Purpose of this Guide: To thoroughly prepare students for the exact types of problems that will be on Exam 2.

Exam Format: Closed-book, closed-notes. The formulae below will be given. Students should have a calculator, pencils, and a straight-edge (if a graph is required).

Given: Required material properties will be given. The following formulae and resources will also be given on exam:

Law of Cosines:
\[ c^2 = a^2 + b^2 - 2ab \cos(\gamma), \]
\[ b^2 = c^2 + a^2 - 2ca \cos(\beta), \]
\[ a^2 = b^2 + c^2 - 2bc \cos(\alpha), \]
\[ \cos(\gamma) = \frac{a^2 + b^2 - c^2}{2ab}. \]

Law of Sines:
\[ \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}. \]

Formulæ:
Normal stress \( \sigma = F/A \), where \( F \) is the normal force on the cut and \( A \) is the area of the cut.
Shear stress \( \tau_{avg} = V/A \), where \( V \) is the shear force on the cut and \( A \) is the area of the cut.
Normal strain \( \varepsilon = \Delta L/L \), where \( \Delta L \) is the change in length and \( L \) is the original length (AKA, gauge length)
Shear strain \( \gamma_{xy} \) = the change in angle of \( x-y \), where \( x-y \) are initially perpendicular or \( \gamma_{xy} = \pi/2 - \theta' \), where \( \theta' \) is the deformed angle between \( x-y \), measured in radians and \( \pi \) is obviously \( \pi \).
Hooke’s “Law” – \( \sigma = E\varepsilon \), where \( E \) = modulus of elasticity (AKA Elastic Modulus or Young’s Modulus)
Poisson’s Ratio \( \nu = -\left(\frac{\varepsilon_{lat}}{\varepsilon_{long}}\right)\), where the specimen is loaded in the long direction, resulting in long strain, as well as lat strain.
Hooke’s Law for Shear: \( \tau = G\gamma \), where \( G \) is the shear modulus of elasticity (AKA, the modulus of rigidity)
The elastic properties \( E \), \( G \), and \( \nu \) are related by:
\[ G = \frac{E}{2(1 + \nu)} \]

Lesson Coverage, Objectives, and Example Problems: The exam covers Lessons 6 to 11. The objectives are given below, with example problems/questions for each objective.

Define shear and normal strain.
1. (3 points) TRUE or FALSE. Normal strain is an angle change, usually measured in radians.
2. (3 points) TRUE or FALSE. The normal strain of orientation AC is the same as the normal strain of AB, below.

Compute normal strain on axially-loaded members for which the extension of the member may be determined by geometry and Employ SOHCAHTOA for right triangles and the Law of Cosines for other triangles to compute lengths in strained members.
3. (15 points) A thin-walled pipe with a 4.00-inch diameter is subjected to air pressure which causes its diameter to increase to 4.10-inches. Determine the normal strain in the walls of the pipe, caused by this diameter-change. (Ans: 0.025 in/in)

4. (20 points) Determine the angle of tilt of rigid beam ABC, relative to its initial position. Give your answer in degrees.

   Given: Beam ABC is rigid. It is pinned at A and supported at B by the pin-ended steel cable BD. Beam ABC supports a uniform load of 1 kip per foot. It is known that the stretched length of Cable BD is 60.5 inches.

5. (30 points) Determine the strain in Cable BC. Give your answer in degrees.

   Given: Beam AB is rigid. It is pinned at A and supported at B by steel cable BC, while a 3 kip vertical load is applied at B. It is known that position B displaces downwards by 0.5 inches.

Compute the average shear and normal strain for any orientation on a plate member for which the initial and final coordinates may be determined.

6. (25 points) Determine the shear strain $\gamma_{xy}$ at A. Given: the triangular plate is fixed at its base and point A is given a horizontal displacement of 5mm.
7. (20 points) Determine the average shear strain $\gamma_{xy}$ for the distorted material shown and determine the shear modulus of elasticity of the material.

Given: The material was originally rectangular, but it distorted as shown due to shear stresses in the x-y plane of 100 MPa.

