Homework (SOLUTION.)

0. (Do not hand in). If we define a term \( M^* = P\delta_1 \), where \( P \) is the axial force and \( \delta_1 \) is the first order deflection, then define \( \alpha = \frac{M^*}{M_{nt}} \), derive a series-solution formula for the final moment (considering second order \( P\delta \) effects), based on the Kurtz Method. This derivation may save you time, in problems 1 and 2.

\[
\begin{align*}
M^* &= 100 \text{ k-in} \\
\delta_1 &= 1'' \text{ in} \\
\Rightarrow \quad M^* &= P\delta_1 = 100(1'') = 600 \text{ k-in} \\
M_{nt} &= 600 \text{ k-in} \\
\Rightarrow \quad \alpha &= \frac{M^*}{M_{nt}} = \frac{600}{600} = 1 \\
\Rightarrow \quad M &= M_{nt} + 100(1') = 16.7 \text{ k-in} \\
\Rightarrow \quad M &= M_{nt} + 100(0.0277') = 2.78 \text{ k-in} \\
\therefore \quad M &= 600 + 100 + 16.7 + 2.78 + \ldots = 719.9 \\
\end{align*}
\]

So, what is happening here?

\[
M = M_{nt} \left( 1 + \frac{M^*}{M_{nt}} + \left(\frac{M^*}{M_{nt}}\right)^2 + \left(\frac{M^*}{M_{nt}}\right)^3 + \ldots \right)
\]

Or if \( \alpha = \frac{M^*}{M_{nt}} \):

\[
M = M_{nt} \sum_{n=0}^{\infty} \alpha^n
\]

Hence, \( \alpha = \frac{100}{600} = 0.1667 \)

\[
\Rightarrow \quad M = 600 \left( 0.1667 + (0.1667)^2 + (0.1667)^3 + \ldots \right) = 600(1.20) = 720
\]
0. (Do not hand in). For which Case (A or B) would the $B_1$ factor be greater?

$B_1$ is greater for Case B because the beam is more flexible, leading to higher deflections, and higher $P\delta$ effect.

1. Use ASD to determine if a W16x31 A992 shape is adequate for PM. Consider 2nd Order Effects using the Kurtz Method for an approximate higher order analysis.

Given:
- The beam is simply-supported over a span of 30 feet, subjected to a uniformly-distributed load (already includes self-weight and slab weight) of 1 kip per foot.
- The beam is continuously braced: LTB cannot occur. Buckling about the weak-axis cannot occur, but it can still buckle about its strong axis.
- The beam is subjected to the 40-kip force that is shown. The 40-kip force is located precisely at the top of the flange.

Be sure to clearly report:
- The applied moment, without including 2nd order effects.
- The deflection due to 1st order moment (compute using the appropriate beam formula from Table 3-23)
- The applied moment, including 2nd order effects (using the Kurtz Method)
- The deflection, including the 2nd order effects.
- The $B_1$ factor, based on your analysis
- The results of AISC Interaction (is it adequate?)
Lesson 34: PM Interaction II

1st Order $M$:

**Distributed Load:** $M = \frac{wL^2}{8} = \frac{1}{8}(30^2) = 112.5 \text{ k-ft} = 1350 \text{ k-in}$

**End $M$:** $M = (40' \times \frac{15.9}{2}) = 318 \text{ k-in}$

$M_{t} = 1350 + 318 = 1668 \text{ k-in}$

1st Order $\delta$:

$EI = (29000) (375) = 10.875 \times 10^6 \text{ kip-in}^2$

For Uniform Load:

$\delta = \frac{5wL^4}{384EI} = \frac{5(12)(360^4)}{384(10.875 \times 10^6)} = 1.676''$

For End Moments (from Strength of Materials):

$M(x) = M$

$b,c$'s:

1. $v = 0$ @ $x = 0$
2. $v = 0$ @ $x = L$

$v(x) = \frac{M}{2EI} x^2 + c_1 x + c_2$

$0 = 0 + 0 + c_2 \Rightarrow c_2 = 0$
\[ a = \frac{M}{2EI} L^2 + C_1 L \]

\[ \Rightarrow C_1 = -\frac{M}{2EI} L \]

\[ \Rightarrow v(x) = \frac{M}{2EI} x^2 - \frac{M}{2EI} L x \]

For \( x = L/2 \):

\[ \sigma = \frac{ML^2}{8EI} - \frac{MLL^2}{4EI} = \frac{ML^2}{8EI} \quad (\uparrow) \]

Plug:

\[ \sigma = \frac{(318)(360)^2}{8EI} = 0.4737'' \]

Total First Order \( \sigma \):

- Uniform Load \( \sigma = 1.676'' \)
- End M \( \sigma = 0.4737'' \)

\[ \sigma_I = 2.150'' \]

\[ P_0 = M^* = (40k)(2.150) = 85.99 \text{ in.-kip} \]

\[ \alpha = \frac{M^*}{M_n} = \frac{85.99}{1668} = 0.05155 \]

\[ M = M_n \sum_{n=0}^{\infty} \alpha^n = 1668 \left( 1 + 0.05155 + 0.05155^2 + \ldots \right) = 1759 \text{ k-in} \]

Axial Resistance:

\[ \left( \frac{KL}{r} \right)_x = \frac{3600}{6.41^2} = 56.16 \Rightarrow \frac{F_{ec}}{A} = 23.77 \text{ ksi (Table 4-21)} \]

\[ \frac{P_0}{A} = A \cdot \frac{F_{ec}}{A} = (9.13)(23.77) = 217 \text{ k} \]
\[ \frac{M_n}{\Delta} = \frac{M_o}{\Delta} = 135 \text{ k-ft} \quad \text{(TABLE 3-2)} \]

\[ M_f = \frac{146.6}{135} = 1.0856 \]

\[ P_f = \frac{40}{217} = 0.1843 \leq 0.2 \]

**Interaction**

\[ \frac{1}{2} (0.1843) + 1.0856 = 1.18 \geq 1 \quad \therefore \text{NG} \]

**Summary**

* 1st Order \( M_n = 1668 \text{ k-ft} = 139 \text{ k-ft} \)

* 1st Order \( \delta = 2.150'' \)

* Full \( \delta = 2.15 \sum \alpha = 2.27'' \)

* \( B_i = \frac{146.6}{139} = 1.055 \)

* Interaction! \( 1.18 \quad \cancel{\text{NG}} \)
2. Use ASD to determine if a W10x45 A992 column is adequate for PM. Consider 2nd Order Effects using the Kurtz Method for an approximate higher order analysis.

Given:
- Simple-braced frame for the plane shown, as well as the plane not shown.
- Uniformly-loaded beam with 4kip/ft is pin-connected to the columns, as shown.
- Simple connections have an eccentricity of 1.5 inches, as shown.
Lesson 34: PM Interaction II

\[ M^* = (59.5 \text{k}')(0.480) = 28.53 \text{kips}\cdot\text{in} \]

\[ \Delta = \frac{28.53}{89.25} = 0.3197 \]

\[ M = M_{nt} \sum_i 0.3197 + 0.3197^2 + 0.3197^3 = 130.8 \text{kips}\cdot\text{in} \]

\[ \frac{M_n}{M} = 50.6 \text{kips}\cdot\text{in} \left( \frac{\text{WEAK AXIS}}{7.34} \right) = 10.9 \text{kips}\cdot\text{ft} \]
\[ \frac{P_n}{\alpha} = 62.3^k \quad (T_4-1 \quad y-y \quad C(ontrols \ @ \ K=30)) \]
\[ P = 59.5^k + S.W = 60.85^k \]
\[ P_\% = \frac{60.85}{62.3} = 0.9767 \]
\[ M_\% = \frac{10.9}{50.6} = 0.2154 \]

**INTERACTION**

\[ 0.9767 + \frac{8}{9} (0.2154) = 1.17 \]