Lesson 8: Column Buckling I

Wednesday, September 13, 2017

Lesson Objectives

1. Determine the controlling slenderness $KL/t$ for a pinned-braced column.
2. Calculate the Euler Elastic Buckling Stress $F_e$ for a column, considering both axes of buckling.
3. Describe the conditions for which the Euler expression accurately predicts column buckling.
4. Calculate the nominal buckling load $P_{cr}$ for a column per AISC equations, considering the controlling axis of buckling.

Today’s Scope – The Simple Braced Frame

Today’s scope is a column in a simple-braced frame. A simple braced frame uses pinned connections and is stabilized by the truss-action of diagonal bracing. Bracing prevents the relative translation of the top and the bottom of the column. In contrast, unbraced frames are characterized by the relative translation of the top and the bottom of the column and will be covered in a subsequent lesson.

Some Nomenclature (for more, see the INDEX tab in the steel manual)

In structural steel design, the letter $F$ indicates a stress, $P$ indicates a compressive force, $T$ indicates a tensile force.

- $F_{cr}$ – critical buckling stress (ksi). When columns are very long (long enough to remain elastic when buckling occurs), then $F_{cr}$ is based on $F_e$, the Euler buckling stress.
- $F_e$ – Euler buckling stress (ksi). The Euler equation predicts the buckling stress, assuming that the column is elastic (generally true for very long columns).
- $F_y$ – yield strength (ksi). This is the maximum elastic stress that steel can sustain, in tension.
- $I_x$ – moment of inertia about the member’s $x$-axis (in$^4$)
- $I_y$ – moment of inertia about the member’s $y$-axis (in$^4$)
- $KL_x$ – effective column length (inches) for buckling about the $x$-axis – this is the distance over which $0$ to $\pi$ of a sine curve is completed by the column’s buckled shape.
- $KL_y$ – effective column length (inches) for buckling about the $y$-axis – this is the distance over which $0$ to $\pi$ of a sine curve is completed by the column’s buckled shape.
- $r_x$ – radius of gyration about the member’s $x$-axis (inches)
- $r_y$ – radius of gyration about the member’s $y$-axis (inches)
- $P_{cr}$ – critical buckling load (kips). This equals the Euler buckling load if the column is very slender and remains elastic during buckling.

$$P_e = \frac{\pi^2 EI}{(KL)^2} = A_y \frac{\pi^2 E}{KL/r_i^2}$$

- $P_n$ – nominal column capacity (kips). This is the same as $P_{cr}$ because all columns in compression fail by buckling.

Reading

- The online version of this handout contains 8 pages of reading
- Textbook (Segui), pp. 109 to 121.

Reference

- AISC Steel Manual Chapter E in the specifications (16.1-31 to 16.1-33)
- AISC Steel Manual Part One (Table 1-1)
- AISC Table 4-22
Table 1-1 (continued) W-Shapes

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W12-W10

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**Notes:**
1. Shape is slender for compression with \( f_y = 50 \) ksi.
2. Shape exceeds compact limit for tension with \( f_y = 50 \) ksi.
3. The actual size, combination and orientation of fastener components should be compared with the prominence of the cross section to ensure compatibility.
4. Shape does not meet the \( A/t_0 \) limit for shear in AISC Specification Section G2.1(a) with \( f_y = 50 \) ksi.
HOMEWORK (Due Friday. Standard assignment. Presentation counts)

Terms: $P_n$ is the nominal buckling strength (units = kips). This is the actual predicted buckling (failure) load of the column. $P_n/\Omega$ is the (ASD) Allowable load, where $\Omega$ is the safety factor ($\Omega = 1.67$); i.e., an engineer would not want the applied (actual) load to exceed the allowable load, as this would be unsafe. $\phi P_n$ is the (LRFD) Design Strength (it is not an allowable load!!! If your column were to actually have this load on it, then you should get the Hell out of the building fast), where $\phi$ is the strength reduction factor ($\phi = 0.9$ for column buckling); i.e., the engineer would not want the factored load to exceed the design strength, as this would be unsafe.

0. (Do this but do not hand in). Go to the steel manual, Part 2 and flip through the pages; this is where you will find the most general information about steel design (LRFD, ASD, fabrication tolerances, materials, etc.). Find page 2-10. This is where the load combinations are found. Go to AISC Tables 2-4 and 2-5 to answer the following:
   a. What is the “preferred ASTM material specification” for W shapes? What is the minimum yield strength for this? What is the minimum tensile strength for this?
   b. What is the “preferred ASTM material specification” for L shapes? What is the minimum yield strength for this? What is the minimum tensile strength for this?
   Note: from now on, unless stated otherwise, you are to assume the “preferred ASTM material specification” for all shapes.

0. (Do this but do not hand in). For a 24-ft long W12x96 column, two possible bracing ideas are shown, A and B. Circle the bracing idea that is “smart.”

0. (Do this but do not hand in). Prove to yourself that “Braces are blind to gravity loads”; i.e., the brace does not have a force in it, due to the vertical load.
   a. Do equilibrium on Point A. From this, determine the force in AB (remember zero-force members?)
   b. Knowing the force in AB, do equilibrium on Joint B. From this determine the force in the brace.

0. (Do this but do not hand in). Let’s suppose that we know that the actual applied load on a column is 100 kips. Let’s also suppose that we know that the $\phi P_n$ of the column is 105 kips. Is the column safe?
0. (Do this but do not hand in). Let’s suppose that we know that the factored load on a column is 100 kips. Let’s also suppose that we know that the $\phi P_n$ of the column is 105 kips. Is the column safe?
1. Compute the allowable buckling stress $F_{cr}/\Omega$ for a column that has a slenderness of 100 if $F_y=50$ksi. Then, go to Table 4-22 in the AISC manual and find the allowable buckling stress $F_{cr}/\Omega$ for a column that has a slenderness of 100 (note: from now on, Table 4-22 is a good table to use to save yourself time. Put a tab there. Avoid hand-calculating $F_{cr}$ unless $F_y$ is an odd specification, not found in the table.)

2. The 6” diameter Extra-Strong Pipe section (Pipe 6 x-strong) is subjected to axial forces $P_{Dead} = 55$ kips and $P_{Live} = 55$ kips. Determine if the section is adequate per ASD. Then determine if the section is adequate per LRFD. Use the “preferred material specification”.

3. Determine if an HSS4x3x3/8 column is adequate per ASD. Then determine if the section is adequate per LRFD.
   - The single-story 1-bay building frame has a Roof Dead Load pressure of 25psf. This includes the proper allowance for all roof weights: fill beams, girders, the roof deck, the roofing material, etc. (it’s all included in the 25 psf number)
   - Snow Load pressure: 50psf (note: that’s a lot. This building must be in Canada)
   - The story height is 18’ but pay close attention to the bracing and note the way it braces the column’s weak-axis. The bay lengths are 30’, each.

Note: the nominal dimensions of the HSS (“hollow structural shape”) are 4x3x3/8, but use the real dimensions and properties given in the AISC Part One Tables. As described in the previous problems (refer to problem 0, above), use the “preferred material specification” for rectangular HSS.

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1 Always use the decimal dimensions of cross-sections, for calculations. The fractional dimensions are for detailers and architects, only. Example: a W16x26 has a depth of 15.7” or 15 3/4”. Engineers use the 15.7”. Detailers and architects use the 15 3/4”.