder](image)

The maximum bending moment in the girder occurs when the minimum distance (ft) between support A and the 10-kip wheel load on the right is most nearly:
A. 27
B. 30
C. 33
D. 35

Solution: See the moving concentrated load set principle stated in the Structural Analysis section of the NCEES Handbook. The maximum moment in the girder occurs when the resultant of the loads and an adjacent load from the set are equidistant to the beam centerline.

The magnitude of the load resultant, \( R \), is the sum of the wheel loads, which equals 40 kips. Treat each wheel load as a point load, \( P_i \), each with distance \( x_i \) to the 10-kip wheel load on the right. Use the center of gravity equation from the Statics section of the NCEES Handbook. The distance of the load resultant \( R \) from the 10-kip wheel load on the right, \( \bar{x} \), is:

\[
\bar{x} = \frac{\sum P_i x_i}{\sum P_i}
\]

\[
\bar{x} = \frac{\sum 5 (18 + 15 + 12 + 9) + 10(3 + 0)}{\sum(5 + 5 + 5 + 5 + 10 + 10)} = 7.5 \text{ ft}
\]

The distance of resultant \( R \) from the 10-kip wheel on the left is
\( 7.5 \text{ ft} - 3 \text{ ft} = 4.5 \text{ ft} \)

For the maximum bending moment in the girder, the adjacent load and the resultant \( R \) must be equidistant from midspan C, as shown in Fig. 12.09.
Problem 12.09: A set of six wheel loads is moving across a 60-ft span girder from point A to point B, as shown in Fig. 12.08. The maximum bending moment in the girder occurs when the wheel load on the right is 35.25 ft from point A, as shown in Fig. 12.09. Which of the following statements is correct?

A. The absolute maximum bending moment occurs under the load resultant R.
B. The absolute maximum bending moment occurs at the midspan.
C. The absolute maximum bending moment occurs under the 10-kip wheel load on the right.
D. The absolute maximum bending moment occurs under the 10-kip wheel load on the left.

Solution: See the principle of moving concentrated load sets in the Structural Analysis section of the NCEES Handbook. The absolute maximum bending moment in the girder occurs under one of the wheel loads adjacent to the resultant (i.e., the second 10-kip wheel load or the first 5-kip wheel load). Since load resultant R, midspan C, and the 10-kip wheel load on the right are not adjacent wheel load locations, options A, B, and C are incorrect. The 10-kip wheel load on the left is the only adjacent load listed in the options.

The answer is D.

Problem 12.10: A unit load moves horizontally from point C to point G over frame ABCDEFG, as shown in Fig. 12.10.
Which of the following is the influence line diagram for the vertical reaction at support A?

A. 

B. 

C. 

D. 

Solution: To determine the influence line for $A_y$, take the moment about point B. Let clockwise moments be positive. Let $x$ be the distance from point C to the point load.

$$\sum M_B = 0 = A_y (20 \text{ ft}) - (1 \text{ kip})(30 \text{ ft} - x)$$

$$A_y = 1.5 - \frac{x}{20}$$

This equation is a straight line, with 1.5 kips as the intercept when $x = 0$. This equation is shown in Option C.

Option D represents the influence line for $B_y$.

Support B has a roller which can translate in the horizontal direction. As such, support B is incapable of developing a horizontal reaction. Option A shows the influence line for reactions $A_x$ and $B_x$, which both equal zero.

Option B is the influence diagram for $A_x$ and $B_x$ if the frame has a hinge instead of a roller support at B and a rotational hinge at midspan E.

The answer is C.

Problem 12.11: A frame is carrying a 24-kip point load, as shown in Fig. 12.11.

