LESSON 12: STATICALLY INDETERMINATE AXIAL-FORCE PROBLEMS

LESSON OBJECTIVES
1. Compute the axial forces in an indeterminate member for which a compatibility expression that describes the geometry of deformation, must be employed. There are three basic classes of problems:
   A. Problems in which a particular deformation is known; e.g., “\( \delta_A = 0 \)” or “\( \delta_C = 0.001 \text{m} \)”. Example 1 is this type.
   B. Problems in which two deformations must be equal or are related to one another; e.g., \( \delta_A = \delta_B \) or \( \delta_A = \delta_B + \delta_C \). Example 2 is this type.
   C. Problems in which a structure or machine is known to undergo rigid-body movement according to a geometric rule; “positions A, B, and C will move but they will remain along a line”. Example 3 is of this type.

PHILPOT
- Philpot 5.5.

IN-CLASS PROBLEMS

1. Compute the end-reactions.
   Given: \( A = 2 \text{ in}^2, E = 30000 \text{ ksi} \)

   \[
   \begin{align*}
   &30 \text{k} \quad 50 \text{k} \\
   &12'' \\
   &6''
   \end{align*}
   \]

2. Problem: Compute forces on steel and concrete, \( F_{st}, F_c \). (4) 34-mm diam bars.
   Given:
   - \( E_{st} = 200 \text{ GPa} \)
   - \( E_c = 25 \text{ GPa} \)
   - \( A_{st} = 3632 \text{ mm}^2 \)

3. Problem:
   Given: Beam is rigid
   Bar areas, \( A_{AB} = A_{EF} = 25 \text{mm}^2 \)
   Bar area, \( A_{CD} = 15 \text{mm}^2 \)
   Steel \( E_{st} = 200 \text{ GPa} \)

   \[
   \begin{align*}
   &0.5 \text{m} \\
   &0.2 \text{m} \quad 0.2 \text{m} \\
   &0.4 \text{m} \\
   &15 \text{kN}
   \end{align*}
   \]

HOMEWORK (DUE MONDAY)
Use mechanical properties from Philpot, as necessary, if the properties are not stated in the problem. Assume all materials are linear-elastic.

1. Compute the member forces \( F_{AC} \) and \( F_{BC} \).
   Given: Steel, \( E_{st} = 200 \text{ GPa} \)
   Rod diameter = 5mm
   Initially, there is a 1mm initial gap at B (between B and B’)

The 3 Laws of Structural Mechanics: 1. Equilibrium (Statics); 2. Constitutive Relations (e.g., Hooke’s Law); 3. Compatibility of Deformations (equations that describe the deformation). Every problem in Structural Mechanics can be solved with these three laws.
1’. (think about it but do not hand in)
Referring to the previous, compute the member forces $F_{AC}$ and $F_{BC}$ with all of the same given information except that the gap at B is 10mm, rather than 1mm.

2. Determine the maximum force $P$ (units: kips) that the reinforced concrete column can support without either the steel yielding or the concrete crushing (i.e., you have to consider two cases)

   Given:
   - Each steel bar has an area of 1 in$^2$.
   - Steel yield stress is 60 ksi
   - Concrete crushing stress is 6 ksi
   - Steel Elastic modulus: Use $E_{steel} = 30000$ ksi
   - Concrete Elastic modulus: Use $E_{concrete} = 4000$ ksi
   - Assume that both materials are linear-elastic because you are assuring that the steel has not yielded and the concrete has not failed by crushing.

3. Determine the reactions at A, B, and C caused by $P=30$ kN. Given: the rigid bar is supported by a pin at A and two linearly elastic wires at B and C. The area of the wire at B is 60 mm$^2$ and the area of the wire at C is 120 mm$^2$.
   Note: it does not matter what the elastic modulus of the wires is, but the wires are of the same material.

4. Determine the normal stresses in the bars due to $P = 60$ kips.
   Given: Bar (1) is steel ($E_1=30000$ksi, $A_1=1.25$in$^2$)
   Bar (2) is bronze ($E_2=15000$ksi, $A_2=3.75$in$^2$)
   Before loading, there is a 0.125-inch gap between the bars.
   The gap will close.
   The materials will remain elastic
   The bearing plate at B is considered rigid
   1 ft = 12 inches

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