8. (15 points) Determine the average normal strain $\varepsilon$ along the BD axis of the previous problem.

9. (25 points) Determine the average shear strain $\gamma_{xy}$, given the deformation shown.

10. (15 points) Determine the average normal strain along diagonal AC of the previous problem.

Graph given experimental stress versus strain data for an engineering material, using two simultaneous horizontal scales – one for low (pre-yield) strain and the other for high strain (with other objectives, below).

Compute the modulus of elasticity of an engineering material by computing the slope of a best-fit line on the elastic portion of the stress-strain plot.

11. (20 points) Determine the modulus of elasticity, if the material is considered to be approximately linear-elastic.

Given: A specimen is originally 1 foot long, has a diameter of 0.5 inches. When subjected to a 1300-lb tensile load, the specimen is found to have elongated by 0.9 inches.

12. (20 points) Determine the modulus of elasticity of this concrete, treating it as an approximately linear-elastic material. Given: the sample of concrete has a cross-section that is 4”x4” square cross-section. It had an original length of 10”. When loaded to 40,000 lbs, as shown, a deformation of 0.0067” was measured over the 10” length.
13. (30 points) Determine the elastic modulus, yield strength, ultimate strength, maximum strain, and toughness for the following engineering material and plot its stress-strain curve on the graph paper below.

**Given:** The material is a 0.505” diameter solid cylinder was tested in tension, recording load and deformations over its 2-inch length.

<table>
<thead>
<tr>
<th>Load (kips)</th>
<th>Deformation (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.000</td>
</tr>
<tr>
<td>8</td>
<td>0.008</td>
</tr>
<tr>
<td>8</td>
<td>0.100</td>
</tr>
<tr>
<td>9</td>
<td>0.200</td>
</tr>
</tbody>
</table>
Define terms: elastic, plastic, linear and nonlinear materials, yield stress, ultimate stress, ductility, brittleness, strain hardening, necking, and percentage elongation.

14. (5 points) Describe the difference between elastic deformation and plastic deformation.

15. (3 points) TRUE or FALSE. Whereas all materials have an ultimate stress, not all materials have a yield stress.

16. (3 points) TRUE or FALSE. Brittness and ductility are antonyms.

17. (5 points) How can one quantify ductility?

18. (8 points) Draw a qualitative stress versus strain graph for an engineering metal such as steel, indicating linear elastic behavior, the yield point, strain hardening, and the ultimate stress.

19. (3 points) TRUE or FALSE. For an engineering metal that exhibits yielding (the distinct transition from linear elastic behavior to plastic behavior at a constant stress), the yield stress and ultimate stress will be the same stress level if no strain hardening is present.

20. (3 points) TRUE or FALSE. Because natural rubber may be stretched up to 800% of its initial length without ever becoming permanently deformed, it is considered to be an extremely ductile material.

The following two questions refer to Figure 1, the stress-strain curve below, for materials 1 and 2.
21. (5 points) Circle the correct statement.
   A. Material 1 has a larger modulus of elasticity $E$ and a larger ultimate stress $\sigma_u$.
   B. Material 1 has a larger modulus of elasticity $E$ and a smaller ultimate stress $\sigma_u$.
   C. Material 1 has a smaller modulus of elasticity $E$ and a smaller ultimate stress $\sigma_u$.
   D. Material 1 has a smaller modulus of elasticity $E$ and a larger ultimate stress $\sigma_u$.

22. (5 points) Circle the correct statement.
   A. Material 1 is more ductile and has a larger yield stress $\sigma_y$.
   B. Material 1 is more ductile and has a smaller yield stress $\sigma_y$.
   C. Material 1 is less ductile and has a smaller yield stress $\sigma_y$.
   D. Material 1 is less ductile and has a larger yield stress $\sigma_y$.

23. (3 points) TRUE or FALSE. (Referring to the stress-strain curve in Figure 2, below), Hooke’s Law is valid between points A and B but it is not valid between points A and C.

24. (3 points) Give three examples of ductile engineering materials.

25. (3 points) Give three examples of brittle materials.