All members have equivalent Young's moduli and moments of inertia. The carryover moment (kip-ft) at support C is most nearly:

A. 60
B. 30
C. 20
D. 0

Solution: Take the moment about joint B due to the cantilever load.

$$M_B = (24 \text{ kips})(5 \text{ ft}) = 120 \text{ kip} - \text{ ft (counterclockwise)}$$

This moment is called the fixed-end moment (FEM), which is distributed between members BC and BD. From the Structural Analysis section of
Problem 12.13: A single degree of freedom system has a mass of 8,000 kg. Its stiffness is 2,000 kN/m. Its fundamental period is most nearly:
A. 2 sec
B. 8 sec
C. 12 sec
D. 0.5 Hertz

Solution: A single degree of freedom system is shown in Fig. 12.12. Its natural frequency is:
$$\omega_n = \frac{K}{m} = \frac{2,000}{8,000} = 0.25 $$

Fig. 12.12

The fundamental period is:
$$T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{0.25} \sqrt{\frac{m}{K}} = \frac{2\pi}{2(3.14)} \sqrt{\frac{8,000}{2,000}} = 12.56 \text{ sec} $$

The answer is C.

Problem 12.14: A car weighing 6,000-lb travels at 35 mph when it collides with a rigid barrier. The car's front end is crushed, buckled, and bent under impact. At a result, the car, which is 18 ft in length before the impact, is 16 ft in length after the impact. The impact force (lb) on the barrier is most nearly:
A. 6,000
B. 10,000
C. 20,000
D. 120,000
Solution: The kinetic energy (KE) of a moving object with mass \( m \) and velocity, \( v \), is:
\[
KE = \frac{1}{2}mv^2
\]
Upon impact, the entire KE is converted into work done that is absorbed in crushing the vehicle. The reduction in length of the car, \( \delta \), due to crushing is called the car crush.
The impact force, \( F \), on a vehicle barrier can be determined by equating its KE with the work done or energy absorbed by the car body at impact as follows:
\[
F\delta = \frac{1}{2}mv^2 = \frac{1}{2}Wv^2
\]
\[
F = \frac{W}{2\delta}v^2
\]
\[
W = 6,000 \text{ lb}
\]
\[
g = 32.2 \text{ ft/sec}^2
\]
\[
v = 35 \text{ mph} = (35)(1.47) = 51.45 \text{ ft/sec}
\]
\[
\delta = 18 - 16 = 2 \text{ ft}
\]
Therefore,
\[
F = \frac{6,000}{(2)(2)(32.2)(51.45)^2} = 123,312 \text{ lbs}
\]
The answer is D.

Problem 12.15: A car weighing 6,000-lb travels at 30 mph when it collides with a non-rigid guard rail barrier. The car’s front end is crushed, and the barrier deflects under impact. The car crush is 16 inches, and the barrier deflects 12 inches at the impact location. The impact force (lb) on the barrier is most nearly:
A. 30,000
B. 80,000
C. 130,000
D. 180,000

Solution: Use the impact force from the previous problem:
\[
F = \frac{1}{2}W\frac{v^2}{g\delta}
\]
The vehicle and the barrier both absorb energy. The distance the car travels is the sum of the car crush, \( \delta_c \), and the barrier deformation, \( \delta_b \):
\[
\delta = \delta_c + \delta_b = 16 + 12 = 28 \text{ in} = 2.33 \text{ ft}
\]
\[
W = 6,000 \text{ lb}
\]
\[
g = 32.2 \text{ ft/sec}^2
\]
\[
v = 30 \text{ mph} = (30)(1.47) = 44.1 \text{ ft/sec}
\]
Therefore,
\[
F = \frac{6,000}{(2)(2.33)(32.2)(44.1)^2} = 77,765 \text{ lb}
\]
The answer is B.

Problem 12.16: A vehicle weighing 72 kips travels at 50 mph on a bridge. The driver applies the brakes and brings the vehicle to a stop in 15 seconds. Assuming the deceleration is uniform, the bridge is level, and friction is adequate, the longitudinal force (lb) exerted on the bridge is most nearly:
A. 0
B. 32.2
C. 11,000
D. 164,000

Solution: When a vehicle accelerates or brakes, longitudinal forces are transmitted from its wheels to the bridge (see Fig. 12.13).

