LEsson Objectives
2. Construct maximum moment envelopes for determinate beams, subjected to loads that can move (e.g., distributed live loads, etc.)

Supplementary Reading: Found on the Online Version of this handout.

Homework
1. Use Muller-Breslau to construct the influence lines for the reaction at B, the shear at C, and the moment at C. Then, determine the range of reactions at B, the maximum positive moment at C, the maximum negative moment at C, and the maximum shear magnitude at C.
   Given: uniform dead load of 0.5 kip/ft, uniform live load of 1 kip/ft, and a single live concentrated load of 10 kips. B is a pin support, while A is a roller support.

2. Draw the maximum moment envelopes (i.e., positive and negative) neatly and to scale for the beam shown, indicating all values, using a 2ft-interval.
   Given:
   1.0 kip/ft distributed dead load
   2.0 kip/ft distributed live load
   A single 10 kip moving point load
   Beam is a reinforced concrete beam (relevant to problem 3).

3. Draw the previous beam, showing the locations for which positive (bottom) and negative (top) reinforcing steel are needed. Note: Reinforced Concrete beams need steel reinforcement on the tension side, only (this will be covered in detail in a subsequent lesson).

1 The irony of this statement is that Dan Gable was known as “the man who never lost.” In 1970, he was famous and on the cover of Sports Illustrated because of his remarkable story: in nearly 300 wrestling matches in high school and in college, he had never lost. Not once. Not ever. With a collegiate record of 181-0, ABC’s Wide World of Sports focused the nation’s attention on Dan Gable, as he met Larry Owings in the NCAA finals, his senior year. Incredibly, Gable lost (meanwhile, the only claim to fame for Larry Owings is that he is the answer to the trivia question, “who is the only man to beat Dan Gable?”). Then he got really motivated. He trained for the 1972 Olympics at 4am because he “couldn’t sleep knowing that, somewhere, the Russians were training.” He won the 1972 Olympic gold medal while never even giving up a point. He celebrated winning the gold medal by immediately going for a 5-mile run. His coaching career was even more dominant than his wrestling career. In 21 years of coaching the Iowa Hawkeyes, he won 21 Big10 titles and 16 National Championships (including 9 in a row, a feat unmatched in any college sport). In the state of Iowa, a common expression is, “Michael Jordan is the Dan Gable of Basketball.”

Internationally, he was so famous that A Des Moines newspaper reporter once wore an Iowa hat to a Russian village during the Cold War, in the mountains near the Siberian border, where an American had never been. A young man stepped out of the crowd, pointed toward the hat and said one of the few English words he knew: “Gable.”
In 1886, Heinrich Müller-Breslau developed a technique for rapidly constructing the shape of an influence line. Referred to as the Müller-Breslau principle, it states that the influence line for a function (reaction, shear, or moment) is to the same scale as the deflected shape of the beam when the beam is acted upon by the function. In order to draw the deflected shape properly, the capacity of the beam to resist the applied function must be removed so the beam can deflect when the function is applied. For example, consider the beam in Fig. 6-12a. If the shape of the influence line for the vertical reaction at A is to be determined, the pin is first replaced by a roller guide as shown in Fig. 6-12b. A roller guide is necessary since the beam must still resist a horizontal force at A but no vertical force. When the positive (upward) force $A_y$ is then applied at A, the beam deflects to the dashed position, which represents the general shape of the influence line for $A_y$, Fig. 6-12c. (Numerical values for this specific case have been calculated in Example 6-1.) If the shape of the influence line for the shear at C is to be determined, Fig. 6-13a, the connection at C may be symbolized by a roller guide as shown in Fig. 6-13b. This device will resist a moment and axial force but no shear. Applying a positive shear force $V_C$ to the beam at C and allowing the beam to deflect to the dashed position, we find the influence-line shape as shown in Fig. 6-13c. Finally, if the shape of the influence line for the moment at C, Fig. 6-14a, is to be determined, an internal hinge or pin is placed at C, since this connection resists axial and shear forces but cannot resist a moment. Fig. 6-14b. Applying positive moments $M_C$ to the beam, the beam then deflects to the dashed position, which is the shape of the influence line, Fig. 6-14c.

The proof of the Müller-Breslau principle can be established using the principle of virtual work. Recall that work is the product of either a linear
displacement and force in the direction of the displacement or a rotational displacement and moment in the direction of the displacement. If a rigid body (beam) is in equilibrium, the sum of all the forces and moments on it must be equal to zero. Consequently, if the body is given an imaginary or virtual displacement, the work done by all these forces and couple moments must also be equal to zero. Consider, for example, the simply supported beam shown in Fig. 6–15a, which is subjected to a unit load placed at an arbitrary point along its length. If the beam is given a virtual (or imaginary) displacement $\delta y$ at the support $A$, Fig. 6–15b, then only the support reaction $A_y$ and the unit load do virtual work. Specifically, $A_y$ does positive work $A_y \delta y$ and the unit load does negative work, $-1 \delta y'$. (The support at $B$ does not move and therefore the force at $B$ does no work.) Since the beam is in equilibrium and therefore does not actually move, the virtual work sums to zero, i.e.,

$$A_y \delta y - 1 \delta y' = 0$$

If $\delta y$ is set equal to 1, then

$$A_y = \delta y'$$

In other words, the value of $A_y$ represents the ordinate of the influence line at the position of the unit load. Since this value is equivalent to the displacement $\delta y'$ at the position of the unit load, it shows that the shape of the influence line for the reaction at $A$ has been established. This proves the Müller-Breslau principle for reactions.
In the same manner, if the beam is sectioned at C, and the beam undergoes a virtual displacement \( \delta y \) at this point, Fig. 6–15c, then only the internal shear at C and the unit load do work. Thus, the virtual work equation is

\[
V_C \delta y - 1 \delta y' = 0
\]

Again, if \( \delta y = 1 \), then

\[
V_C = \delta y'
\]

and the shape of the influence line for the shear at C has been established.