26. (5 points) Describe and illustrate the Poisson’s effect.

27. (3 points) TRUE or FALSE. If the Poisson’s ratio, $\nu$ and the Shear Modulus of Elasticity, $G$ are given, then one may determine the Elastic Modulus $E$, from this given information.
General Knowledge of Common Engineering Materials – Steel, Aluminum, Concrete, Wood

Structural Steel (typical steel)
- Tensile Yield stress ~ 36 to 50 ksi (~ 250 to 340 MPa)
- Tensile Ultimate strength ~ 55 to 65 ksi (~ 380 to 450 MPa)
- E: 30000 ksi (200 GPa)
- An extremely ductile material. Strain at fracture can be as high as 25 to 30%
- An extremely tough material
- Like all ductile metals, its shear strengths are approximately 60% of its tensile strengths

Aluminum 6061 Alloy (the most common alloy)
- Tensile Yield stress ~ 36 to 42 ksi (~ 250 to 290 MPa)
- Tensile Ultimate strength ~ 45 to 50 ksi (~ 310 to 340 MPa)
- E: 9700 ksi (70 GPa)
- A ductile material but not as ductile as typical steel. Strain at fracture can be as high as 12%
- A tough material but not as tough as typical steel.
- Like all ductile metals, its shear strengths are approximately 60% of its tensile strengths

Typical Concrete (note: the material tested in lab was unusually strong and atypical)
- Yield stress: not applicable. Only metals possess a yield point
- Compressive Ultimate strength ~ 3 to 5 ksi (~ 20 to 35 MPa)
- E: 4000 ksi (28 GPa)
- Virtually no ductility
- Toughness extremely low
- Tensile strength: ~ zero. The tensile strength is typically about 10% of the compressive strength, but this tiny strength is generally considered negligible.

Typical Wood (southern yellow pine)
- Yield stress: not applicable. Only metals possess a yield point
- Compressive Ultimate strength ~ 7 to 9 ksi (~ 45 to 60 MPa)
- E: 1500 ksi (10 GPa)
- High ductility
- Toughness high

Solve problems for unknown, stress, force, strain, or deformation, utilizing, $\sigma=\varepsilon E$, $\sigma=F/A$, and statics (including multiple FBD’s).

28. (15 points) The tension member shown below has a cross-sectional area of 5 in$^2$. Initially, it was 20 feet long. Determine the new length of the tension member after a load of 350 kips has been applied.

Given: The tension member is made from an engineering metal that has the stress-strain curve shown below.

![Stress-Strain for Engineering Metal](image)
29. (20 points) A thin-walled steel pipe (E=29000 ksi) with a 4.00-inch diameter is subjected to air pressure which causes its diameter to increase to 4.005-inches. Determine the normal stress in the walls of the pipe, caused by this diameter-change. (Ans: 36.25 ksi)

30. (20 points) Determine how much the wire stretches when the horizontal force of 2.5 kips acts on the pole. Given: the rigid pole is supported by a pin at C and a high-strength steel (E=29000 ksi) wire at point B. The diameter of the wire is 0.2 inches. (Ans: 0.304 inches)

31. (30 points) Determine the angle of tilt of the rigid beam when P=80kN is applied. Given: The rigid beam is supported by members AB (40-mm diameter) and CD (80-mm diameter), both made from a polyester resin whose $\sigma-\epsilon$ curves are given below. (Ans: 0.708°)

32. (35 points) Determine the applied load P to cause $\theta=44.9^\circ$. Given: The A-36 steel (E=200000 MPa) guy wire AB has a cross-sectional area of 10mm². When unstretched, $\theta=45^\circ$. The link is considered to be rigid.
33. (30 points) Determine the cross-sectional area of Cable BC. Give your answer in units of in².

Given: Beam AB is rigid. It is pinned at A and supported at B by a high strength steel cable (E=30000 ksi) BC, while a 3 kip vertical load is applied at B. It is known that position B displaces downwards by 0.1 inch. It is also known that the cable BC remains elastic.