Fig. 12.13

The magnitude of the force can be determined using Newton’s law of motion:
\[
F = ma = \frac{W}{g} \frac{dv}{dt}
\]
\[
W = \text{weight of vehicle}
\]
\[
g = \text{acceleration of gravity (32.2 ft/sec}^2\text{)}
\]
Solution: Assume the member forces in both members AC and BC are compressive, and consider the equilibrium at joint C (Fig. 12.16):
\[ \sum F_x = F_{CB}\cos 60^\circ + F_{CA}\cos 30^\circ = 0 \]
\[ \sum F_y = F_{CB}\sin 60^\circ + F_{CA}\sin 30^\circ = 0 \]

From the first equation, \( F_{CB}\cos 60^\circ = -F_{CA}\cos 30^\circ \)
\[ F_{CB} = -\frac{F_{CA}(0.866)}{0.5} = -1.73 \ F_{CA} \]
Substituting the value of \( F_{CB} \) into the second equation,
\[ -1.73F_{CA}\sin 60^\circ + F_{CA}\sin 30^\circ = 30 \]
\[ (-1.73)(F_{CA})(0.866) + 0.5F_{CA} = 30 \]
\[ F_{CA} = -30 \]
The negative sign for \( F_{CA} \) indicates that the assumption is incorrect, and member CA is in tension.
\[ F_{CB} = -1.73(-30) = 51.9 \text{ kips (compression)} \]
The answer is D.

Problem 12.21: A truss carries a vertical load, \( P \), of 90 kips. The forces in the truss members AC and BC are 90 (tension) and 155.7 (compression), respectively. The truss geometry and member lengths are shown in Fig. 12.15. All members have pinned joints and have a standard steel pipe cross-section area of 5 in\(^2\). The vertical deflection (in) of the
Problem 12.22: A moment of 100 ft-kips is applied at the non-fixed end of a propped cantilever. The induced moment (ft-kips) at the other end is most nearly:
A. 200  
B. 100  
C. 50  
D. 0

Solution: Fig. 12.17 shows the applied moment, $M_c$.

Fig 12.17

Support B is the fixed end of the propped cantilever. The induced moment at support B, $M_B$, is called the carry-over moment. The associated carry-over factor is $\frac{1}{2}$.

$M_B = \frac{1}{2} M_A = \frac{100}{2} = 50 \text{ ft} - \text{ kips}$

The answer is C.

Problem 12.23: A beam carries a point load, as shown in Fig. 12.18.

Fig. 12.18

Therefore, $\Delta_c = 0.812 \text{ in}$

The positive sign denotes displacement in the direction of the applied load.

The answer is C.
Assuming clockwise moment is positive, the moments (ft-kips) at ends A and B, respectively, are most nearly:

A. 0, 0
B. 333, −167
C. 167, −333
D. −333, 167

Solution: The fixed-end moments (FEM) for a beam carrying a point load are:

\[ FEM_{AB} = \frac{Pab^2}{L^2} \]
\[ FEM_{BA} = \frac{Pa^2b}{L^2} \]

\[ P = 50 \text{ kips} \]
\[ a = 15 \text{ ft} \]
\[ b = 30 \text{ ft} \]
\[ L = 45 \text{ ft} \]

Therefore,

\[ FEM_{AB} = \frac{(50)(15)(30)^2}{(45)^2} = 333 \text{ ft} - \text{kips} \]
\[ FEM_{BA} = \frac{(50)(30)(15)^2}{(45)^2} = 166 \text{ ft} - \text{kips} \]

A fixed-end moment is considered positive if it is acting clockwise on the support. The FEM at A is counterclockwise and, therefore, it is negative. The FEM at B is clockwise and, therefore, it is positive. Using this sign convention, the FEMs are:

\[ FEM_{AB} = -333 \text{ ft} - \text{kips} \]
\[ FEM_{BA} = 167 \text{ ft} - \text{kips} \]

The answer is D.

