1. (15 points) When running at a steady state, the motor causes accelerations that make people nearby uncomfortable. Determine the steady state acceleration that is caused by the motor, reporting your answer in g's.

Given: the machine is supported by a W10x33 beam. The machine weighs 11000 lbs. The machine contains an eccentric weight of 50 lbs that spins with an eccentricity of 3 inches at 300 rpm. Assume the beam self-weight is negligible. Assume 5% damping and that the beam is elastic.

Machine with 50 lb eccentric weight at 3-inch eccentricity, spinning at 300 rpm.

\[ M = \frac{11}{386} = 0.0285 \]

\[ K = \frac{48(29000)(1.71)}{2003} = 29.754 \ m/\pi \]

\[ P_o = \left(\frac{50}{386}\right) \left(\frac{1}{3}\right)^2 = 384/29.754 = 0.384 \]

\[ \frac{P_o}{K} = 0.01289'' = u_{ST} \]

\[ \text{PEAK } U = R_d \left(\frac{P_o}{K}\right) = 0.1150'' \]

\[ U(t) = (0.1150) \cos (\omega \ t) \]

\[ \omega = \sqrt{\frac{K}{m}} = 32.33 \text{ rad/s} \]

\[ \omega = \frac{\text{300 rpm}}{360} = \frac{\pi}{3} \text{ rad/s} \]

\[ r = \frac{P_o}{\omega_m^2} = 0.9717, \ z = 0.05 \]

\[ R_d = \frac{1}{\sqrt{(1-r^2)^2 + (2z \ r)^2}} = 8.92 \]

\[ \ddot{U}(t) = (0.1150)(10^2) = 113.5 \text{ in/s}^2 = 0.2949g \]

29.4%g is huge
2. (15 points) Determine the maximum bending stress in a column due to the inelastic collision of the artillery shell with the building roof and explain why damping can be ignored.

Given: at time $t=0$, the structure is struck by a 50-lb artillery shell, traveling at 1000 feet per second. Floor beams are considered infinitely stiff. All beam-column connections are rigid. All foundations are pinned. Columns are W10x33, bent about their strong axes. Bracing consists of redundant (tension-only) pin-ended cross-bracings ($A = 1\text{in}^2$). All material is steel. The roof weight is 100 kips. Ignore column self-weight. Assume the structure remains elastic.

**Diagram:**

- 50-lb artillery shell traveling at 1000 feet per sec.
- W10x33 Columns, typ.
- $I = 171\text{in}^4$
- Weight = 100 kips
- 30-ft
- 40-ft, typ.

**Analysis:**

\[
\text{FRAME } K_f = 4 \left( \frac{361}{360} \right) = 1.275 \text{ k/lb}
\]

\[
\text{BRACE } K_b = \frac{AF \cos^2\theta}{L} = \frac{29000}{600^2} (0.64) = 30.93 \text{ k/lb}
\]

**SYSTEM**

\[
K = 1.275 + 30.93 = 32.21 \text{ k/lb}
\]

\[
\omega_n = \sqrt{\frac{K}{m}} = 11.15 \text{ rad/s}
\]

**Conservation of Momentum:**

\[
\frac{m}{2} v_0 = (50 \text{lb})(12000 \text{ ft/s}) = (100000) v_0 = v_0 = 5.997 \text{ ft/s}
\]

**Consider Undamped b/c Peak U Occurs At First 1/4-Cycle (Negligible Energy Loss)**

\[
U(t) = v_0 \cos \omega_n t + \frac{v_0}{\omega_n} \sin \omega_n t
\]

**Max Amplitude:**

\[
U = 0.5378
\]

\[
5.146\text{k-ft} = (30') (0.175) = M_{\text{max}}
\]

\[
\rho = 0.3189 \frac{(0.5378)}{0.1715} = 1.81 \text{k-ft}
\]

\[
\sigma = \frac{Mc}{I} = \frac{5.146(12\%)/(5)}{171} = 1.81 \text{kpsi}
\]
3. (15 points) Determine the time required for block A to move 1 meter downward if it is released from rest.

\[
\begin{align*}
\sum F_x &= ma \\
16.99 - T &= 2a \\
T &= 16.99 - 2a \\
\text{Subst} & \Rightarrow 16.99 - 2a = 4.905 + 3a \\
12.09 &= 5a \\
& \Rightarrow a = 2.418 \text{ m/s}^2 \\

x &= \frac{1}{2} at^2 \\
1 &= \frac{1}{2} (4.03)t^2 \\
& \Rightarrow t = 0.704 \text{ sec}
\end{align*}
\]
4. (10 points). A 386-lb bungee-jumper jumps off of a bridge. Determine how far the bungee stretches, \( x \) (inches).

**Given – The Bungee:**

The bungee is 500 inches long, with an area \( A = 10 \text{ in}^2 \), made from a material with \( E = 100 \text{ psi} \).