Solve problems for unknown, stress, force, strain, or deformation, utilizing, \( \tau = G \gamma \), \( \tau = \frac{V}{A} \), and statics (e.g., shear spring problems).

34. (20 points) Two blocks of rubber, each with a width \( w = 3 \) inches, are bonded to rigid supports and to the movable plate AB. Knowing that a force of magnitude \( P = 7 \) kips causes plate AB to move \( \delta = 0.125 \) inches, determine the shear modulus \( G \) for the material. Assume that the small-angle assumption may be used.
Solve problems for unknown, stress, force, strain, or deformation, utilizing Poisson’s ratio \( v = -\left(\frac{\varepsilon_{\text{lat}}}{\varepsilon_{\text{long}}}\right) \)

35. (20 points) The high-strength steel bolt shown was initially 1.000 inches in diameter. After tensioning the nut, the diameter was found to be 0.999 inches. If \( E = 30000 \) ksi, and the Poisson’s ratio \( v = 1/3 \), determine the tensile force in the bolt. Assume that the bolt remains elastic.

36. (25 points). Determine the force \( P \).

Given: Prior to applying the load \( P \), the steel bar (\( E = 29000 \) ksi, \( v = 0.25 \)) was 2.000 inches wide, 0.500 inches thick, and 25.000 inches long. After applying the load \( P \), the width was reduced to 1.999 inches.

(Ans: \( P = 58 \) kips)

Compute or estimate the elastic modulus, yield strength (if applicable), ultimate strength, maximum strain (AKA, Percentage Elongation or Contraction), and toughness of engineering materials from experimental data.

37. (35 points) Determine the elastic modulus, yield strength, ultimate strength, maximum strain, and toughness for the following engineering material and plot its stress-strain curve. Given: The material (cross-sectional area = 0.2 in\(^2\)) was tested in tension, recording load and deformations over a 2-inch length.

38. (3 points) Which material has a higher Modulus of Elasticity, Steel or Aluminum?

39. (3 points) Which material has a higher Modulus of Elasticity, Concrete or Wood?

40. (3 points) Which material has higher ductility in tension, Steel or Concrete?

41. (3 points) Which material has higher ductility in tension, Steel or Aluminum?

42. (3 points) TRUE or FALSE. Typical concrete (such as that tested in lab) has a higher compressive strength than typical wood (such as that tested in lab).

43. (3 points) Circle the closest answer:
   A. Typical concrete is approximately 2 to 3 times stronger than typical wood in compression.
   B. Typical concrete has approximately the same strength as typical wood in compression.
   C. Typical wood is approximately 2 to 3 times stronger than typical concrete in compression.
   D. Typical wood is approximately 20 to 30 times stronger than typical concrete in compression.

44. (3 points) Circle the closest answer:
   A. Steel and concrete have approximately the same stiffness.
   B. Steel is approximately 10 times stiffer than typical concrete.
   C. Typical concrete is approximately 10 times stiffer than steel.
   D. Steel is approximately 100 times stiffer than typical concrete.

45. (3 points) TRUE or FALSE. The tensile strength of steel is greater than the shear strength of steel.

Derive or Memorize the formula that relates deformation, \( \delta \), to \( P, L, A, \) and \( E \).

Compute the deformation of a prismatic axially-loaded bar due to discrete loads.
46. (20 points) Determine deformation $\delta_B$ and specify its direction (left or right)
Given: Member has a fixed support at A. Area $A = 1\text{in}^2$ and $E=10000\text{ksi}$ for all. It is known to be elastic.

47. (5 points) Determine the magnitude of internal force between B and C and indicate whether it is in compression or tension.
Given: Member has a fixed support at A. It is known to be elastic.

48. (10 points) Determine the internal axial force present in the rod due to its own weight at a distance of 10000 inches below point A.
Given: Rod AB weighs 2 lbs per inch and has a cross-sectional area of 10 in$^2$. It is 100000 inches long and hangs from a pin at A. It is known to be elastic.