Problem 12.24: A fixed-end beam spans 30 ft. It carries a uniformly-distributed load (UDL) of 1 kip/ft and a 10-kip concentrated load at its midspan. See Fig. 12.19.

Assuming counter-clockwise moment is positive, the bending moment (ft-kips) at support A is most nearly:

A. −112.5
B. −37.5
C. 37.5
D. 112.5

Solution: Use the method of superposition. Treat the UDL and the point load as loads that act alone, and then combine their effects. For a uniformly distributed load, \( w \), the fixed-end moments are:

\[ FEM_{AB} = \frac{wl^2}{12} \]

For point load, \( P \), at mid-span, the distance \( a = b = 0.5L \)

\[ FEM_{AB} = \frac{Pab^2}{L^2} = \frac{P(0.5L)(0.5L)^2}{L^2} = \frac{PL}{8} \]

For a combined UDL and point load,

\[ FEM_{AB} = \frac{wl^2}{12} + \frac{PL}{8} \]

\[ FEM_{AB} = \frac{(1)(30)^2}{12} + \frac{(10)(30)}{8} = 75 + 37.5 = 112.5 \text{ ft} - \text{kips} \]

The FEM at A is counterclockwise. Therefore, it is positive.

The answer is D.
Problem 12.25: A W24 × 68 propped cantilever beam spans 25 ft, as shown in Fig. 12.20.

The moment (ft-kips) needed to rotate the beam end by 2° is most nearly:
A. 25,000
B. 2,100
C. 200
D. 2

Solution: For W24 × 68, \( I = 1,830 \text{ in}^4 \)
The rotation of a propped cantilever can be determined by the equation:
\[
i = \frac{ML}{4EI}
\]
\[
j = 29,000 \text{ ksi}
\]
\[
i = 25 \text{ ft} = 300 \text{ in}
\]
\[
i = \frac{(2°)(\pi \text{ rad})}{180°} = 0.035 \text{ rad}
\]
\[
i = \frac{ML}{4EI} = 0.035
\]
\[
\gamma = \frac{(0.035)(4)(29,000)(1,830)}{300} = 24,760 \text{ kip - in} = 2,060 \text{ kip - ft}
\]
The answer is B.

Problem 12.26: Consider the fixed-end beam shown in Fig. 12.21.

Its mid-span bending moment (ft-kips) is most nearly:
A. 48
B. 96
C. 144
D. 192

Solution: The bending moment at each end for a beam carrying a uniformly distributed load is:
\[
FEM_{AB} = FEM_{EA} = \frac{wL^2}{12}
\]
The moment at the mid-span can be determined by using the mid-span bending moment for a simply-supported condition and superimposing the fixed end moments on it, as shown in Fig. 12.22.

The mid-span bending moment for a simply-supported beam under a UDL is:
\[
M_e = \frac{wL^2}{8}
\]

Since the bending moment due to the UDL is a sagging moment, and the moments to keep the ends fixed are hogging moments, the bending moment along the beam span is the difference between the two. For the fixed-end beam, the bending moment at its mid-span, point C, is:
\[
M_e = \frac{wL^2}{8} - \frac{wL^2}{12} = \frac{wL^2}{24} = \frac{(2)(24)^2}{24} = 48 \text{ ft - kips}
\]
The answer is A.
Problem 12.27: A cantilever beam carries a uniformly distributed load (UDL) of 1 kip/ft. The cantilever spans 24 ft. It is propped at its free end to reduce its tip deflection to zero. The vertical force (kip) needed to propped the cantilever is most nearly:

A. 6
B. 9
C. 12
D. 24

Solution: The deflection caused by the UDL is downward, as shown in Fig. 12.23. Therefore, the tip deflection is not zero. The tip deflection is:

$$\delta_{\text{tip}} = \frac{wL^4}{2EI}$$

For the tip deflection to be zero, we need:

$$wL^4 = 2EI$$

The tip deflection is:

$$\delta_{\text{tip}} = \frac{wL^4}{2EI}$$

The cracking action requires an upward force, so we negate the beam deflection so that the net deflection is zero (see Fig. 12.23). For a cantilever under a point load at a distance, $a$, from the fixed end, the tip deflection is:

$$\delta_{\text{tip}} = \frac{Pa^2}{4EI}$$

For the load, $P$, located at the tip of the cantilever, the distance $a = L$. Therefore,

$$\delta_{\text{tip}} = \frac{PL^2}{4EI}$$

Since the UDL, $w$, and point load, $P$, are in opposite directions, equate the two deflections numerically:
\[ \delta_p = \delta_{\text{max}} \]
\[ P = \frac{3wL}{8} = \frac{(3)(1)(24)}{8} = 9 \text{ kips} \]

*The answer is B.*

**Problem 12.28:** An L-beam carries a 50-kip load at the midpoint of its 6-in wide ledge, as shown in Fig. 12.24. The torsion (in-kips) on the beam is most nearly:

A. 0  
B. 130  
C. 390  
D. 740

**Solution:** A beam subjected to a twisting couple perpendicular to its cross-section is said to be in torsion. Determine the center of gravity of the beam. The magnitude of torsion depends upon the force and the eccentricity with respect to the beam’s centroid (Fig. 12.25):

Torsional moment or Torque = (load)(eccentricity)
The eccentricity is defined as the shortest distance between the load and centroid (CG) axis. Use the formula to compute the C.G. of the entire beam section:
\[ x_c = \frac{\int x dA}{A} \]

Divide the beam into two rectangles (Fig. 12.25). The equation is:
\[ x = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2} \]

Calculate the beam section centroid distance, \( x \), from the bottom left corner, A. Using inch-units, calculations are tabulated as shown:
Problem 12.30: A high-strength steel rod with a cross-sectional area of 4 in$^2$ passes through a circular concrete element with a diameter of 24 in. The diameter of the opening that the steel rod passes through is 4 in. The rod is fitted at each end with a steel cap plate, a washer, and a nut, as shown in Fig. 12.27.

Fig. 12.27

The nuts are tightened until the tensile stress in the rod reaches 55 ksi. Assuming the modular ratio of steel to concrete is 8, the stress (ksi) in the concrete section is most nearly:

A. 0.125
B. 0.5
C. 6.00
D. 6.875

Solution: Consider equilibrium: Pull on steel rod = Push on concrete:

\[ P_{\text{steel}} = A_c \sigma_c = 4(55) = 220 \text{ kips} \]

Concrete area \( A_c = \frac{\pi}{4} (D_c^2 - D_i^2) = \frac{\pi}{4} (24^2 - 4^2) = 440 \text{ in}^2 \)

Concrete stress \( = \frac{220 \text{ kips}}{440 \text{ in}^2} = 0.5 \text{ ksi} \)

The answer is B.

Problem 12.31: Two aluminum planar parallel rods, each 10 mm in diameter and 5-m long, carry 90-kN load as shown in Fig. 12.28.
Since both rods have the same area, length, and $E$, the tensile force ratio is:
\[ \frac{T_1}{T_2} = 0.23 \]
\[ T_1 = 0.23T_2 = 540 - 4.33T_2 \]
\[ (0.23 + 4.33)T_2 = 540 \]
\[ T_2 = 118.4 \, kN \]

The answer is $D$.