**Assumptions:**

Neglect air resistance. Ignore the bungee self-weight. Assume that the jumper falls vertically, without any horizontal oscillation. Assume that the bungee is perfectly elastic, without energy loss until after the bungee stretches to its lowest position (note: if there was *never* any energy loss, he would be able to return to the bridge after each oscillation).

\[
U_{\text{top}} = \frac{386}{m} \left( 500 + x \right) = 193000 + 386x
\]

\[
U_{\text{bot}} = \frac{1}{2} k x^2 = x^2
\]

\[
k = \frac{AE}{L} = \frac{1000}{500} = 2 \text{ lb/in}
\]

\[
U_{\text{top}} = U_{\text{bot}}
\]

\[
193000 + 386x = x^2
\]

\[
x = 386 \pm \sqrt{386^2 + 4(193000)} \Rightarrow x = 673 \text{ in}
\]

WE ARE ASKING A LOT OF THIS BUNGEE
5. (10 points). Compute the maximum horizontal displacement of the floor, relative to the ground, due to the ground-shaking (units: inches).

The frame shown has a horizontal frame stiffness \( k = 30 \text{ kip/inch} \), a weight of 20 kips at the floor level and is subjected to harmonic horizontal shaking at the ground with a frequency of 5 hertz, reaching a peak base acceleration of 0.5g. Damping is 5%.

\[
\begin{align*}
\omega &= \frac{20}{386} = 0.05181 \\
\omega_n &= \sqrt{\frac{30}{0.05181}} = 24.06 \text{ rad/s} \\
\gamma &= \frac{5/2}{10\pi} = 1.3057 \\
\gamma &= \frac{\omega}{\omega_n} = \frac{10\pi}{24.06} = 1.3057 \\
\ddot{\gamma} &= 193.9/2 \\
\dot{\gamma} &= \frac{\sqrt{1+(2\pi)^2}}{\sqrt{(1-\dot{\gamma})^2 + (2\pi)^2}} = 1.407 \\
\ddot{\gamma} &= 1.407(193) = 271.5 \text{ m/s}^2 = 0.7035g \\
\ddot{\gamma} &= 20(0.7035) = 14.07^k \\
\omega &= \frac{k}{30} = 0.469^" \\
\text{Relative Disp} \ U_r &= \frac{14.07}{30} = 0.469^"
\end{align*}
\]
7. (20 points) Use response spectrum analysis to draw the column moment diagrams due to El Centro (refer to D-V-A plots on page 1).

Given: The acceleration response spectrum is given, below. The mode shapes and periods are also given, as are story weights. All columns are W10x33 (neglect their self-weights) and all floor are considered infinitely stiff. Assume 5% damping and that the structure remains elastic.

\[ M = \begin{bmatrix} 0.2591 \\ 0 \\ 0.2591 \end{bmatrix} \]

\[ M_1 = \begin{bmatrix} 1 & 1.627 \\ 0 & 0.2591 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.2591 \\ 0 \\ 0.2591 \end{bmatrix} \begin{bmatrix} 1 \\ 1.627 \end{bmatrix} = 0.9449 \]

\[ M_2 = \begin{bmatrix} 1 & -0.617 \\ 0 & 0.2591 \\ 0 & 0.2591 \end{bmatrix} \begin{bmatrix} 0.2591 \\ 0 \\ 0.2591 \end{bmatrix} \begin{bmatrix} -0.617 \\ 1 \\ -0.617 \end{bmatrix} = 0.3577 \]

\[ \Gamma_1 = \begin{bmatrix} 1 & 1.627 \\ 0 & 0.2591 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} 0.2591 \\ 0 \\ 0.2591 \end{bmatrix} \begin{bmatrix} 1 \\ 1.627 \end{bmatrix} = 0.6806 \]

\[ \Gamma_2 = \begin{bmatrix} 1 & -0.617 \\ 0 & 0.2591 \\ 0 & 0.2591 \end{bmatrix} \begin{bmatrix} 0.2591 \\ 0 \\ 0.2591 \end{bmatrix} \begin{bmatrix} -0.617 \\ 1 \end{bmatrix} = 0.3577 \]

MODE 1: \[ u_1 = (0.7203)(1.36) = 2.593" \Rightarrow u_1 = 2.61" \]

MODE 2: \[ u_2 = (0.7203)(1.627)(3.6) = 4.219" \Rightarrow u_2 = 4.22" \]
\[ u_1 = 2.61'' \]

\[ DV_1 = \frac{1}{2}(39.86)(2.61) = 52.02 \]

\[ \sum M = (52.02)(12) - 2M \]

\[ \Rightarrow M = 312 \text{ k-ft} \]

\[ 4.22 - 2.61 = 1.61'' \]

\[ V_2 = \frac{1}{2}(39.86)(1.61) = 32.09 \text{ k} \]

\[ \sum M = 32.09(12) - 2M \]

\[ \Rightarrow M = 192.52 \text{ k-ft} \]