49. (10 points) Determine the maximum length that the hanging rod can have without yielding the rod due to its own self-weight.
Given: Rod AB weighs 2 lbs per inch and has a cross-sectional area of 10 in$^2$. The rod material yields at a stress of 50 ksi.
50. (35 points) Determine how much rod AB stretches ($\delta_{AB}$) due to its own weight.

Given: Rod AB weighs 2 lbs per inch. It is 100000 inches long and hangs from a pin at A. Rod AB is made from a material with a modulus of elasticity $E = 10,000,000$ psi. It has a cross-sectional area $A = 10$ in$^2$. It is known to be elastic.

51. (20 points) Determine the internal axial force at a point midway between A and B.

Given: Fixed support at B. The member is subjected to a distributed frictional force $w(x) = (1/8)x^2$, where the origin for $x$ is at point A, and units are kips and feet. It is known to be elastic.
52. (20 points) Determine the reaction at point B.
Given: Fixed support at B. The member is subjected to a distributed frictional force \( w(x) = \frac{1}{8}x^2 \) kips/ft, where the origin for \( x \) is at point A, and units are kips and feet. It is known to be elastic.

\[ w = \frac{1}{8}x^2 \text{ kips/ft} \]

53. (30 points) Determine \( \delta_A \)
Given: Fixed support at B. The member is subjected to a distributed frictional force \( w(x) = 0.0000723x^2 \) kips/ft, where the origin for \( x \) is at point A, and units are kips and inches. The member has \( A = 1 \text{ in}^2 \) and \( E = 29000 \text{ ksi} \). It is known to be elastic.

\[ w = 0.0000723x^2 \]

54. (30 points) Determine \( \delta_B \).
Given: The 100 inch long bar shown is made from a material with \( E = 10000 \text{ ksi} \). It is subjected to a point load of 10 kips at point B. The cross sectional area of the bar varies linearly from 2 in\(^2\) at point A to 1 in\(^2\) at point B. It is known to be elastic.

55. (20 points) Determine the internal axial force at a point midway between A and B.
Given: Circular concrete pile is embedded in the soil. The pile is 400-inches long and is subjected to a vertical load at point A of 50 kips, directed downward. The vertical force is resisted by a distributed frictional force \( f(x) \) that varies linearly from 0 kips/inch at the bottom to \( f_{\text{max}} \) at the top. It is known to be elastic.
Compute the deformation of a simple truss or machine due to the axial deformation of its bars.

56. (25 points). Determine how far roller B moves due to the 5 kip load at point C.

Given: the truss consists of three pin-connected steel members (E=29000 ksi), each with a cross-sectional area A=1 in.². It is known to be elastic.
Answers to Selected Problems

1. F
2. F
3. 0.025
4. 0.999 degrees
5. 0.00502 inches
6. 0.00884 rad
7. 0.0142 rad, G=7060 MPa
8. -0.006797
9. 0.02 rad
10. 0.00963
11. 88.28 ksi
12. E=3731 ksi

13. Elastic deformation returns to zero deformation upon load removal. Plastic deformation is deformation that remains, upon load removal.


15. T

16. Max plastic strain

17. F. This is elastic strain, not plastic strain.

18. D
22. C  
23. T  
24. Gold, copper, mild steel  
25. Glass, ceramic, masonry  

26.  
27. T  
28. L=21.06 feet  
29. 36.25 ksi  
30. 0.304”  
31. 0.709 degrees  
32. 2466 N  
33. A=0.166 in²  
34. G=1.815 ksi  
35. Force in bolt is 70.69 kips  
36. 58 KIPS  
37. E=10000 ksi, yield = 50 ksi, ultimate = 55 ksi, toughness = 5.00 ksi  
38. Steel  
39. Concrete  
40. Steel  
41. Steel  
42. F  
43. C  
44. B  
45. T  
46. 0.075 inches to the left.  
47. 10 kips, tension  
48. 180 kips, tension  
49. 250,000 inches  
50. 100 inches  
51. 0.3333 kips  
52. 2.666 kips  
53. 0.0000230 inches  
54. 0.0693 inches  
55. 12.5 kips  
56. 0.0166 inches