**Problem 12.32:** A steel shaft has a moment of inertia of 0.00055 m$^4$ about its centroid. The shaft is simply supported at its ends, which are 12 m apart. It carries a uniformly distributed load and has a maximum deflection of 12 mm. The shaft is propped at its midspan so that its midspan deflection is reduced to 2 mm. The force (kN) needed for propping is most nearly:

A. 25  
B. 30  
C. 35  
D. 40

**Solution:** The propping force ($P$) is an upward load that is applied at the shaft’s mid-span (Fig. 12.30). It is in the direction opposite to the applied load. The upward deflection under $P$ is:

\[ y_2 = y_1 - 2mm = 12 - 2 = 10 \, mm = 0.01 \, m \]

Mid-span deflection of a beam subjected to a point load $P$ at midspan is given by:
Problem 12.33: The plan view of a three-story reinforced building is shown in Fig. 12.31.

![Building Plan View](image)

All columns are 2-ft squares and are centered on the grid lines. The concrete floors and roof are one-way slabs supported on the beams. The floor system weighs 120 psf, including the dead load of the columns.

Problem 12.34: The plan view of a three-story building is shown in Fig. 12.31. All columns are 2-ft squares and are centered on the grid lines. The concrete floors and roof are one-way slabs supported on the beams. The floor live load is 50 psf, and the roof live load is 15 psf. Assuming no live load reduction, the service live load, LL, (kips) on an interior
column at its base is most nearly:
A. 54
B. 140
C. 180
D. 240

**Solution:** Determine the tributary area of the column at each floor level. The tributary area of a column is the area adjoining the column which sheds its load to the column. It is shown in Fig. 12.32.

Tributary area of an interior column/level = \((30)(40) = 1,200 \text{ ft}^2\)

The column live load/floor = (Tributary area)(Unit live load)
The unit live load, \(L\), on the floor level differs from the unit live load on the roof. The slab at the ground level rests on the ground, so no load is transferred to the column from the ground. The live load at the column base is computed using the table:

<table>
<thead>
<tr>
<th>Level</th>
<th>Trib. Area (ft²)</th>
<th>Unit LL (lb/ft²)</th>
<th>Floor Load (lb)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td>1200</td>
<td>15</td>
<td>((1200)(15) = 18,000) lb</td>
</tr>
<tr>
<td>3rd Floor</td>
<td>1200</td>
<td>50</td>
<td>((1200)(50) = 60,000) lb</td>
</tr>
<tr>
<td>2nd Floor</td>
<td>1200</td>
<td>50</td>
<td>((1200)(50) = 60,000) lb</td>
</tr>
<tr>
<td>Ground</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td></td>
<td></td>
<td><strong>138,000 lb = 138 kips</strong></td>
</tr>
</tbody>
</table>

**The answer is B.**

**Problem 12.35:** For an interior column in the previous problem, the ultimate load (kips) at the column base is most nearly:

A. 600
B. 750
C. 1,200
D. 1,700

**Solution:** The service loads are factored to determine the ultimate load, \(U\). Two ultimate load combinations to be considered in the NCEES reference book are:

\(U = 1.4D\)
\(U = 1.2D + 1.6L\)

The larger of the factored loads using the combinations governs.

\(U_1 = (1.4)(432) = 605 \text{ kips}\)
\(U_2 = 1.2(432) + 1.6(138) = 739 \text{ kips} > U_1\)

Therefore, \(U_2\) governs.

\(U = 739 \text{ kips}\)

**The answer is B.**

**Problem 12.36:** For a corner column in the building shown in Fig. 12.31, the service load (kips) at the column base is most nearly:

A. 120
B. 150
C. 160
D. 300

**Solution:** The tributary length of a corner column in a particular direction is the distance from the center of the bay to the edge of the building. The edge is located beyond the grid line. Therefore, consider the column width:

\[\text{Tributary length} = \frac{\text{grid length}}{2} + \frac{\text{column width}}{2}\]

The grid lengths are 30 and 40 ft. The column width is 2-ft, and the floor extends to the edge of exterior column (Fig. 12.33).