Lastly, assume a hinge or pin is introduced into the beam at point C, Fig. 6–15d. If a virtual rotation \( \delta \phi \) is introduced at the pin, virtual work will be done only by the internal moment and the unit load. So

\[
M_C \delta \phi - 1 \delta y' = 0
\]

Setting \( \delta \phi = 1 \), it is seen that

\[
M_C = \delta y'
\]

which indicates that the deflected beam has the same shape as the influence line for the internal moment at point C (see Fig. 6–14).

Obviously, the Müller-Breslau principle provides a quick method for establishing the shape of the influence line. Once this is known, the ordinates at the peaks can be determined by using the basic method discussed in Sec. 6–1. Also, by simply knowing the general shape of the influence line, it is possible to locate the live load on the beam and then determine the maximum value of the function by using statics. Example 6–12 illustrates this technique.
EXAMPLE 6-9

For each beam in Fig. 6-16a through 6-16c, sketch the influence line for the vertical reaction at A.

Solution

The support is replaced by a roller guide at A and the force $A_y$ is applied.

![Diagram (a)](image)

Fig. 6-16

Again, a roller guide is placed at A and the force $A_y$ is applied.

![Diagram (b)](image)

A double-roller guide must be used at A in this case, since this type of support will then transmit both a moment $M_A$ at the fixed support and axial load $A_x$, but will not transmit $A_y$.

![Diagram (c)](image)
For each beam in Fig. 6-17a through 6-17c, sketch the influence line for the shear at B.

Solution

The roller guide is introduced at B and the positive shear $V_B$ is applied. Notice that the right segment of the beam will not deflect since the roller at A actually constrains the beam from moving vertically, either up or down. [See support (2) in Table 2-1.]

Placing the roller guide at B and applying the positive shear at B yields the deflected shape and corresponding influence line.

Again, the roller guide is placed at B, the positive shear is applied, and the deflected shape and corresponding influence line are shown. Note that the left segment of the beam does not deflect, due to the fixed support.
EXAMPLE 6-11

For each beam in Fig. 6-18a through 6-18c, sketch the influence line for the moment at B.

Solution
A hinge is introduced at B and positive moments $M_B$ are applied to the beam. The deflected shape and corresponding influence line are shown.

![Deflected Shape and Influence Line](image)

Fig. 6-18

Placing a hinge at R and applying positive moments $M_B$ to the beam yields the deflected shape and influence line.

![Deflected Shape and Influence Line](image)

With the hinge and positive moment at B the deflected shape and influence line are shown. The left segment of the beam is constrained from moving due to the fixed wall at A.

![Deflected Shape and Influence Line](image)
Determine the maximum positive moment that can be developed at point $D$ in the beam shown in Fig. 6-19d due to a concentrated moving load of 4000 lb, a uniform moving load of 300 lb/ft, and a beam weight of 200 lb/ft.

**Solution**

A hinge is placed at $D$ and positive moments $M_D$ are applied to the beam. The deflected shape and corresponding influence line are shown in Fig. 6-19b. Immediately one recognizes that the concentrated moving load of 4000 lb creates a maximum positive moment at $D$ when it is placed at $D$, i.e., the peak of the influence line. Also, the uniform moving load of 300 lb/ft must extend from $C$ to $E$ in order to cover the region where the area of the influence line is positive. Finally, the uniform weight of 200 lb/ft acts over the entire length of the beam. The loading is shown on the beam in Fig. 6-19c. Knowing the position of the loads, we can now determine the maximum moment at $D$ using statics. In Fig. 6-19d the reactions on $BE$ have been computed. Sectioning the beam at $D$ and using segment $DE$, Fig. 6-19e, we have

$$\sum M_D = 0; \quad -M_D - 5000(5) + 4750(10) = 0$$

$$M_D = 22500 \text{ lb} \cdot \text{ft} = 22.5 \text{ k} \cdot \text{ft}$$

**Ans.**
This problem can also be worked by using numerical values for the influence line as in Sec. 6-1. Actually, by inspection of Fig. 6-19b, only the peak value \( h \) at \( D \) must be computed. This requires placing a unit load on the beam at \( D \) in Fig. 6-19a and then solving for the internal moment in the beam at \( D \). Show that the value obtained is \( h = 3.33 \). By proportional triangles, \( h'/(10 - 5) = 3.33/(15 - 10) \) or \( h' = 3.33 \). Hence, with the loading on the beam as in Fig. 6-19c, using the areas and peak values of the influence line, Fig. 6-19b, we have:

\[
M_D = 500 \left[ \frac{3}{2} (25 - 10)(3.33) \right] + 4000(3.33) - 200 \left[ \frac{3}{2} (10)(3.33) \right] \\
= 22500 \text{ lb} \cdot \text{ft} = 22.5 \text{ k} \cdot \text{ft}
\]

Ans.