Lesson 10 – Forced Harmonic Motion I (AKA “motors”)  

LESSON OBJECTIVES

1. Plot the transient and steady-state parts of forced harmonic response in undamped SDOF’s, for given conditions.
2. Identify the components of the displacement response: static deformation $u_s$, dynamic response factor $R_d$, the steady state term, and the transient term.
3. Compute and Plot the steady-state displacement and forces on a damped SDOF.
4. Study the effects of damping and driving frequency on a SDOF, subjected to harmonic loading.

Reading: Chopra Chapter 3, up to and including 3.2.1.

Frequency Terms:

$\omega_n =$ the natural frequency of the SDOF. This is the frequency that the SDOF system would vibrate at, in free vibration.

$\omega_d =$ the damped natural frequency of the SDOF.

$\omega =$ the forcing frequency. This is the frequency of the motor or other machinery that imparts a force on the structure.

If $\omega = \omega_n$, the structure is said to be in forced resonance. Large displacements and forces result. If the structure is undamped, infinite displacements result (in theory. In reality, it would develop damping due to damage).

Homework (Due Monday, September 26, 2016)

1. Assuming no damping, plot the complete displacement response of the rigid frame structure for 15 seconds of time, for each of the following cases:
   - In each case, the motor delivers a dynamic force of 5 kips, but the $\omega$ varies; i.e., $P(t)=5\sin\omega t$.
     a. Motor runs at 40 rpm
     b. Motor at 80 rpm
     c. Motor at 120 rpm
     d. Motor at 130 rpm
     e. Motor at 160 rpm
     f. Motor at 200 rpm
   - Then, considering the previous 6 $\omega$’s plot the peak dynamic force $ku$ versus $\omega$ (think about the fact that the static driving force is 5 kips).

Treat the horizontal girder as infinitely stiff. Assume the W10x33 columns have negligible weight and that they are bent about their strong axis. The floor has the distributed load shown. Be sure to show all calculations on a separate page.

2. Develop an Excel spreadsheet to study the effects of damping and driving frequency on the previous frame. Varying the damping ratio and driving frequency, obtain the maximum dynamic driving force $ku$. Study:
   - $\zeta=0.05$; Motor at 40, 80, 120 130, 160, 200 rpm
   - $\zeta=0.10$; Motor at 40, 80, 120 130, 160, 200 rpm
   - $\zeta=0.20$; Motor at 40, 80, 120 130, 160, 200 rpm
   - $\zeta=0.30$; Motor at 40, 80, 120 130, 160, 200 rpm

Plot one graph of dynamic driving force $ku$ versus driving frequency $\omega$, but this plot will have four curves: one for each damping ratio. Mark the position of $ku = 5$ kips and note the fact that, while resonant effects are normally understood to result in amplification, there are significant portions of the graph for which the dynamic force is actually less than static amplitude of 5 kips.