LESSON OBJECTIVES
1. List the components that resist lateral-torsional buckling: Weak Axis bending (EI_y), Pure torsion (GJ), and Warp Torsion.
2. Describe why open sections (like I-shapes, channels, angles) stink in lateral-torsional buckling, but closed sections (square tubes, rectangular tubes, and round tubes) are awesome in LTB.
3. Estimate the polar moment of inertia for an open section and use this to Construct a simplified M_L versus L curve by ignoring the warp-torsional component and considering the beam to be either pure plastic (reaching M_p) or elastic (predicted by the M_r equation, below).

The Polar Moment of Inertia J (the torsional constant)
Here’s the deal: In strength of materials, you dealt with torsion on circular sections – solid circular sections and hollow circular section. The Polar Moment of Inertia J (the torsional constant) is simple, for these. The torsional constant J is definitely not simple for solid rectangles. Fortunately, the torsional constant IS simple for solid rectangles if the width b is significantly greater than the thickness t. For these, \( J = \frac{1}{3}bt^3. \) For general open shapes, composed of several thin plates (such as a wide-flange shape), J is simply the summation of \( \frac{1}{3}bt^3 \) for all of the plates. See below.

Closed Shapes: hollow rectangular and circular tubes. These shapes have exceptional torsional stiffness. J is given by the formulae on the lower left.

Thin, solid rectangular shape: The formulae for the J of general solid rectangular plates is beyond the scope of this course. However, if the plate is relatively thin (b/t greater than 10), then \( J = \frac{1}{3}bt^3 \) (see formula to the left). This formula still provides reasonable results if b/t is somewhat less than 10.

Open shapes: e.g., angles, channels, wide-flange I-shapes, etc. These shapes are very flexible in torsion. The exact solution for the J of general shapes is beyond the scope of this course. However, if the open shape is composed of relatively thin plates (typical for angles, channels, I-shapes), then the formula for J is the summation of the J’s for all of the plates: For these, J can be determined using the formulae to the left. Most notably, for all open sections (such as I shapes):

\[ J = \frac{1}{3} \sum bt^3 \]

Where b is the length of the long dimension of each plate and t is the thickness of the short dimension of each plate. The J for an I-shape, therefore, is the summation of 3 terms: a web bt and two flange bt.

Example: Compute J for the ridiculous-looking open section shown, consisting of the four plates:

Solution:
\[ J \approx \frac{1}{3} \left[ (10)\left(\frac{1}{2}\right)^3 + (6)\left(\frac{1}{4}\right)^3 + (7)\left(\frac{1}{8}\right)^3 + (4)\left(\frac{3}{8}\right)^3 \right] = 0.523in^4 \]

If you can compute J for this silly section, you can compute J for a W-shape or any other open shape (approximately). Using the other formulae, you can compute J for other shapes, including closed shapes (hollow rectangles, etc).

General Statement: All open shapes are terrible in torsion, while all closed shapes are excellent in torsion.
ELASTIC LATERAL-TORSIONAL BUCKLING:
If the resistance of warp-torsion is conservatively ignored, then the moment resistance with respect to LTB is:

\[ M_{cr} = \frac{\pi}{L_b} \sqrt{EI_y GJ} \]

The Nominal Moment Resistance \( M_n \).

If we ignore warp torsional resistance and inelastic behavior, then the nominal moment resistance \( M_n \) is simply the lesser of:

\begin{itemize}
  \item The limit state of material strength. In the case of a perfectly ductile steel beam, this is the fully plastic moment \( M_p \). In the case of a less ductile beam (e.g., a wooden beam, a fiberglass composite beam, etc), it would not be possible to reach fully-plastic, so this limit state of material strength might correspond with the extreme-fiber reaching a limiting value.
  \item Elastic Lateral-Torsional Buckling: \( M_{cr} = \frac{\pi}{L_b} \sqrt{EI_y GJ} \)
\end{itemize}

**HOMEWORK** (Due Wednesday. Standard homework assignment. Presentation counts.)

For all problems: “ignore warp torsion and inelastic behavior”; i.e., simplify Lateral-Torsional Buckling by considering the problem to either be limited by material strength (typically, this is the fully-plastic moment) or be limited by elastic lateral-torsional buckling, while ignoring the strength contribution of warp torsional resistance. Whereas Wednesday’s lesson will cover actual AISC specifications for structural steel, this homework uses simplified theory; today’s lesson is not applicable for actual steel design in the real world, but is intended to equip you with the ability to understand LTB in a simplified way.

1. An I-shaped beam is made of Nylon (\( E = 600 \text{ ksi}, \ G = 215 \text{ ksi}, \ \sigma_{yield} = 4 \text{ ksi} \)) and used as an unbraced cantilever beam, as shown. As described above, the limit states are either fully plastic bending or elastic lateral-torsional buckling. Assuming self-weight is negligible, determine:
   \begin{itemize}
     \item a. The maximum moment \( M_n \) that it can resist if the unbraced length \( L_b = 10'' \)
     \item b. The maximum end-force \( P \) that it can resist if \( L_b = 10'' \)
     \item c. The maximum moment \( M_n \) that it can resist if \( L_b = 100'' \)
     \item d. The maximum end-force \( P \) that it can resist if \( L_b = 100'' \)
   \end{itemize}

2. Situation: you are marooned on an island. You find an I-shaped Aluminum beam. Needing a cantilever beam for the purpose of hoisting large amounts of cantaloupes, you need to investigate the relationship between the unbraced cantilever length and the moment that it can resist, so you remember today’s lesson and realize that, ignoring warp torsion and inelastic effects, the nominal moment resistance \( M_n \) will be the lesser, considering only two limit states: Fully Plastic Bending \( (M_p) \) and Elastic LTB. Given:
   \( E = 10000 \text{ ksi}, \ G = 3750 \text{ ksi}, \ F_y = 40 \text{ ksi} \).

\[ 1'' \times 1/8'' \text{ Flange} \]
\[ 2'' \times 1/8'' \text{ Web} \]
\[ 1'' \times 1/8'' \text{ Flange} \]

\[ 3'' \times 1/2'' \text{ Flange} \]
\[ 15 \times 1/4'' \text{ Web} \]
\[ 3'' \times 1/2'' \text{ Flange} \]
Determine (units: kip-ft vs ft):
   a. $M_n$ for $L_b=0$ ft, 2 ft, 10 ft, 20 ft.
   b. Plot $M_n$ (y-axis) vs $L_b$ (x-axis) neatly with a pencil on engineering paper. It should look like this:

3. Situation: Same as problem 2, but you have now discovered a rectangular Aluminum tube shape (same material properties). You note that this beam has the same area (and, therefore, weight), as the I-shape (strange island, isn’t it?).

Determine (units: kip-ft vs ft):
   a. $M_n$ for $L_b=0$ ft, 2 ft, 10 ft, 100 ft.
   b. Plot $M_n$ (y-axis) vs $L_b$ (x-axis) neatly with a pencil on engineering paper.

4. Situation: The beams of problem 2 and 3 have the same weight per foot, but which is better?
   a. If $L_b = 2$ ft, which beam is stronger (I-shape or the rectangular tube?)?
   b. If $L_b = 10$ ft, which beam is stronger (I-shape or the rectangular tube?)?